Unit 1 - Introduction

The function of Power System is to convert energy from other forms to electricity and distribute it to the consumers. The primary objective of an electric Power system is to deliver the power (or) Energy demanded by all the consumers, in full during all the time by generating the required power. The Power system components are

* Generating station
* Transmission system
* Distribution system

The power system analysis is required for different purposes. They are

(a) To monitor the voltage at various buses, real and reactive power flow between all the buses.

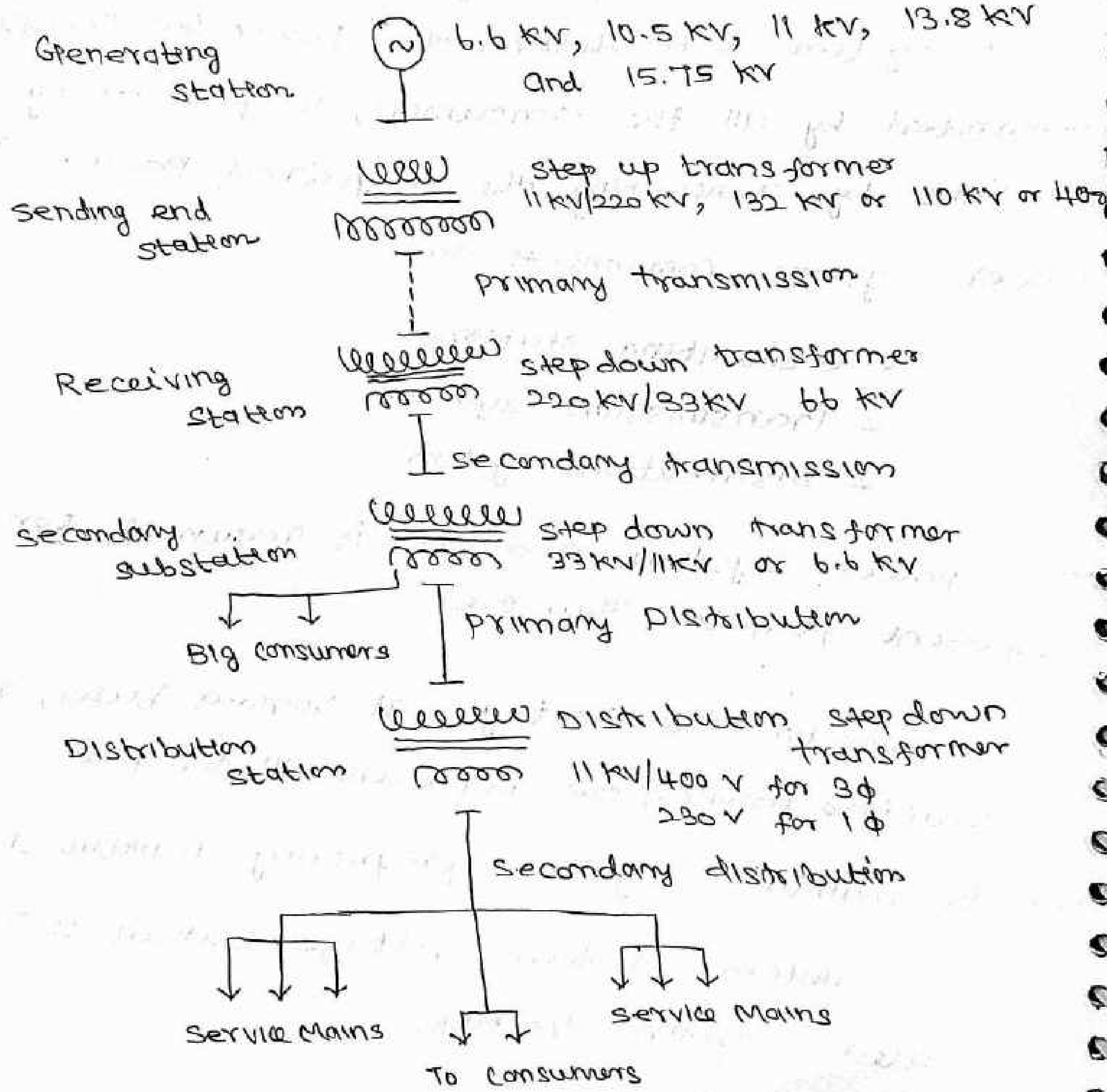
(b) To maintain system frequency within ± 0.5 %

(c) To maintain system voltage within ± 5 % of rated system voltage

(d) To study the ability of the system for large disturbances and small disturbances

(e) To analyze the system under different fault conditions (3ϕ fault, LG, LL, LLG faults)

Structure of Power System



Components of an Electric power system

Generators
* Mechanical energy into electrical energy

Transformers * one circuit to other circuit without change of frequency

Transmission line
* Transform power from one location to other

Control equipments
* protection devices

Primary Transmission
110 kV, 132 kV, 220 kV, 400 kV
765 kV 3ϕ 3 wire system

Primary distribution

11 kV or 6.6 kV
3ϕ, 3 wire system

Secondary Transmission
33 kV or 66 kV feeders
3ϕ, 3 wire

Secondary distribution

400 V - 3 phase
230 V - single phase

Vertically Integrated power system

* Generation
* Transmission
* Distribution

Deregulated Power System

* Pool market
* Spot market
* Bilateral market

Various studies in power system:

* Load flow
* Short circuit studies
* Stability studies
* Economic dispatch
* Electromagnetic Transient Analysis
* load frequency control

Need for system planning and operational studies:

Power Balance equation

$$P_D = \sum_{i=1}^{n} P_{Gi} \quad , i=1,2\ldots N$$

* when this relation satisfied, it gives good economy and security.

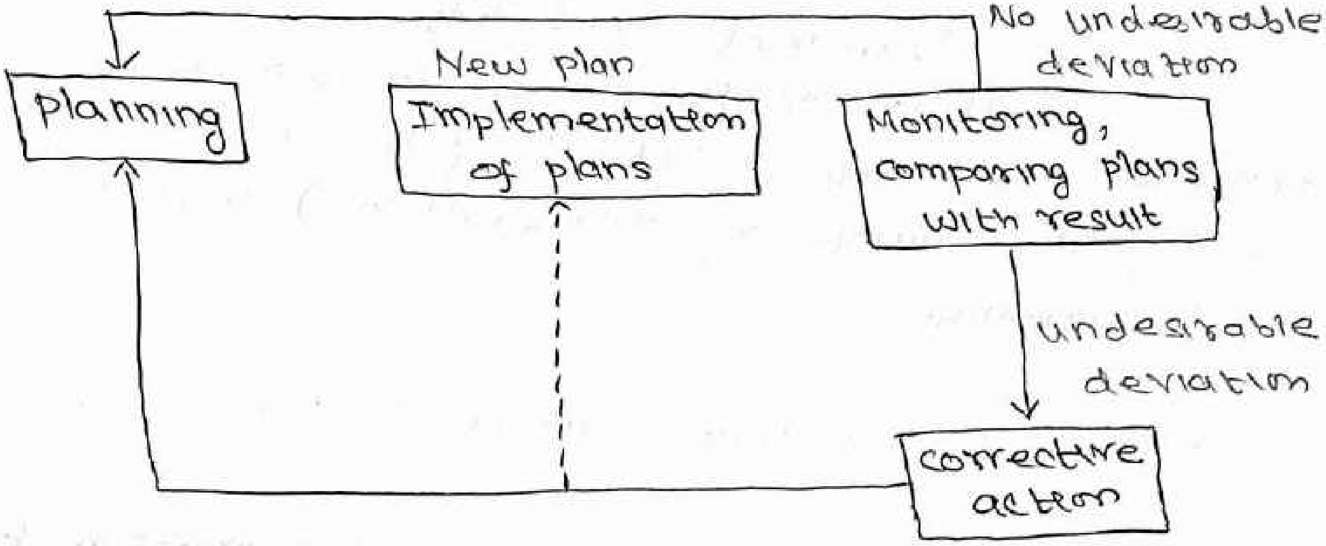
* planning and Operational analysis covers the maintenance of G-T-D facility. planning for future expansion of a power system is essential.

* The following analysis are more important for planning and operation of the system.

Load flow analysis

Short circuit analysis

Transient analysis



* These studies simulate the behavior of the system under normal and abnormal operating conditions.

* To overcome the deficiences remedial planning actions and corrective operational control actions are taken.

Load flow Analysis - Power flow studies - determine the voltage, current, active and reactive power flow

Short circuit studies - precisely determine the current and voltages at different locations of the system corresponding to difference type of fault.

Transient stability Analysis - ability of PS to continue to operate after a changes occur on the system.

