## Problem Set 3

1. A three-phase transmission line operating at 33 kV and having a resistance and reactance of $5 \Omega$ and $20 \Omega$ respectively is connected to a generating station busbar through a 5 MVA 6.6 / 33 kV step up transformer which has a reactance of $6 \%$. Connected to the busbar are two alternators, one of 10 MVA, 6.6 kV having 10 \% reactance and another of $5 \mathrm{MVA}, 6.6 \mathrm{kV}$ having $7.5 \%$. Calculate the short circuit MVA when a three phase fault occurs at (i) the high voltage terminals of the transformer (ii) load end of the transmission line.
2. A 25 MVA, 13.8 kV alternator with $X_{d}{ }^{"}=15 \%$ is connected through a transformer to a busbar that supplies four identical synchronous motors. Each motor has $X_{d}=\mathbf{2 0} \%$ on a base of 5 MVA and 6.9 kV . The three-phase ratings of the transformer are 25 MVA, 13.8 / 6.9 kV with a leakage reactance of $10 \%$. The bus voltage at the motors is 6.9 kV when a symmetrical three phase fault occurs at the terminals of one of the motors. Determine
i) the subtransient current in the fault
ii) the subtransient current flowing in the circuit breaker connected between the faulted motor and 6.9 kV busbar.
3. For the network with the following data construct the bus impedance matrix.

Element No. Between buses p.u. impedance

| 1 | 0 | 1 | j 0.3 |
| :--- | :--- | :--- | :--- |
| 2 | 2 | 3 | j 0.5 |
| 3 | 4 | 1 | j 0.44 |
| 4 | 0 | 3 | j 0.3 |
| 5 | 2 | 1 | j 0.6 |
| 6 | 1 | 3 | j 0.4 |

4. Consider the power network shown. Values marked are p.u. impedances. Determine its bus impedance matrix.

5. Calculate the impedance matrix of the network shown.

6. A power system has the following transmission line data
From bus To bus Line impedance in p.u.

| 1 | 2 | j 0.1 |
| :--- | :--- | :--- |
| 2 | 3 | j 0.0625 |
| 3 | 5 | j 0.05 |
| 1 | 4 | j 0.05 |

Generator 1 with reactance j 0.1 p.u. is connected at bus 2. Generator 2 having reactance j 0.125 p.u. is connected at bus 5 . Using building algorithm, construct the bus impedance matrix of the generatortransmission network.
7. The bus impedance matrix of a four bus power system, with values in p.u., is given by

$$
\left.\mathbf{Z}_{\text {bus }}=\mathbf{j} \begin{array}{c} 
\\
1 \\
2 \\
3 \\
4
\end{array} \begin{array}{cccc}
1 & 2 & 3 & 4 \\
0.15 & 0.08 & 0.07 & 0.04 \\
0.08 & 0.15 & 0.09 & 0.06 \\
0.07 & 0.09 & 0.12 & 0.05 \\
0.04 & 0.06 & 0.05 & 0.13
\end{array}\right]
$$

Generators are connected to buses 1 and 2 and their subtransient reactances were included while computing $Z_{\text {bus }}$. A three phase fault occurs at bus 3 with a fault impedance of j 0.08 p .u. Find the subtransient current in the fault. If the subtransient reactance of the generator in bus 2 is 0.2 p.u., determine the subtransient fault current supplied by the generator. Also find the voltage at bus 4.
8. For the system shown below, find the fault current in all the branches, when a three phase fault with fault impedance j 0.0785 p.u. occurs at bus 2. Mark the currents in the one-line diagram. All the reactances and voltages are in p.u.

9. A 5 bus transmission network is shown below. Marked values are the line impedances in per unit on a 100 MVA system base.


Generators are connected at buses 1 and 3 and rated 270 and 225 MVA respectively. The generator subtransient reactances plus the reactances of the transformers connecting them to the buses are each 0.3 p.u. on the generator rating as base. The turns ratios of the transformers are such that the voltage base in each generator circuit is equal to the voltage rating of the generator. Using the bus impedance matrix, find the subtransient current in a three phase fault at bus 4 and the current coming to the faulted bus over each line.
10. In the power system described in Problem 5, symmetrical three phase fault occurs at bus 4. Using the bus impedance matrix, calculate the fault current, voltages at all the buses and the currents in all the elements.
11. A generating station has two alternators of 3 MVA, $7 \%$ subtransient reactance and 4.5 MVA, 8 \% subtransient reactance. The circuit breaker in the outgoing feeder is rated for 150 MVA. It is intended to extend the system by including one more alternator of 7.5 MVA, 7.5 \% subtransient reactance. Find the reactance necessary which should be connected in series with the third alternator to protect the system. Assume the system voltage as 3.3 kV .
12. A generating station has three generators each of 10 MVA capacity with $10 \%$ subtransient reactance. Generators are connected to a common busbar through reactors of $\mathbf{8 \%}$. If symmetrical three phase fault develops on the busbar of one of the generators, calculate the short circuit MVA and compare it with the case when there are no external reactors used.
13. Four generators in a power station are arranged in a ring busbar with suitable reactors as shown. Calculate the short circuit MVA if there is symmetrical three phase fault at point F. All the reactances are referred to their respective MVA rating. Select 600 MVA as the base value.


## ANSWERS

1. (i) 55.556 MVA
(ii) 27.278 MVA
2. (i) 16720 A
(ii) 14640 A
3. 

$\left.\begin{array}{ll} & \\ Z_{\text {bus }} & =j \\ 1 \\ 2 \\ 3\end{array} \begin{array}{cccc}1 & 2 & 3 & 4 \\ 0.19925 & 0.1455 & 0.1007 & 0.19 y 25 \\ 0.1455 & 0.4231 & 0.1545 & 0.1455 \\ 0.1007 & 0.1545 & 0.19925 & 0.1007 \\ 0.19925 & 0.1455 & 0.1007 & 0.63925\end{array}\right]$
4.

$$
\mathrm{Z}_{\text {bus }}=\mathrm{j} \begin{aligned}
& 1 \\
& 2 \\
& 3 \\
& 4
\end{aligned}\left[\begin{array}{llll}
0.71660 & 0.60992 & 0.53340 & 0.58049 \\
0.60992 & 0.73190 & 0.64008 & 0.69659 \\
0.53340 & 0.64008 & 0.71660 & 0.66951 \\
0.58049 & 0.69659 & 0.66951 & 0.76310
\end{array}\right]
$$

5. 

|  |  |
| :---: | :---: |
|  | 1 |
| $Z_{\text {bus }}$ |  |$=j$| 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: |
| 2 | 3 |  |  |  |
|  | 4 |  |  |  |
| 5 |  |  |  |  |\(\left[\begin{array}{ccccc}5.1 \& 5.0 \& 5.0 \& 5.1 \& 5.0 <br>

5.0 \& 5.0 \& 5.0 \& 5.0 \& 5.0 <br>
5.0 \& 5.0 \& 5.0625 \& 5.0 \& 5.0625 <br>
5.1 \& 5.0 \& 5.0 \& 5.15 \& 5.0 <br>
5.0 \& 5.0 \& 5.0625 \& 5.0 \& 5.1125\end{array}\right]\)
6.

|  |  |
| :--- | :--- |
|  | 1 |
|  | 1 |
| $Z_{\text {bus }}=j$ | 2 |
|  | 3 |
|  | 4 |
| 5 |  |\(\left[\begin{array}{cccccc}0.112082 \& 0.055798 \& 0.55494 \& 0.090224 \& 0.055251 <br>

0.055798 \& 0.066727 \& 0.052762 \& 0.050333 \& 0.041591 <br>
0.055798 \& 0.052762 \& 0.084031 \& 0.056861 \& 0.059046 <br>
0.090224 \& 0.050333 \& 0.056861 \& 0.110170 \& 0.062083 <br>
0.055252 \& 0.041591 \& 0.059046 \& 0.062083 \& 0.073011\end{array}\right]\)
7. Subtransient current in the fault $I_{f}{ }^{\prime \prime}=-j 5.0$ p.u.

Current supplied by the generator in bus $2=-\mathrm{j} 2.25$ p.u.
$V_{4(F)}=0.75$ p.u.
8.

9. Subtransient current in the fault $=-\mathbf{j} 4.30885$ p.u.

$$
\mathrm{i}_{3-4}=-\mathrm{j} 2.05324 \text { p.u.; } \quad \mathrm{i}_{5-4}=-\mathrm{j} 2.25563 \text { p.u. }
$$

10. $\quad V_{1(F)}=0.181047$ p.u.; $\quad V_{2(F)}=0.543133$ p.u.
$V_{3(F)}=0.483879$ p.u.; $\quad V_{4(F)}=0 \quad V_{5(F)}=0.436480$ p.u.;
$\mathrm{i}_{2-1}=-\mathrm{j} 3.62086$ p.u.; $\mathrm{i}_{2-3}=-\mathrm{j} 0.94806$ p.u. $\quad \mathrm{i}_{3-5}=-\mathrm{j} 0.94798$ p.u.
$\mathrm{i}_{1-4}=-\mathrm{j} 3.62094$ p.u.; $\quad \mathrm{i}_{5-4}=-\mathrm{j} 5.456$ p.u.
$\mathrm{i}_{\mathrm{g} 1}=-\mathrm{j} 4.56867$ p.u.; $\quad \mathrm{i}_{\mathrm{g} 2}=-\mathrm{j} 4.5081$ p.u.
11. $0.105 \Omega$
12. 158.73 MVA 300 MVA
13. 1370.8 MVA

## PROBLEM SET 4

1. In an unbalanced circuit the three line currents are measured as

$$
\begin{aligned}
& I_{a}=7.0311 \angle 59.85^{\circ} \\
& I_{b}=4.3733 \angle 203.84^{0} \\
& I_{c}=2.6810 \angle 160.75^{\circ}
\end{aligned}
$$

Obtain the corresponding sequence components of currents and draw them to scale.
2. For the sequence components calculated in Problem 1, find the corresponding phase components of line currents and verify the results graphically.
3. A three phase transmission line has the phase impedance of
$z_{a, b, c}=j\left[\begin{array}{ccc}21 & 6 & 6 \\ 6 & 21 & 6 \\ 6 & 6 & 21\end{array}\right]$
Calculate its sequence impedances.
4. A $\mathbf{2 0}$ MVA, 13.8 kV alternator has the following reactances:
$X_{1}=0.25$ p.u. $\quad X_{2}=0.35$ p.u. $\quad X_{g 0}=0.04$ p.u. $\quad X_{n}=0.02$ p.u.
A single line to ground fault occurs at its terminals. Draw the interconnections of the sequence networks and calculate
i) the current in each line
ii) the fault current
iii) the line to neutral voltages
iv) the line to line voltages