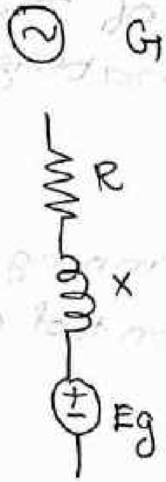
Stability - steady state

Transient stability

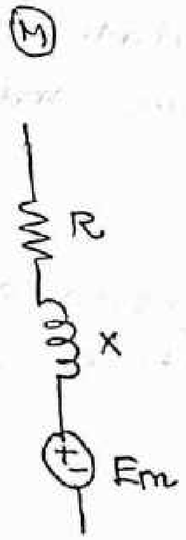
ability of power s/m to remain in synchronism following relatively (slow load change or contenual change in generation.) under large disturbances

Components and equivalent circuit of power system

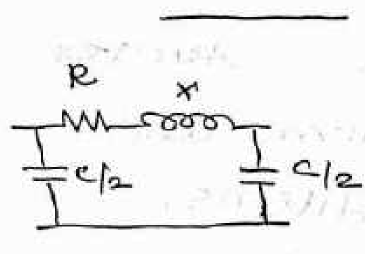
Generator



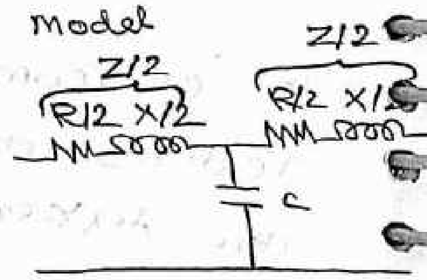
Motor



Transmission line

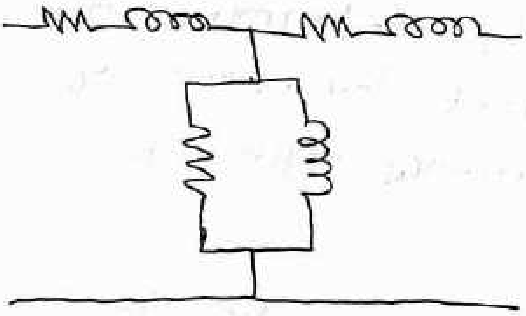


π model

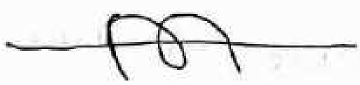


T model

Transformer



Current transformer



Potential transformer

2-winding transformer

3-winding transformer

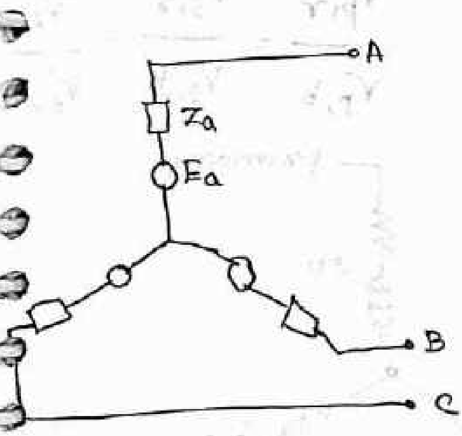
Single line Diagram – diagrammatic representation of Power System in which the components are represented by their symbols and interconnection them are shown by a straight line.

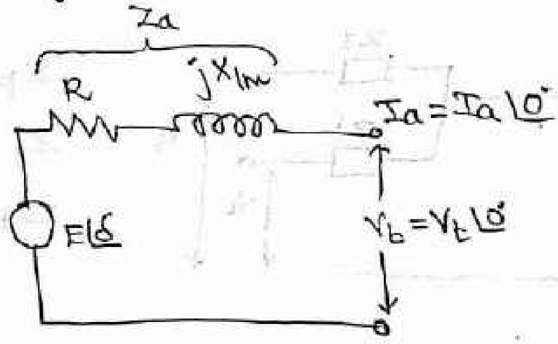
Purpose of SLD: To represent power system network in simplest form without loosing significance about the system.

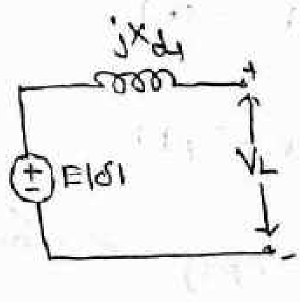
Per phase Analysis:

Balanced 3ϕ s/m – per phase basis analysis – considering one of 3ϕ line and neutral

Per phase representation of Generator



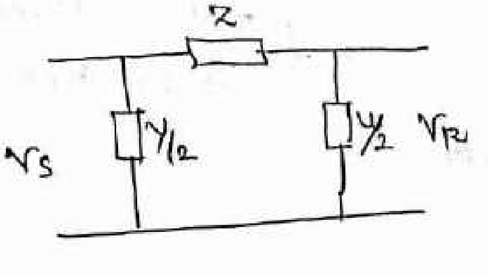


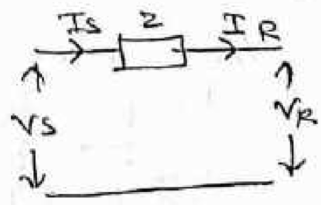


$V_a = E - I_a (R + jX_{Lm})$

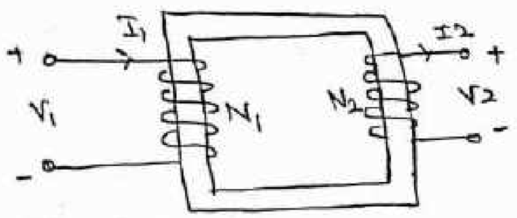
$X_{Lm} = X_l + X_m$

Transmission line





Two Winding Transformer

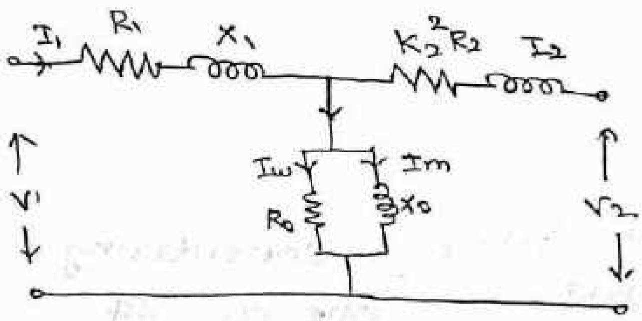


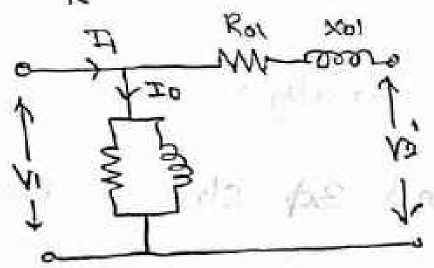
$$K = \frac{E_2}{E_1} = \frac{N_2}{N_1} = \frac{V_2}{V_1} = \frac{I_1}{I_2}$$

$$R_{01} = R_1 + R_2' = R_1 + \frac{R_2}{K^2}$$

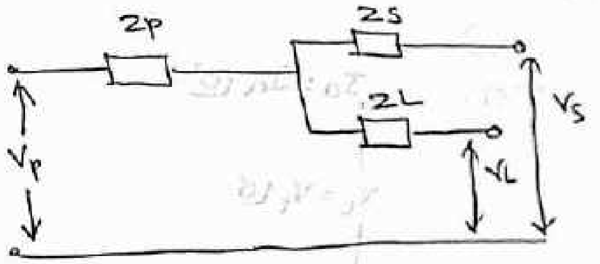
$$X_{01} = X_1 + X_2' = X_1 + \frac{X_2}{K^2}$$

$$I_2' = \frac{I_2}{K}$$





Three winding Transformer

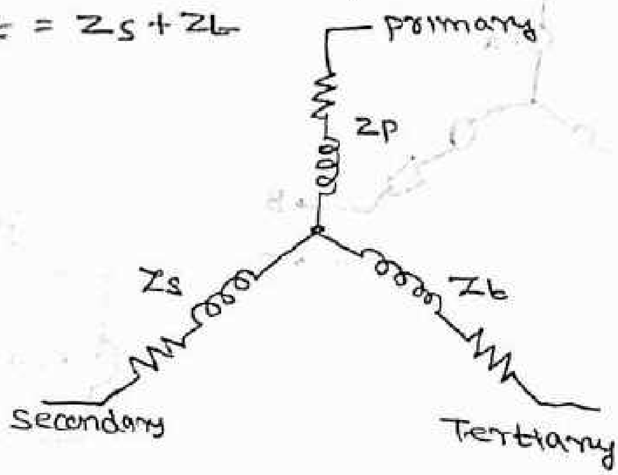


$$Z_{ps} = Z_p + Z_s$$

$$Z_{pt} = Z_p + Z_t$$

$$Z_{st} = Z_s + Z_t$$

$$\frac{V_{p,r}}{V_{p,b}} = \frac{V_{s,r}}{V_{s,b}} = \frac{V_{t,r}}{V_{t,b}}$$



$$Z_p = \frac{1}{2}(Z_{ps} + Z_{pt} - Z_{st})$$

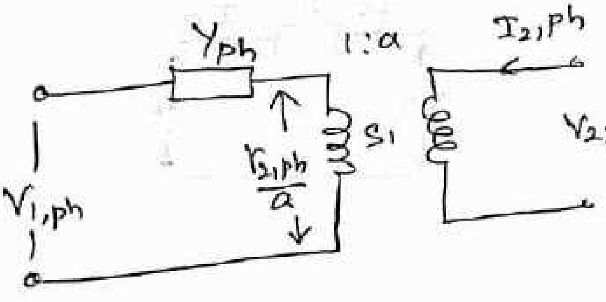
$$Z_s = \frac{1}{2}(Z_{ps} + Z_{st} - Z_{pt})$$

$$Z_t = \frac{1}{2}(Z_p + Z_{st} - Z_{ps})$$

Two winding Transformer with tap

If, $a = 1$ nominal trx.

$a \neq 1$ trx with tap.



$S_1 = - S_2$

$\frac{V_2}{a} I_1^* = - V_2 I_2^*$; $I_2^* = + \frac{-I_1^*}{a}$

$I_1 = \left[V_1 - \frac{V_2}{a}\right] y = V_1 y - \frac{V_2}{a} y$

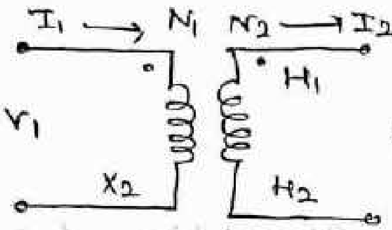
$I_2 = \frac{-y}{a^*} V_1 + \frac{V_2}{|a|^2} y$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y & -y/a \\ -y/a & y/a^2 \end{bmatrix}$$

Singular matrix

Non-symmetrical matrix

Auto-transformer



$\frac{V_1}{V_2} = \frac{N_1}{N_2} = a$ $\frac{I_2}{I_1} = \frac{N_1}{N_2} = a$

$V_H = V_1 + V_2$

$V_H = V_2 + \frac{N_1}{N_2} V_2$

$V_2 = V_L$

$= (1+a) V_L$

$\frac{V_H}{V_L} = 1 + a$

For Ideal Transformer

$N_2 I_2 = N_1 I_1$

$I_L = \frac{N_1 + N_2}{N_2} I_1$

$\frac{I_L}{I_H} = 1 + a$

Load Representation

(i) constant current

Voltage vary

frequency vary

$I_1 = I_2$

$I = \frac{P - jQ}{V^*}$

$V = |V| \angle \theta$

$\theta = \tan^{-1}\left(\frac{Q}{P}\right)$

(ii) constant power

MVAR } constant
MW }

$P_1 = P_2 \quad Q_1 = Q_2$

Per Unit Analysis - Representation

* Ratio of the actual value of any quantity to the base value of same quantity

$$\text{Per unit value} = \frac{\text{Actual value}}{\text{Base value}}$$

Advantages of P.U system

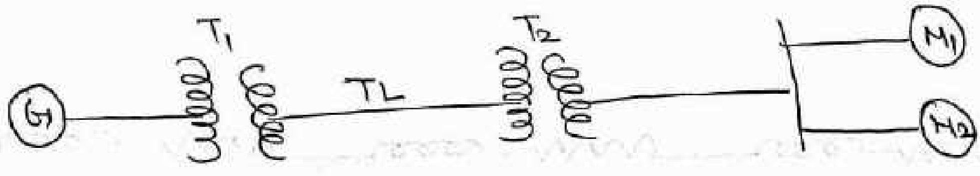
* Manufacture's - specify ratings in per unit.
* Reduce complexity
* Computerized analysis and simulation of complex power s/m problems
* circuit law are valid, bcoz $\sqrt{3}$ and 3 are eliminated
* 3ϕ transformer, p.u analysis is same for star and delta connected network

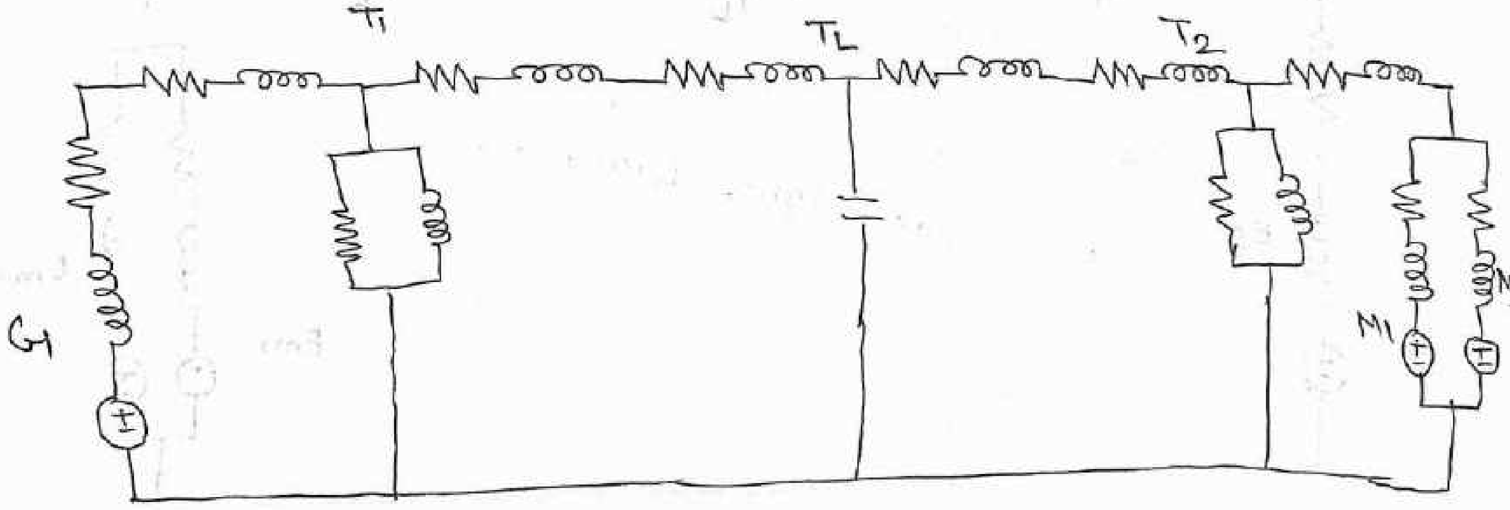
Base value the voltage, current and impedance rating of components of PS are expressed with reference to a common value.

Selection of Base Values

* Base MVA choosen - Same MVA will be used in all parts of s/m.

* Rated voltage of largest section - Base kV depends on turns ratio.





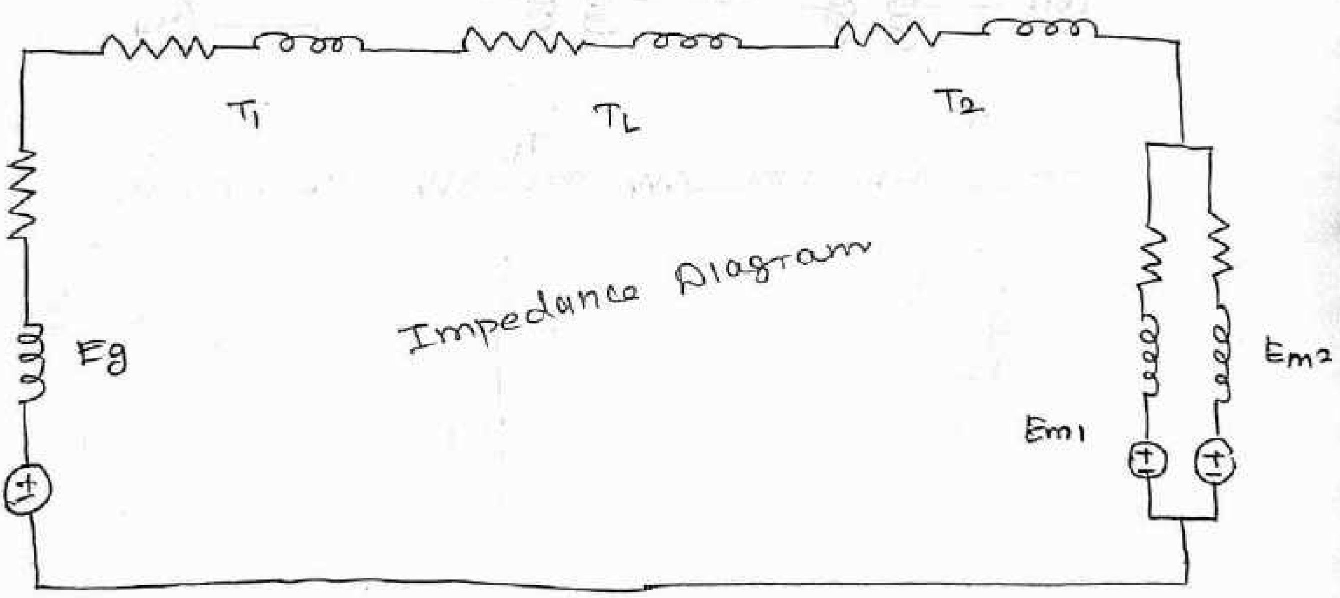
Impedance diagram - equivalent circuit of power s/m - various components - represented - simplified equivalent circuit.