Denoting the neutral point as $n$ and the ground as $o$, draw the phasor diagram of line to neutral voltages.
5. Repeat Problem 4 for line to line fault.
6. Repeat Problem 4 for double line to ground fault.
7. Repeat Problem 4 for symmetrical three phase fault.
8. Consider the alternator described in Problem 4. It is required to limit the fault current to $\mathbf{2 5 0 0} \mathrm{A}$ for single line to ground fault. Find the additional reactance necessary to be introduced in the neutral.
9. Two synchronous machines are connected through three-phase transformers to the transmission line as shown.


The ratings and reactances of the machines and transformers are:
Machines 1 and 2: 100 MVA, $20 \mathrm{kV}, X_{1}=X_{2}=20 \%, X_{m 0}=4 \%, X_{n}=5 \%$
Transformers $T_{1}$ and $T_{2}: 100$ MVA, $20 \Delta / 345 Y k V, X=8 \%$
On a chosen base of 100 MVA, 345 kV in the transmission line circuit, the line reactances are $X_{1}=X_{2}=15 \%$ and $X_{0}=50 \%$. Draw each of the three sequence networks and find $Z_{\text {bus }}{ }^{0}, Z_{b u s}{ }^{1}$ and $Z_{\text {bus }}{ }^{2}$.
10. The one-line diagram of a power system is shown below.


The following are the p.u. reactances of different elements on a common base.

Generator 1: $\quad X_{g 0}=0.075 ; \quad X_{n}=0.075 ; \quad X_{1}=X_{2}=0.25$
Generator 2: $X_{g 0}=0.15 ; \quad X_{n}=0.15 ; \quad X_{1}=X_{2}=0.2$
Generator 3: $\quad \mathrm{X}_{\mathrm{g} 0}=0.072 ; \quad \mathrm{X}_{1}=\mathrm{X}_{2}=0.15$

Transformer 1: $\quad X_{0}=X_{1}=X_{2}=\mathbf{0 . 1 2}$
Transformer 2: $\quad X_{0}=X_{1}=X_{2}=0.24$
Transformer 3: $\quad X_{0}=X_{1}=X_{2}=\mathbf{0 . 1 2 7 6}$
Transmission line 2-3 $\quad X_{0}=0.5671 ; \quad X_{1}=X_{2}=0.18$
Transmission line 3-5 $\quad X_{0}=0.4764 ; \quad X_{1}=X_{2}=0.12$
Draw the three sequence networks and determine $Z_{\text {bus }}{ }^{0}, Z_{\text {bus }}{ }^{1}$ and $Z_{\text {bus }}{ }^{2}$.
11. The single line diagram of a small power system is shown below.


Generator: 100 MVA, $20 \mathrm{kV}, \mathrm{X}_{1}=\mathrm{X}_{2}=\mathbf{2 0} \%, \mathrm{X}_{\mathrm{g} 0}=4 \%, \mathrm{X}_{\mathrm{n}}=5 \%$
Transformers $T_{1}$ and $T_{2}$ : 100 MVA, $20 \Delta / 345 Y k V, X=10 \%$
On a chosen base of 100 MVA, 345 kV in the transmission line circuit, the line reactances are:

From $T_{1}$ to $P: X_{1}=X_{2}=20 \% ; X_{0}=50 \%$
From $T_{2}$ to $P: X_{1}=X_{2}=10 \% ; X_{0}=30 \%$
A bolted single lone to ground occurs at P. Determine
i) fault current $\mathrm{I}_{\mathrm{fA}}, \mathrm{I}_{\mathrm{fB}}$ and $\mathrm{I}_{\mathrm{fC}}$.
ii) currents flowing towards $\mathbf{P}$ from $\mathrm{T}_{1}$.
iii) currents flowing towards $P$ from $\mathbf{T}_{2}$.
iv) current supplied by the generator.

Note that the positive sequence current in $\Delta$ winding of transformer lags that in $Y$ winding by $30^{\circ}$; the negative sequence current in $\Delta$ winding leads that in $Y$ winding by $30^{\circ}$.
12. In the power system described in Problem 10, a single line to ground fault occurs at bus 2 with a fault impedance of j0.1. Determine the bus currents at the faulted bus and the voltages at buses 1 and 2.

## ANSWERS

1. $I_{a 0}=I_{b 0}=I_{c 0}=2 \angle 120^{\circ} ; I_{a 1}=3.5 \angle 30^{\circ} ; I_{b 1}=3.5 \angle 270^{\circ} ; I_{c 1}=3.5 \angle 150^{\circ}$

$$
\mathrm{I}_{\mathrm{a} 2}=3 \angle 60^{\circ} ; \quad \mathrm{I}_{\mathrm{b} 2}=3 \angle 180^{\circ} ; \quad \mathrm{I}_{\mathrm{c} 2}=3 \angle 300^{\circ}
$$