Reactance Diagram - represented by reactances - used for fault calculations

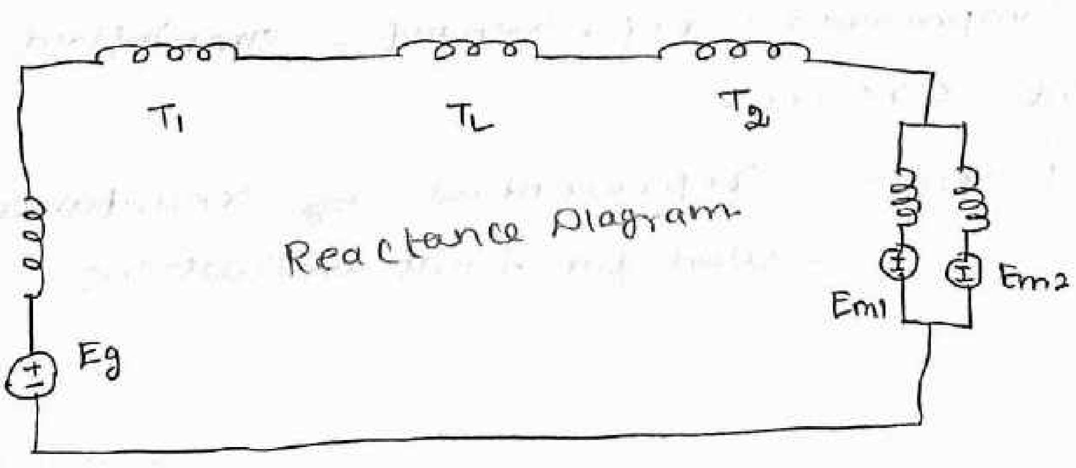
Reactance Diagram - obtained from impedance diagram if we omit all static loads, all resistance, shunt branches of trx's and capacitance of Tr. line

Assumptions to made Reactance diagram

* Neutral reactances are neglected
* resistance, static loads, capacitance neglected
* shunt branches in equivalent ckt of transformer are neglected



Impedance Diagram



Reactance Diagram

Per Unit Impedance - Single phase system

$P = VI$

$I = \frac{P}{V} = \frac{KVA_b}{KV_b}$

P.U Voltage $= \frac{V}{V_b}$ p.u P.U current $= \frac{I}{I_b}$ p.u

Base Impedance $= Z_b = \frac{V_b}{I_b}$ ohm $= \frac{V_b}{I_b} \times \frac{1000}{1000}$

$Z_b = \frac{KV_b}{I_b} \times 1000$

Base current $I_b = \frac{KVA_b}{KV_b} \times \frac{1000}{1000}$

$= \frac{MVA_b}{KV_b} \times 1000$ amps

$Z_b = \frac{KV_b \times 1000}{\frac{MVA_b \times 1000}{KV_b}}$

$$\boxed{Z_b = \frac{(KV_b)^2}{MVA_b}}$$

$Z_{p.u} = \frac{Z_{actual}}{Z_{Base}}$

$= \frac{Z_{actual}}{KV_b^2 / MVA_b}$ $\therefore$ $\boxed{Z_{p.u} = \frac{Z_{actual} \times MVA_b}{KV_b^2}}$

Per unit Impedance - 3φ system

Base current $I_b = \frac{kVA_b}{\sqrt{3}\, kV_b} \times \frac{1000}{1000}$

$$I_b = \frac{MVA_b \times 1000}{\sqrt{3}\, kV_b}$$

$$Z_b = \frac{kV_b}{\sqrt{3} I_b} \times 1000 = \frac{kV_b}{\sqrt{3} (I_b)} \times 1000$$

Subsitute I_b value here

$$\therefore \boxed{Z_b = \frac{kV_b^2}{MVA_b}}$$

Change of Base values:

$$Z_{p.u(old)} = \frac{Z_{act}}{Z_{base}} = \frac{Z_{actual}}{\frac{kV_b^2(old)}{MVA_{b(old)}}}$$

$$Z_{pu(old)} = Z_{act} \times \frac{MVA_b(old)}{kV_b^2(old)}$$

$$Z_{pu(new)} = Z_{act} \times \frac{MVA_b(new)}{kV_b^2(new)}$$

$$\therefore Z_{act} = \frac{Z_{pu(old)} \times kV_b^2(old)}{MVA_b(old)}$$

old values → given values

$$Z_{p.u(new)} = \frac{Z_{p.u\,old} \times KV_{b(old)}^2 \times MVA_{b(new)}}{MVA_{b(old)} \times KV_{b}^2(new)}$$

$$\therefore Z_{p.u(new)} = Z_{p.u\,old} \times \left[\frac{KV_{b(old)}}{KV_{b(new)}}\right]^2 \times \frac{MVA_{b(new)}}{MVA_{b(old)}}$$

Similarly

$$X_{pu,new} = X_{pu\,old} \times \left[\frac{KV_{b(old)}}{KV_{b(new)}}\right]^2 \times \frac{MVA_{b(new)}}{MVA_{b(old)}}$$

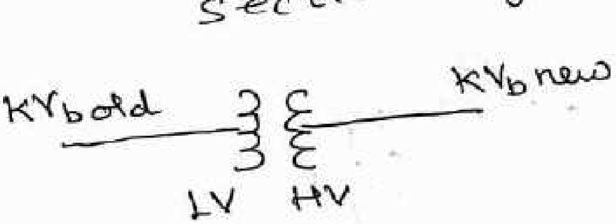
Procedure to form P.U. Impedance (or) Reactance diagram :

Step 1 : Choose a common base KV and base MVA for the entire section.

Step 2 : Divide the s/m into various section different voltage level

Step 3 : Base MVA will be remaining the same for all the sections of the system

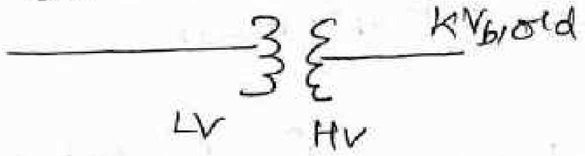
Step 4 : Find out the base KV for each and every section by using the relation.



$$\frac{\text{Base KV on HT side}}{\text{Base KV on LT side}} = \frac{\text{Voltage on HT side}}{\text{Voltage on LT side}}$$

$$KV_{b,new} = KV_{b,old} \times \frac{\text{HT side rating}}{\text{LT side rating}}$$

$KV_{b,new}$



$$\frac{\text{Base kV on LT side}}{\text{Base kV on HT side}} = \frac{\text{Voltage on LT side}}{\text{Voltage on HT side}}$$

$$KV_{b,new} = KV_{b,old} \times \frac{\text{LT side rating}}{\text{HT side rating}}$$

Step 5: Calculate per unit value

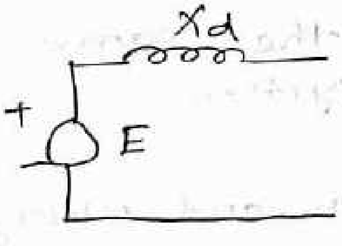
$$Z_{pu} = \frac{Z_{act}}{Z_b} \qquad Z_b = \frac{kV_b^2}{MVA_b}$$

Step 6: Convert pu value to New base values

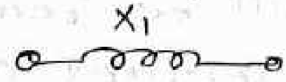
$$Z_{pu,new} = Z_{pu,old} \times \left[\frac{KV_{b(old)}}{KV_{b,new}}\right]^2 \times \left[\frac{MVA_{b,new}}{MVA_{b,old}}\right]$$

Reactance Diagram's for power S/m Components.

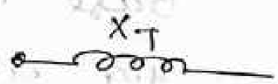
Alternator



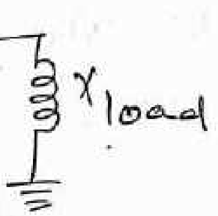
Transmission line



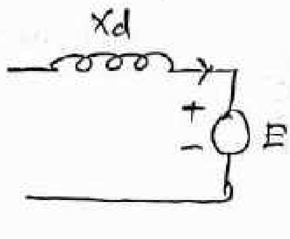
Transformer



Load



Synchronous motor



A generator has a reactance of 10 ohm. Find the pu value for the base values of 11 kV, 5 MVA.

$Z_{act} = 10\,\Omega \quad kV_b = 11\,kV \quad MVA_b = 5\,MVA$

$$Z_{pu} = \frac{Z_{act}}{Z_b} \qquad Z_b = \frac{kV_b^2}{MVA_b} = \frac{11^2}{5} = 24.2\,\Omega$$

$$= \frac{10}{24.2} = 0.4132\ pu$$

(2) A 3ϕ generator with rating 1000 kVA, 33 kV has its armature resistance and synchronous reactance as 20 Ω/p and 70 Ω/p. Calculate pu impedance of the generator.

$kV_b = 33\,kV \quad kVA_b = 1000\,kVA$

$MVA_b = 1\,MVA.$

$Z_{act} = (20 + j70)$ ohm/phase

$$Z_b = \frac{kV_b^2}{MVA_b} = \frac{33 \times 33}{1} = 1089\,\Omega$$

$$\therefore Z_{pu} = \frac{20 + j70}{1089} = 0.0184 + j\,0.064\ pu$$

(3) If base MVA is 50, base kV is 22 kV.

(a) find base current

For 1ϕ system,

$$I_b = \frac{MVA_b \times 1000}{kV_b} = \frac{50 \times 1000}{22} = 2272.22\ \text{Amps}$$

For 3ϕ system

$$I_b = \frac{MVA_b \times 1000}{\sqrt{3}\ KV_b} = \frac{50 \times 1000}{\sqrt{3} \times 22} = 1312.159 \text{ Amps}$$

b) what is the new pu impedance, if the new base MVA is twice the old base MVA?

$$MVA_{b,new} = 2 \times MVA_{b,old}$$
$$= 2 \times 50 = 100 \text{ MVA.}$$

$$KV_b = 22$$

$$Z_b = \frac{KV_b^2}{MVA_b} = \frac{22^2}{100} = 4.84\ \Omega$$

(4) A generator rated 500 MVA, 22 kV. Its Y-connected winding has a reactance of 1.1 pu. Find the actual value of the reactance of winding.

$MVA_b = 500$ MVA $\quad KV_b = 22$ kV

$$Z_b = \frac{KV_b^2}{MVA_b} = \frac{22 \times 22}{500} = 0.968\ \Omega$$

$$X_{pu} = \frac{X_{actual}}{X_{base}} = \frac{X_{actual}}{0.968}$$

$$X_{actual} = 1.1 \times 0.968$$
$$= 1.0648$$

(5) The new base values are 100 MVA, 20 kV. Then find the p.u value of reactance diagram of generator winding on the specified base.

$$X_{pu,new} = X_{pu,old} \times \left[\frac{KV_{b,old}}{KV_{b,new}}\right]^2 \times \left[\frac{MVA_{b,new}}{MVA_{b,old}}\right]$$

$$= 1.1 \times \left(\frac{22}{20}\right)^2 \times \frac{100}{500} = 0.2662 \text{ pu}$$

3) A generator has a reactance of j0.2 pu under the base voltage Voltage of 13 kV and base MVA of 10. Find the new pu value of the generator when referred to base value of 11 kV and 15 MVA.

$X_{pu,old} = j0.2$ $\quad KV_{b,old} = 13\ kV$

$MVA_{b,old} = 10\ MVA$

$KV_{b,new} = 11\ kV$ $\quad MVA_{b,new} = 15\ MVA$

$$X_{pu,new} = X_{pu,old} \times \left[\frac{KV_{b,old}}{KV_{b,new}}\right]^2 \times \frac{MVA_{b,new}}{MVA_{b,old}}$$

$$= j0.2 \times \left[\frac{13}{11}\right]^2 \times \frac{15}{10}$$

$$= j0.419\ pu$$

4) A 120 MVA, 19.5 kV generator has a synchronous reactance of 0.15 pu and it is connected to a transmission line through a transformer rated 150MVA 230/18 kV (Y/Δ) with X = 0.1 pu

(a) Calculate the p.u reactance by taking generator ratings as base value.

$X_{pu,old} = 0.1\ pu$

$KV_{b,old} = 18$ $\quad MVA_{b,old} = 150$

$KV_{b,new} = 19.5$ $\quad MVA_{b,new} = 120$

New p.u reactance of the transformer $(X_{pu,new})$ = 0.0682 pu

(b) Calculate the p.u reactance by taking transformer rating as base values.

$MVA_{b,new} = 150\ MVA$ $\quad X_{p.u\,(old)} = 0.1\ pu$

$KV_{b,new} = 18\ KV$

$$X_{pu,new} = 0.1 \times \left(\frac{19.5}{18}\right)^2 \times \frac{150}{120}$$

$$= 0.22\ pu$$

(7) The three phase ratings of a three-winding transformer are:

primary Y-connected 132 KV, 20 MVA

Secondary Y-connected 33 KV, 15 MVA

Tertiary Δ-connected 11 KV, 10 MVA

Neglecting resistance, the leakage impedances are:

Z_{ps} = 8 % on 20 MVA, 132 KV base

Z_{pt} = 10 % on 20 MVA, 132 KV base

Z_{st} = 9 % on 15 MVA, 33 KV base

Find the p.u impedances of per-phase equivalent circuit for a base of 20 MVA, 132 KV in primary ckt

$KV_{base} = 132$ $\quad MVA_{base} = 20$

$$Z_{p.u,new} = Z_{p.u,given} \times \left[\frac{KV_{b,given}}{KV_{b,new}}\right]^2 \times \frac{MVA_{b,new}}{MVA_{b,given}}$$

$$Z_{ps,pu} = j0.08 \times \left(\frac{132}{132}\right)^2 \times \frac{20}{20} = j0.08\ pu$$

$$Z_{pt,pu} = j0.1 \times \left(\frac{132}{132}\right)^2 \times \frac{20}{20} = j0.1\ pu$$

4.1.23

$$KV_{b,new} = KV_{b,old} \times \frac{\text{LT side rating}}{\text{HT side rating}} = 132 \times \frac{33}{132}$$

$$= 33\ KV$$

$$Z_{st} = j0.09 \times \left(\frac{33}{33}\right)^2 \times \left(\frac{20}{15}\right) = j0.12\ pu$$

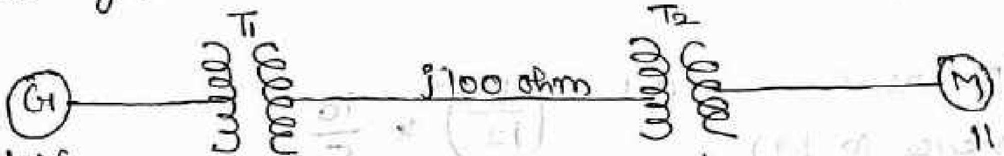
$$Z_p = \frac{1}{2}\left[Z_{ps} + Z_{pt} - Z_{st}\right] = \frac{1}{2}\left[j0.08 + j0.1 - j0.12\right]$$

$$= j0.03\ pu$$

$$Z_s = \frac{1}{2}\left[Z_{ps} + Z_{st} - Z_{pt}\right] = j0.05\ pu$$

$$Z_t = \frac{1}{2}\left[Z_{pt} + Z_{st} - Z_{ps}\right] = j0.07\ pu$$

Draw the reactance diagram for the sample power system given below



G	T1	T2	M
11 KV	11/128 KV	128/12 KV	11 KV
8 MVA	5 MVA	10 MVA	15 MVA
X = 0.1 pu	X = 10%	X = 15%	X = 0.1 pu
	X = 0.1 pu	X = 0.15 pu	

Choose a base KV of 12 KV and base MVA of 10 MVA on the generator side.