2. $I_{a}=7.0311 \angle 59.85^{\circ} ; I_{b}=4.3733 \angle 203.84^{\circ} ; I_{c}=2.681 \angle 160.75^{\circ}$
3. $Z_{0}=j 33 ; Z_{1}=Z_{2}=j 15$
4. $I_{a}=-j 3586.1 A \quad I_{b}=0 \quad I_{c}=0 \quad I_{f}=-j 3586.1 A$

$$
\begin{aligned}
& \mathrm{V}_{\mathrm{a}}=0 \quad \mathrm{~V}_{\mathrm{b}}=8.0694 \angle-102.22^{\circ} \mathrm{kV} \quad \mathrm{~V}_{\mathrm{c}}=8.0694 \angle 102.22^{\circ} \mathrm{kV} \\
& \mathrm{~V}_{\mathrm{ab}}=8.0694 \angle 77.78^{0} \mathrm{kV} ; \mathrm{V}_{\mathrm{bc}}=15.7724 \angle-90^{\circ} \mathrm{kV} ; \mathrm{V}_{\mathrm{ca}}=8.0694 \angle 102.22^{\circ} \mathrm{kV}
\end{aligned}
$$

5. $I_{a}=0 \quad I_{b}=-2415.5 A \quad I_{c}=2415.5 A \quad I_{f}=-2415.5 A$
$\mathrm{V}_{\mathrm{a}}=9.2948 \mathrm{kV} \quad \mathrm{V}_{\mathrm{b}}=-4.6474 \mathrm{kV} \quad \mathrm{V}_{\mathrm{c}}=-\mathbf{- 4 . 6 4 7 4 \mathrm { kV }}$
$\mathrm{V}_{\mathrm{ab}}=13.9422 \mathrm{kV} \quad \mathrm{V}_{\mathrm{bc}}=0 \quad \mathrm{~V}_{\mathrm{ca}}=13.9422 \angle 180^{\circ} \mathrm{kV}$
6. $\mathrm{I}_{\mathrm{a}}=0 \quad \mathrm{I}_{\mathrm{b}}=4020.95 \angle 132.22^{\circ} \mathrm{A} \quad \mathrm{I}_{\mathrm{c}}=4020 \quad \angle 47.78^{\circ} \mathrm{A} \quad \mathrm{I}_{\mathrm{f}}=5956.08 \quad \angle 90^{\circ} \mathrm{A}$ $\mathrm{V}_{\mathrm{a}}=5.6720 \mathrm{kV} \quad \mathrm{V}_{\mathrm{b}}=0 \quad \mathrm{~V}_{\mathrm{c}}=0$
$\mathrm{V}_{\mathrm{ab}}=5.6720 \mathrm{kV} \quad \mathrm{V}_{\mathrm{bc}}=0 \quad \mathrm{~V}_{\mathrm{ca}}=5.6720 \angle 180^{\circ} \mathrm{kV}$
7. $I_{a}=3347 \angle-90^{\circ} A \quad I_{b}=3347 \angle 150^{\circ} A \quad I_{c}=3347 \angle 30^{\circ} A$
$I_{f}=3347 \angle-90^{\circ} A$
$\mathrm{V}_{\mathrm{a}}=\mathrm{V}_{\mathrm{b}}=\mathrm{V}_{\mathrm{c}}=0$
$\mathrm{V}_{\mathrm{ab}}=\mathrm{V}_{\mathrm{bc}}=\mathrm{V}_{\mathrm{ca}}=\mathbf{0}$
8. $0.9655 \Omega$
9. Zero sequence network:


Positive sequence network:


Negative sequence network:


Reference

$$
\begin{aligned}
& \mathrm{Z}_{\text {bus }}{ }^{0}=\mathrm{i} \begin{array}{c} 
\\
\\
\\
\\
\\
3 \\
2
\end{array}\left[\begin{array}{cccc}
1 & 2 & 3 & 4 \\
0.19 & 0 & 0 & 0 \\
0 & 0.08 & 0.08 & 0 \\
0 & 0.08 & 0.58 & 0 \\
0 & 0 & 0 & 0.19
\end{array}\right] \\
& 12 \\
& 3 \\
& 4 \\
& Z_{\text {bus }}{ }^{1}=Z_{\text {bus }}{ }^{2}=\mathrm{i} \begin{array}{l}
1 \\
2 \\
3
\end{array}\left[\begin{array}{llll}
0.1437 & 0.1211 & 0.0789 & 0.0563 \\
0.1211 & 0.1696 & 0.1104 & 0.0789 \\
0.0789 & 0.1104 & 0.1696 & 0.1211 \\
0.0563 & 0.0789 & 0.1211 & 0.1437
\end{array}\right]
\end{aligned}
$$

10. 

Zero sequence network:


Negative sequence network:


$$
Z_{b u s}{ }^{1}=Z_{b u s}{ }^{2}=
$$

1

| 1 | 0.167640 |
| :---: | :---: |
| 2 | 0.128108 |
| 3 | 0.068808 |
| 4 | 0.031276 |
|  | 0.048041 |
| 5 | 0.025959 |
| 6 |  | 6

2
3
0.128108
0.189500
0.101836
0.046289
0.071101
0.068808
$0.031276 \quad 0.048041$
0.025959
$\begin{array}{llll}0.101836 & 0.046289 & 0.071101 & 0.038419\end{array}$
$\begin{array}{llll}0.151377 & 0.068808 & 0.105690 & 0.057109\end{array}$ $\begin{array}{llll}0.068808 & 0.140367 & 0.048041 & 0.025959\end{array}$ $\begin{array}{lllll}0.105690 & 0.048041 & 0.157574 & 0.085145\end{array}$ $\begin{array}{llll}0.057109 & 0.025959 & 0.085145 & 0.114956\end{array}$ 0.038419
11. $I_{f A}=-j 2.4195$ p.u.; $\quad I_{f B}=0 ; \quad I_{f C}=0$
$I_{A}=-j 1.9356$ p.u.; $I_{B}=j 0.4839$ p.u.; $I_{C}=j 0.4839$ p.u.
$I_{A}=-j 0.4839$ p.u.; $I_{B}=-j 0.4839$ p.u.; $I_{C}=-j 0.4839$ p.u.
$I_{a}=-j 1.3969$ p.u.; $I_{B}=j 1.4969$ p.u.; $I_{C}=0$
12. $\quad I_{2 a}=j 3.828162$ p.u.; $\quad I_{2 b}=0 ; \quad I_{2 c}=0$
$V_{2 a}=0.382815$ p.u.; $V_{2 b}=0.950352 \angle 245.68^{0}$ p.u.
$\mathrm{V}_{2 \mathrm{c}}=0.950352 \angle 114.32^{0}$ p.u.
$\mathrm{V}_{1 \mathrm{a}}=0.673054$ p.u.; $\mathrm{V}_{1 \mathrm{~b}}=0.929112 \angle 248.76^{0}$ p.u.
$V_{1 c}=0.929112 \angle 111.236^{0}$ p.u.

## PROBLEM SET 5

1. A $60-\mathrm{Hz}$ four pole turbogenerator rated $500 \mathrm{MVA}, 22 \mathrm{kV}$ has an inertia constant of $\mathrm{H}=7.5 \mathrm{MJ} / \mathrm{MVA}$. The electrical power developed is 400 MW when the input less the rotational losses is $\mathbf{7 4 0 0 0 0} \mathbf{h p}$. Find (a) the kinetic energy stored in the rotor at synchronous speed and (b) the angular acceleration in elec. deg. / $\mathrm{s}^{2}$ and rpm / $\mathrm{s}^{2}$.
[ 3750 MJ $\quad 437.76$ elec. deg. $/ \mathrm{s}^{2} \quad 36.48 \mathrm{rpm} / \mathrm{s}^{2}$ ]
2. If the acceleration computed for the generator described in Prob. 1 is constant for a period of $\mathbf{1 5}$ cycles, find the speed in rpm at the end of $\mathbf{1 5}$ cycles and the change in $\delta$ in electrical degrees in that period. Assume that the generator is synchronized with a large system and has no acceleration torque before the 15-cycle period begins.
[1809.12 rpm; 13.68 elec. deg.]
3. The single-line diagram shows a generator connected through parallel transmission lines to an infinite bus. The machine is delivering 0.8 per unit power and both the terminal voltage and infinite bus voltages are 1.0 per unit. (i) Calculate the machine's internal voltage (ii) Find the operating rotor angle. (iii) Determine the power angle equation for the system.


$$
\left[1.0353 \angle 25.16^{0} ; \quad 25.16^{\circ} ; \quad P_{e}=1.8824 \sin \delta\right]
$$

4. The single-line diagram shows a generator connected through parallel transmission lines to an infinite bus. The machine with $\left|E^{\prime}\right|=1.0352$ is delivering 0.8 p.u. A three-phase fault occurs at a point on one of the transmission lines at a distance of $30 \%$ of the line length away from the sending end terminal. Inertia constant of the machine is 5.0 MJ / MVA. For the faulted condition determine the power angle equation and the swing equation.

$\left[P_{e}=0.5752 \sin \delta ; \quad \frac{d^{2} \delta}{d t^{2}}=36 f(0.8-0.5752 \sin \delta)\right]$
5. A 60 Hz generator having $\mathrm{H}=6.0 \mathrm{MJ} / \mathrm{MVA}$ is delivering power of 1.0 p.u. to an infinite bus through a purely reactive network when the occurrence of a fault reduces the generator output power to zero. The maximum power that could be delivered is 2.5 p.u. When the fault is cleared the original network condition again exist. Determine the critical clearing angle and critical clearing time.
[ $\delta_{\mathrm{cc}}=89.374^{0}=1.5599$ rad.; $\quad t_{\mathrm{ccT}}=\mathbf{0 . 2 7 0 4} \mathbf{~ s e c}$. ]
6. A synchronous generator is supplying $60 \%$ of $P_{\max }$ to an infinite bus through a reactive network. A fault occurs which increases the reactance of the network between the generator internal voltage and the infinite bus by $400 \%$. When the fault is cleared the maximum power that can be delivered is $80 \%$ of the original maximum value. Determine the critical clearing angle.
[ $61.68^{0}$ ]
7. If the synchronous generator of Prob. 6 has an inertia constant of $\mathbf{H}=\mathbf{6} \mathbf{~ M J}$ / MVA and $P_{m}$ (equals to $0.6 P_{\text {max }}$ ) is 1.0 p.u., find the critical clearing time. Use $\Delta t=0.05 \mathrm{sec}$ to plot the necessary swing curve.
[0.194 s ]
8. A 60 Hz single generator having inertia constant of $5 \mathrm{MJ} / \mathrm{MVA}$ is connected to an infinite bus through transformer and transmission system as shown. Input to the generator $=0.8$ p.u.