$MVA_{b,new} = 10$ MVA — Remain same for all s/m

Section 1 : $KV_{b,new} = 12$ KV

Section 2 :

KV_b on HT side of T_1 ($KV_{b,new}$) $= KV_b$ on LT side $\times \frac{V.R \text{ on HT}}{V.R \text{ on LT}}$

$$= 12 \times \frac{128}{11}$$

$$= 139.636 \simeq 140\ KV$$

Section 3 :

KV_b on HT side of T_2 ($KV_{b,new}$) = KV_b on HT side T_2 ($KV_{b,old}$) × $\frac{\text{LT Voltage}}{\text{HT Voltage}}$

$$= 140 \times \frac{12}{128}$$

$$= 13.125 \text{ kV}$$

For Generator

$$X_{pu,new} = X_{pu,old} \times \left[\frac{KV_{b,old}}{KV_{b,new}}\right]^2 \times \left[\frac{MVA_{b,new}}{MVA_{b,old}}\right]$$

$$= j0.1 \times \left(\frac{11}{12}\right)^2 \times \frac{10}{3}$$

$$= j0.28 \text{ pu}$$

For Transformer 1

$X_{pu,new}$ (with reference to LT) $= j0.1 \times \left(\frac{11}{12}\right)^2 \times \frac{10}{5}$

$$= j0.168 \text{ pu}$$

(or)

$X_{pu,new}$ (with reference to HT) $= j0.1 \times \left[\frac{128}{140}\right]^2 \times \left(\frac{10}{5}\right)$

$$= j0.168 \text{ pu}$$

For Transmission line.

$$X_{base} = \frac{KV_b^2}{MVA_b} = \frac{140 \times 140}{10} = 1960 \, \Omega$$

$$X_{p.u} = \frac{X_{actual}}{X_{base}} = \frac{j100}{1960}$$

$$= j0.051 \text{ pu}$$

For transformer 2,

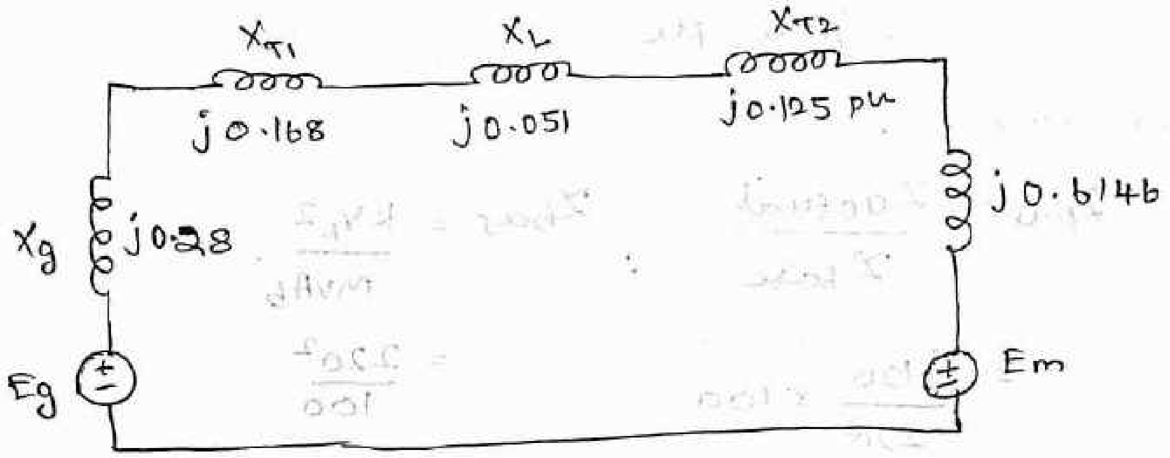
$$X_{pu, new} = j0.15 \times \left[\frac{128}{140}\right]^2 \times \frac{10}{10}$$

$$= 0.125j \text{ pu}$$

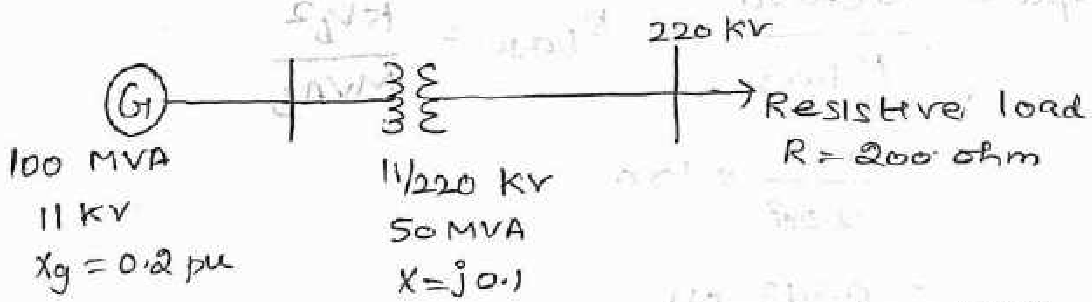
For motor

$$X_{pu, new} = j0.1 \times \left[\frac{11}{13.125}\right]^2 \times \frac{10}{15}$$

$$= j0.6146 \text{ pu}$$



For the system shown in below figure, obtain the impedance diagram



$MVA_b = 100$ MVA $= MVA_{b,new}$

Section 1: $KV_b = 11$ KV

Section 2:

$$KV_{b\,new} \text{ (HT side)} = KV_{b,old} \times \frac{\text{HT voltage rating}}{\text{LT voltage rating}}$$

$$= 11 \times \frac{220}{11} = 220 \text{ KV}$$

For Generator:

$$Z_{pu,new} = Z_{p.u\ old} \times \left[\frac{KV_{b,old}}{KV_{b,new}}\right]^2 \times \frac{MVA_{b,new}}{MVA_{b,old}}$$

$$= j0.2 \times \left[\frac{11}{11}\right]^2 \times \frac{100}{100}$$

$$= j0.2\ pu$$

For Transformer:

$$Z_{pu,new} = j0.1 \times \left[\frac{11}{11}\right]^2 \times \frac{100}{50}$$

$$= j0.2\ pu$$

Transmission line:

$$Z_{p.u} = \frac{Z_{actual}}{Z_{base}} \qquad Z_{base} = \frac{KV_b^2}{MVA_b}$$

$$= \frac{j120}{220^2} \times 100 \qquad = \frac{220^2}{100}$$

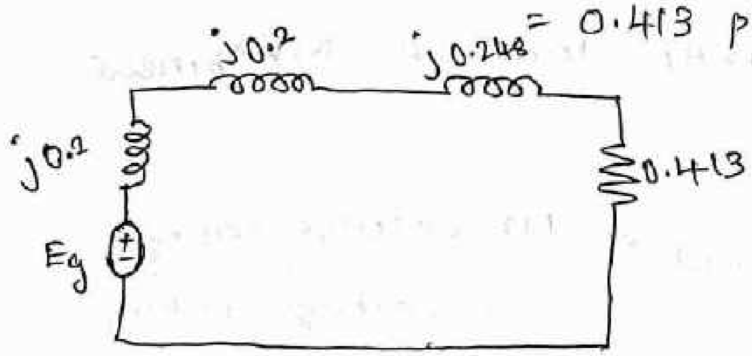
$$= j0.248\ pu$$

For Resistive load:

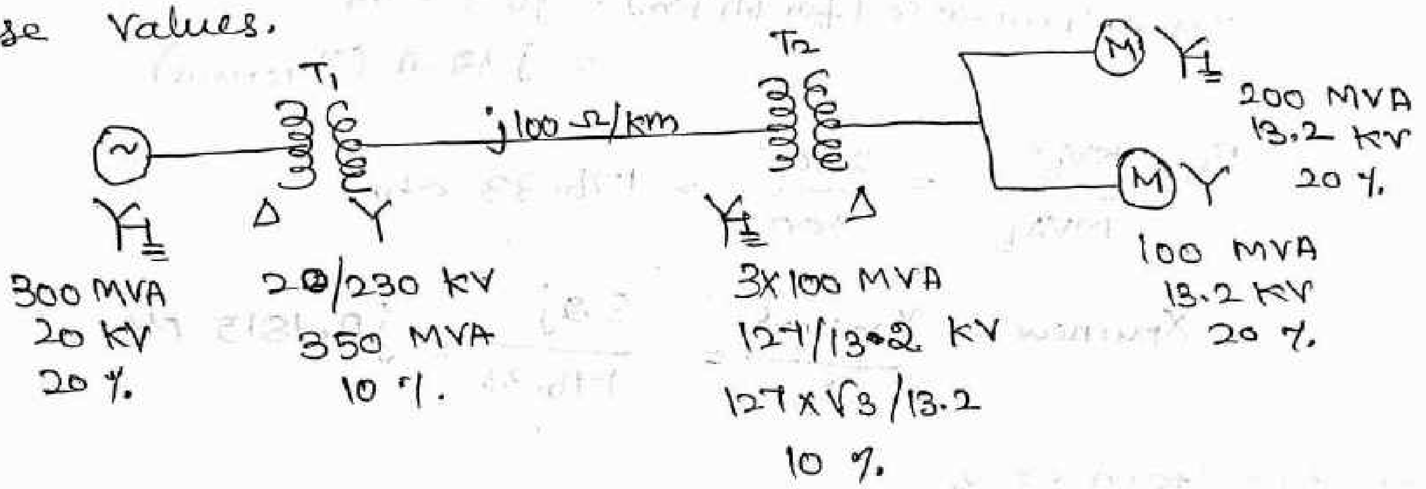
$$R_{pu} = \frac{R_{actual}}{R_{base}} \qquad R_{base} = \frac{KV_b^2}{MVA_b}$$