Before the fault generator has voltage $\left|E^{\prime}\right|=1.0352$ p.u. and maximum power output is $1.882 \mathrm{p} . \mathrm{u}$. During fault generator output power is $0.575 \mathrm{sin} \delta$. The fault that occurs in one of the transmission is cleared at 4.5 cycles by the simultaneous opening of breakers at both ends.(i) Determine the power angle equation for the post fault period. (ii) Obtain the swing curve up to 0.25 s taking $\Delta t=0.05 \mathrm{~s}$.
8. A 60 Hz synchronous generator having inertia constant $\mathrm{H}=\mathbf{4} \mathrm{MJ} / \mathrm{MVA}$ is connected to infinite bus through transformer and a transmission network. Mechanical input power is 1.0 p.u. Its prefault and during fault maximum power output are 1.6667 p.u. and 0.4167 p.u. respectively. Using Modified Euler's method obtain its swing curve up to 0.2 s taking time step of 0.05 s .
[ $0,36.87^{0}$
$0.05,37.88^{0}$
$0.1,45.46^{0}$
$0.15,57.82^{0}$
$0.2,74.24^{0}$ ]
9. Repeat Problem 8 using $4^{\text {th }}$ order RK method.
$\left[0,36.87^{0}\right.$
$0.05,39.40^{0}$
$0.1,46.87^{0}$
$0.15,59.03^{0}$
$0.2,76.66^{0}$ ]

## Problem Set 1

1. A $\mathbf{1 2 0}$ MVA, 19.5 kV generator has $X_{s}=0.15$ per unit and is connected to a transmission line by a transformer rated 150 MVA, $230 \mathrm{Y} / 18 \Delta \mathrm{kV}$ with X $=0.1$ per unit. If the base to be used in the calculation is $\mathbf{1 0 0}$ MVA, 230 kV for the transmission line, find the per unit values to be used for the transformer and the generator reactances.
2. A transformer's three phase rating is $5000 \mathrm{kVA}, 115 / 13.2 \mathrm{kV}$, and its impedance is $0.007+\mathrm{j} 0.075$ per unit. The transformer is connected to a short transmission line whose impedance is $0.02+\mathbf{j} 0.1$ per unit on a base of 10 MVA, 13.2 kV . The line supplies a three phase load rated 3200 kW , 13.2 kV with a lagging power factor of 0.8 . Draw the per unit impedance diagram taking the base as 10 MVA, 13.2 kV at the load. If the high tension voltage is maintained at 115 kV , find the per unit and the actual load voltage.
3. The one-line diagram of an unloaded power system is shown below.


Reactances of the two sections of transmission line are shown in the diagram. The generators and transformers are rated as follows:

Generator 1: 20 MVA, $13.8 \mathrm{kV}, \mathrm{X"}=\mathbf{0 . 2}$ per unit
Generator 2: 30 MVA, $18 \mathrm{kV}, \mathrm{X}^{\prime \prime}=0.2$ per unit
Generator 3: 30 MVA, $20 \mathrm{kV}, \mathrm{X}^{\prime \prime}=0.2$ per unit
Transformer $\mathrm{T}_{1}$ : 25 MVA, 220Y/13.8 $\mathrm{LkV}, \mathrm{X}=10 \%$
Transformer $\mathrm{T}_{2}$ : Single-phase units each rated 10 MVA, $127 / 18 \mathrm{kV}$,

$$
X=10 \%
$$

Transformer $\mathrm{T}_{3}$ : 35 MVA, 220Y/22Y kV, X = 10\%

Draw the reactance diagram with all reactances marked in per unit and with letters to indicate points corresponding to the one-line diagram. Choose a base of 50 MVA, 13.8 kV in the circuit of generator 1 .
4. Draw the impedance diagram of the power system shown below.


Mark impedances in per unit. Neglect resistance and use a base of 50 MVA, 138 kV in the 40- $\Omega$ line. The ratings of the generator, motors and transformers are:

Generator 1:
Generator 2:
Synchronous motor 3:
Three phase Y-Y transformers: $\mathbf{2 0}$ MVA, 138Y/20Y kV, X = 10\%
Three phase Y- $\Delta$ transformers:15 MVA, $138 \mathrm{Y} / 13.8 \Delta \mathrm{kV}, \mathrm{X}=10 \%$
5. The p.u. impedance diagram of a power system is shown below.


Compute its bus admittance matrix.
6. For the network with the following data determine the bus admittance matrix.

| Element No. | Between buses |  | p.u.impedance |
| :---: | :---: | :---: | :---: |
| 1 | 0 | 1 | j 0.3 |
| 2 | 2 | 3 | j 0.5 |
| 3 | 4 | 1 | j 0.44 |
| 4 | 0 | 3 | j 0.3 |
| 5 | 2 | 1 | j 0.6 |
| 6 | 1 | 3 | j 0.4 |

7. For the network shown in Fig. 1.21, neglect the mutual coupling and compute $Y_{\text {bus }}$ matrix using the bus incidence matrix and the primitive admittance matrix. Take the orientation of elements as from 1 to 2; from 3 to 1; from 3 to 2; from 4 to 2; from 3 to 4; from 0 to 1 and from 0 to 4.


Fig. 1.21 One-line diagram of a 4-bus power system
8. For the network shown in Fig. 1.21, neglect the mutual coupling and compute $Y_{\text {bus }}$ matrix by adding one element at a time.
9. For the network shown in Fig. 1.21, neglect the mutual coupling and compute $Y_{\text {bus }}$ matrix using the rules for $Y_{i \mathrm{i}}$ and $\mathrm{Y}_{\mathrm{ij}}$ elements.
10. For the network shown in Fig. 1.21, include the mutual coupling and compute $Y_{\text {bus }}$ matrix using the bus incidence matrix and the primitive admittance matrix. Take the orientation of elements as from 1 to 2; from 3 to 1; from 3 to 2; from 4 to 2; from 3 to 4; from 0 to 1 and from 0 to 5.
11. Consider the network shown in Fig. 1.21. Find the bus admittance matrix corresponding to the coupled group; compute the bus admittance matrix of the network excluding the coupled group and hence find the $Y_{\text {bus }}$ matrix of the full network.
12. i) Determine the bus admittance corresponding to the off-nominal tap setting transformer.
ii) Obtain the bus admittance matrix of the transmission system with the following data.

## Line data

| Line No. | Between buses | Line Impedance | HLCA | Off nominal turns ratio |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 4-3 | j 0.125 | 0 | 0.95 |
| 2 | 1-6 | j 0.5 | j 0.015 | --- |
| 3 | 2-3 | j 1.2 | 0 | --- |
| 4 | 2-5 | j 0.75 | 0 | --- |
| 5 | 1-4 | j 0.32 | j 0.01 | --- |
| 6 | 4-6 | j 0.4 | j 0.075 | --- |
| 7 | 6-5 | j 0.25 | 0 | -- |

## Shunt capacitor data

Bus No. $4 \quad$ Admittance j 0.05

## ANSWERS

1. 0.06667 p.u.; 0.1467 p.u.
2. $\quad 0.9317$ p.u.; 12.2984 kV

3. 


4.

5. $\quad Y_{\text {bus }}=\left[\begin{array}{ccc}-j 26.6667 & j 10 & j 10 \\ j 10 & -j 33.3333 & j 10 \\ j 10 & j 10 & -j 20\end{array}\right]$
6. $\quad Y_{\text {bus }}=\left[\begin{array}{cccc}-j 9.7727 & j 1.6667 & j 2.5 & j 2.2727 \\ j 1.6667 & -j 3.6667 & j 2 & 0 \\ j 2.5 & j 2 & -j 7.8333 & 0 \\ j 2.2727 & 0 & 0 & -j 2.2727\end{array}\right]$

7 to 9: $\quad Y_{\text {bus }}=\left[\begin{array}{cccc}-\mathrm{j} 5.5 & \mathrm{j} 2.5 & \mathrm{j} 2 & 0 \\ \mathrm{j} 2.5 & -\mathrm{j} 11.5 & \mathrm{j} 4 & \mathrm{j} 5 \\ \mathrm{j} 2 & \mathrm{j} 4 & -\mathrm{j} 7.6 & \mathrm{j} 1.6 \\ 0 & \mathrm{j} 5 & \mathrm{j} 1.6 & -\mathrm{j} 7.4\end{array}\right]$

10 and $11 \quad Y_{\text {bus }}=\left[\begin{array}{cccc}-j 5.9390 & j 3.9634 & j 0.9756 & 0 \\ j 3.9634 & -j 12.3780 & j 3.4146 & j 5 \\ j 0.9756 & j 3.4146 & -j 5.9902 & j 1.6 \\ 0 & j 5 & j 1.6 & -j 7.4\end{array}\right]$

## 12.

(3) (4)
$\mathrm{Y}_{\text {bus }}=\begin{gathered}(3)\end{gathered}\left[\begin{array}{cc}-\mathrm{j} 8 & \mathrm{j} 8.4211 \\ \mathrm{j} 8.4211 & -\mathrm{j} 8.8643\end{array}\right]$
$Y_{\text {bus }}=\left[\begin{array}{cccccc}-\mathrm{j} 5.1 & 0 & 0 & \mathrm{j} 3.125 & 0 & \mathrm{j} 2 \\ 0 & -\mathrm{j} 2.1667 & \mathrm{j} 0.8333 & 0 & \mathrm{j} 1.3333 & 0 \\ 0 & \mathrm{j} 0.8333 & -\mathrm{j} 8.8333 & \mathrm{j} 8.4211 & 0 & 0 \\ \mathrm{j} 3.125 & 0 & \mathrm{j} 8.4211 & -\mathrm{j} 14.3543 & 0 & \mathrm{j} 2.5 \\ 0 & \mathrm{j} 1.3333 & 0 & 0 & -\mathrm{j} 5.3333 & \mathrm{j} 4 \\ \mathrm{j} 2 & 0 & 0 & \mathrm{j} 2.5 & \mathrm{j} 4 & -\mathrm{j} 8.41\end{array}\right]$

## Problem Set 2

1. Fig. 2.7 shows the one-line diagram of a simple three-bus power system with generation at bus 1 . The voltage at bus 1 is $\mathrm{V}_{1}=1.0 \angle 0^{0}$ per unit.The scheduled load at buses 2 and 3 are marked on the diagram. Line impedances are marked in per unit on a 100-MVA base.