$$= \frac{200}{220^2} \times 100$$

$$= 0.413\ pu$$



A 300 MVA, 20 kV, 3ϕ generator has a subtransient reactance of 20 %. The generator supplies 2 synchronous motor thru a 64 km transmission line having transformer at both ends as shown in figure. In this T_1 is 3ϕ transformer and T_2 is made of 3 single ϕ transformer of rating 100 MVA, 127/13.2 kV, 10 % reactance. The series reactance of transmission line is 0.5 ohm/km. Draw the reactance diagram with all the reactances marked in pu. Select the generator rating as base values.



Section 1: $kV_{b,new} = 20\ kV$ $MVA_{b,new} = 300\ MVA$

Section 2:

$$kV_{b,new}\ T_1\ (\text{HT side}) = kV_b \text{ on LT side} \times \frac{\text{HT Voltage}}{\text{LT Voltage}}$$

$$= \frac{230}{20} \times 20 = 230\ kV$$

Section 3

$$kV_{b,new}\ T_2\ (\text{HT side}) = kV_b \text{ on HT Side} \times \frac{\text{LT Voltage}}{\text{HT Voltage}}$$

$$= \frac{13.2}{220} \times 230$$

$$= 13.8\ kV$$

For generator:

$$X_{pu,new} = X_{pu,old} \times \left[\frac{kV_{b,old}}{kV_{b,new}}\right]^2 \times \frac{MVA_{b,new}}{MVA_{b,old}}$$

$$= j0.2\ pu$$

For transformer 1:

$$X_{pu,new} = X_{pu,old} \times \left[\frac{KV_{b,old}}{KV_{b,new}}\right]^2 \times \frac{MVA_{b,new}}{MVA_{b,old}}$$

$$= j0.1 \times \left[\frac{20}{20}\right]^2 \times \frac{300}{350}$$

$$= j0.0857 \text{ pu}$$

For transmission line:

Reactance of tr. line = $j0.5\ \Omega/km$

Total reactance (for 64 km) = $j0.5 \times 64$

$= j32\ \Omega$ (X_{actual})

$$Z_b = \frac{KV_b^2}{MVA_b} = \frac{230^2}{300} = 176.33 \text{ ohm}$$

$$X_{pu,new} = \frac{X_{actual}}{Z_b} = \frac{32j}{176.33} = j0.1815 \text{ pu}$$

For transformer 2:

$$X_{pu,new} = X_{pu,old} \times \left[\frac{KV_{b,old}}{KV_{b,new}}\right]^2 \times \frac{MVA_{b,new}}{MVA_{b,old}}$$

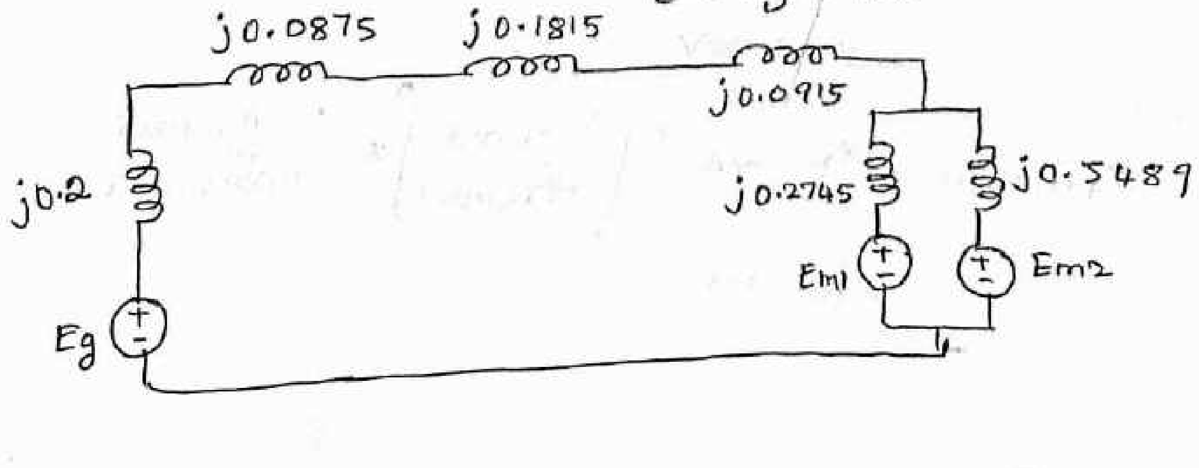
$$= j0.1 \times \left[\frac{220}{230}\right]^2 \times \frac{300}{300} = j0.0915 \text{ pu}$$

For motor 1

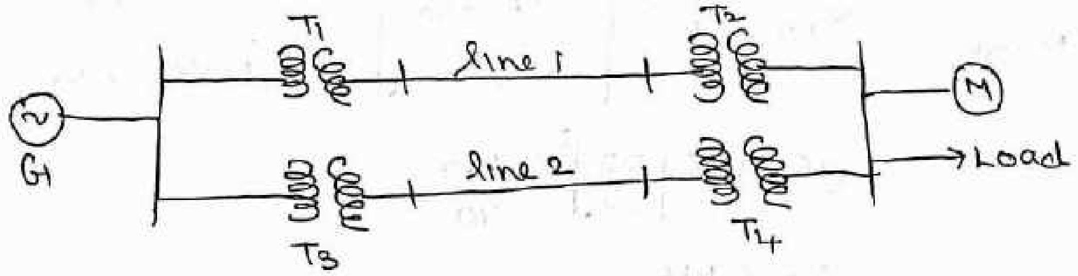
$$X_{pu,new} = j0.2 \times \left[\frac{13.2}{13.8}\right]^2 \times \frac{300}{200} = j0.2745 \text{ pu}$$

For motor 2

$$X_{pu,new} = j0.2 \times \left[\frac{13.2}{13.8}\right]^2 \times \frac{300}{100} = j0.5489 \text{ pu}$$



The single line diagram of a 3Φ S/m is shown below:



Select a common base of 100 MVA and 22 KV on the generator side. Draw an impedance diagram with given data

Generator : 90 MVA, 22 KV, $X = 18\% = j0.18$

Transformer 1 : 50 MVA, 22/220 KV, $X = 10\%$

Transformer 2 : 40 MVA, 220/11 KV $X = 6\%$

Transformer 3 : 40 MVA 22/110 KV $X = 6.4\%$

Transformer 4 : 40 MVA 110/11 KV $X = 8\%$

Motor : 66.5 MVA, 10.45 KV $X = 18.5\%$

Load : 3Φ load, 57 MVA 0.6 pf lagging 10.45 KV

line 1 : j 48.4 ohm

line 2 : j 65.43 ohm

$MVA_{b,new} = 100$ MVA

Section 1 :

$KV_{b,new} = 22$ KV

Section 2 :

KV_b on HT side of section 2 $= \frac{220}{22} \times 22 = 220$ KV

Section 3 :

KV_b on LT side of section 3 $= \frac{11}{220} \times 220 = 11$ KV

Section 4 :