Fig. 2.7 One-line diagram for Problem 1
(a) Assuming a flat start using Gauss-Seidel method determine $\mathrm{V}_{2}$ and $\mathrm{V}_{3}$. Perform two iterations. Take acceleration factor as 1.2.
(b) If after several iterations the bus voltages converge to $\mathrm{V}_{2}=(0.9-\mathrm{j} 0.1) \mathrm{pu}$ and $\mathrm{V}_{3}=(0.95-\mathrm{j} 0.05)$ pu determine the line flows, line losses, transmission loss and the slack bus real and reactive power. Construct a power flow diagram and show the direction of the line flows.
2. Fig. 2.9 shows the one-line diagram of a simple three-phase power system with generation at buses 1 and 3 . The voltage at bus 1 is $\mathrm{V}_{1}=1.025 \angle 0^{0}$ per unit. Voltage magnitude at bus 3 is fixed at 1.03 pu with a real power generation of $\mathbf{3 0 0} \mathbf{~ M W}$. A load consisting of $\mathbf{4 0 0} \mathbf{~ M W}$ and $\mathbf{2 0 0}$ Mvar is taken from bus 2. Line impedances are marked in per unit on a 100-MVA base.


Fig. 2.8 One-line diagram for Problem 2
3. Consider the two-bus system shown in Fig. 2.9. Base $=100$ MVA. Starting with flat start, using Newton-Raphson method, obtain the voltage at bus 2 at the end of first and second iteration.


Fig. 2.9 One-line diagram for Problem 3
4. Consider the power system with the following data. Perform power flow analysis for the power system with the data given below, using Newton Raphson method, and obtain the bus voltages at the end of first two iterations.

Line data (p.u. quantities)
Line No. Between buses Line impedances

| 1 | $1-2$ | $0+j 0.1$ |
| :--- | :--- | :--- |
| 2 | $2-3$ | $0+j 0.2$ |
| 3 | $1-3$ | $0+j 0.2$ |

Bus data (p.u. quantities)

| Bus <br> No | Type | Generator |  | Load |  | V | ठ | $\mathbf{Q}_{\text {min }}$ | $\mathbf{Q}_{\text {max }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | P | Q | P | Q |  |  |  |  |
| 1 | Slack | --- | --- | 0 | 0 | 1.0 | 0 | -- | --- |
| 2 | P-V | 5.3217 | --- | 0 | --- | 1.1 | --- | 0 | 3.5 |
| 3 | P-Q | 0 | 0 | 3.6392 | 0.5339 | --- | --- | --- | --- |

5. Redo the problem using Fast Decoupled Power Flow method.

## ANSWERS

1. (a)

$$
\begin{array}{ll}
V_{2}{ }^{(1)}=0.9232-\mathrm{j} 0.096 ; & V_{3}{ }^{(1)}=0.9491-\mathrm{j} 0.0590 \\
V_{2}{ }^{(2)}=0.8979-\mathrm{j} 0.1034 ; & V_{3}{ }^{(2)}=0.9493-j 0.0487
\end{array}
$$

(b)


Transmission loss $=S_{L 1-2}+S_{L 1-3}+S_{L 2-3}=j 0.6+j 0.4+j 0.1=j 1.1 p u$
i.e. 110 Mvar

Slack bus power $S_{L}=S_{12}+S_{13}=(3+j 3)+(4+j 4)=(7+j 7) p u$
i.e. 700 MW and 700 Mvar
2. At the end of first iteration
$\mathrm{V}_{1}=1.025+\mathrm{j} 0 ; \quad \mathrm{V}_{2}=1.0025-\mathrm{j} 0.05 ; \quad \mathrm{V}_{3}=1.02989+\mathrm{j} 0.01521$

## At the end of second iteration

$V_{1}=1.025+j 0 ; \quad V_{2}=1.00008-j 0.0409 ; \quad V_{3}=1.02978+j 0.0216$
3. At the end of first iteration $\left|V_{2}\right|=1-0.2=0.8$ and $\delta_{2}=0-0.1=-0.1$
$\mathrm{V}_{2}=0.8 \angle-5.73^{0}$
At the end of second iteration $\quad\left|V_{2}\right|=0.8-0.07736=0.7226$

$$
\delta_{2}=-0.1-0.035=-0.135
$$

Thus $\mathrm{V}_{\mathbf{2}}=0.7226 \angle-7.735^{0}$
4.

At the end of first iteration $\quad V_{1}=1.0 \angle 0^{0} \quad V_{2}=1.1 \angle 14.37^{\circ} \quad V_{3}=0.9964 \angle-12.33^{0}$
At the end of second iteration

$$
V_{1}=1.0 \angle 0^{0} \quad V_{2}=1.1 \angle 14.943^{0} \quad V_{3}=0.912 \angle-14.456^{0}
$$

5. At the end of first iteration, bus voltage

$$
\begin{aligned}
& V_{1}=1.0 \angle 0^{0} \\
& V_{2}=1.1 \angle 13.84^{0} \\
& V_{3}=0.9186 \angle-13.93^{0}
\end{aligned}
$$

At the end of second iteration, bus voltages

$$
\begin{aligned}
& V_{1}=1.0 \angle 0^{0} \\
& V_{2}=1.1 \angle 14.79^{\circ} \\
& V_{3}=0.9799 \angle-14.57^{0}
\end{aligned}
$$