KV_b on HT side of section 4 $= \frac{110}{11} \times 11 = 110$ KV

Generator:

$$X_{pu,new} = X_{pu,old} \times \left[\frac{KV_{b,old}}{KV_{b,new}}\right]^2 \times \frac{MVA_{b,new}}{MVA_{b,old}}$$

$$= j0.18 \times \left[\frac{22}{22}\right]^2 \times \frac{100}{90}$$

$$= j0.2 \text{ pu}$$

For Transformer-1

$$X_{pu,new} = j0.1 \times \left(\frac{22}{22}\right)^2 \times \frac{100}{50}$$

$$= j0.2 \text{ pu}$$

For line 1

$$X_{pu} = \frac{X_{act}}{X_{base}}$$

$$X_{base} = \frac{220^2}{100}$$

$$= 484 \text{ ohm}$$

$$X_{pu} = \frac{j48.4}{484}$$

$$= j0.1 \text{ pu}$$

For line 2

$$X_{base} = \frac{110^2}{100}$$

$$= 110 \text{ ohm}$$

$$X_{pu} = \frac{j65.43}{110}$$

$$= j0.54 \text{ pu}$$

Transformer 2

$$X_{pu,new} = j0.06 \times \left[\frac{220}{220}\right]^2 \times \frac{100}{40}$$

$$= j0.15 \text{ pu}$$

Transformer 3

$$X_{pu,new} = j0.064 \times \left[\frac{22}{22}\right]^2 \times \frac{100}{40}$$

$$= j0.16 \text{ pu}$$

Transformer 4

$$X_{pu,new} = j0.08 \times \left[\frac{110}{110}\right]^2 \times \frac{100}{40}$$

$$= j0.2 \text{ pu}$$

For motor:

$$X_{pu,new} = j0.185 \times \left[\frac{10.45}{11}\right]^2 \times \frac{100}{66.5}$$

$$= j0.25 \text{ pu}$$

Load:

Power = 57 MVA   p.f = 0.6 lag   Voltage = 10.45 kV

$S = VI^*$

$S^* = V^* I$

$S^* = V^* \frac{V}{Z} = \frac{V^2}{Z}$

$S^* = \frac{|V|^2}{Z}$

$Z = \frac{|V|^2}{S} = \frac{(10.45)^2}{57 \angle -\cos^{-1}(0.6)}$

$= \frac{(10.45)^2}{57 \angle -53.13}$

$= 1.916 \angle 53.13$

$= 1.149 + j1.533 \text{ ohm}$

$S = P + jQ$

$= VI\cos\phi + jVI\sin\phi$

$= VI \angle\phi$

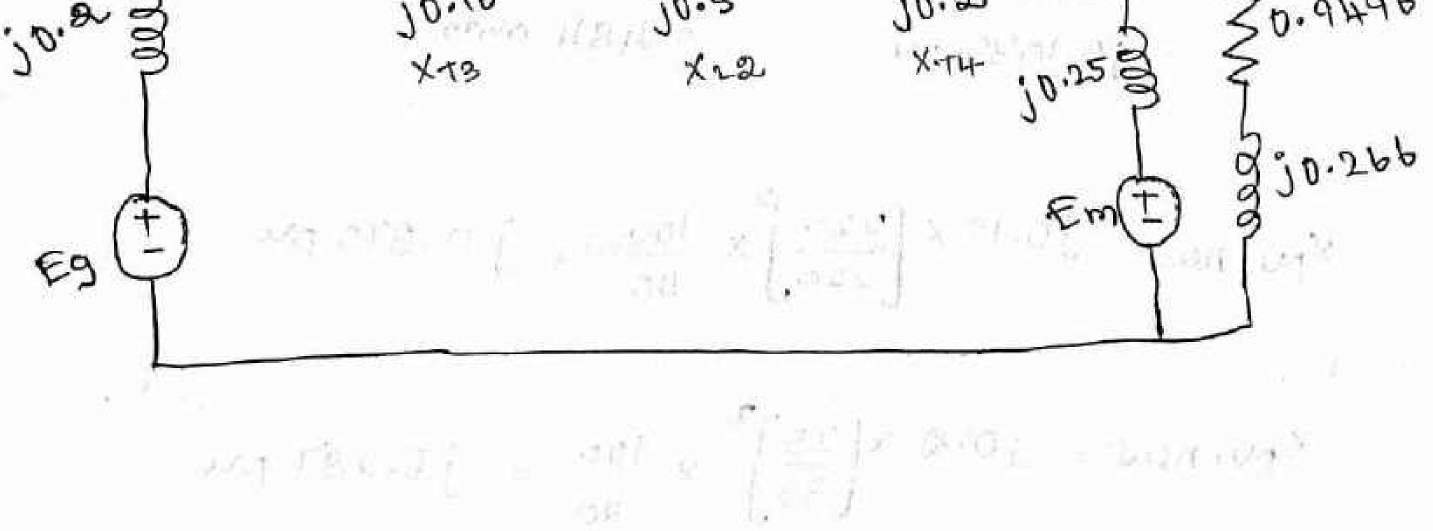
$Z_{pu} = \frac{Z_{act}}{Z_b}$

$Z_b = \frac{11^2}{100}$

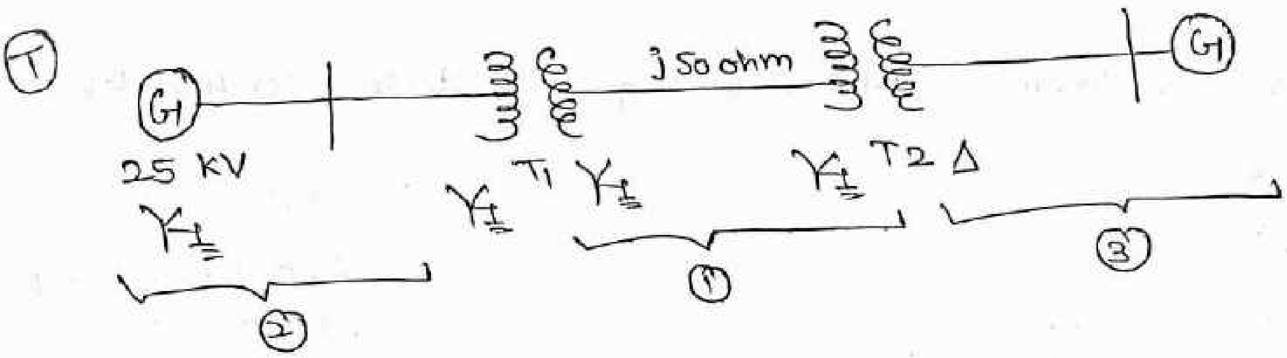
$= 1.21 \text{ ohm}$

$Z_{pu} = \frac{1.149 + j1.533}{1.21}$

$Z_{pu} = 0.9496 + j1.266 \text{ pu}$



Draw the pu reactance diagram for the power s/m shown. Neglect resistance and use a base of 100 MVA, 220 KV in 50 ohm line. The ratings of the generator, motor and transformer are



Generator : 40 MVA, 25 KV X = 20 %

Motor : 50 MVA, 11 KV, X = 30 %.

Transformer 1: 40 MVA, 33/220 KV X = 15 %.

Transformer 2: 30 MVA, 11/220 X = 15 %.

Section 1 : $KV_{b,new} = 220$ KV

Section 2 : KV_b on LT side $= 220 \times \frac{33}{220} = 33$ KV

Section 3 : KV_b on LT side $= \frac{11}{220} \times 220 = 11$ KV

Transmission line

$$X_{pu} = \frac{j50}{484} = j0.1033 \text{ pu}$$

$$X_b = \frac{220^2}{100} = 484 \text{ ohm}$$

Transformer 1

$$X_{pu,new} = j0.15 \times \left[\frac{220}{220}\right]^2 \times \frac{100}{40} = j0.375 \text{ pu}$$

Generator:

$$X_{pu,new} = j0.2 \times \left[\frac{25}{33}\right]^2 \times \frac{100}{40} = j0.287 \text{ pu}$$

Transformer 2:

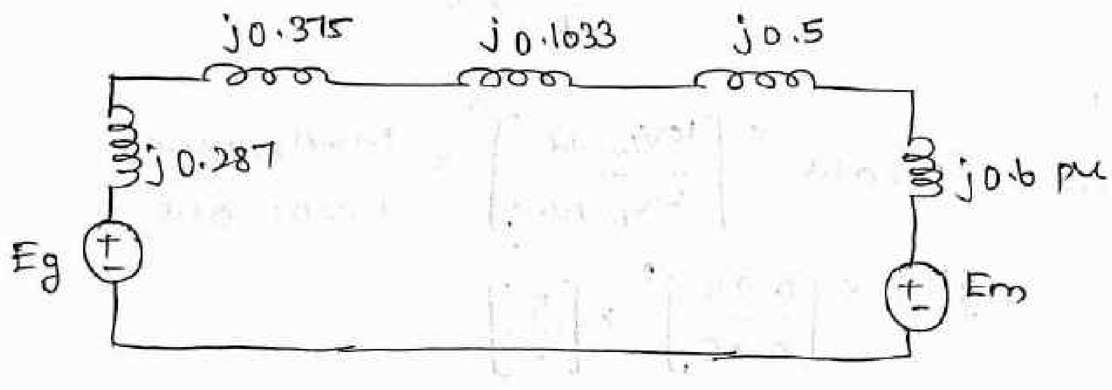
$X_{pu,new} = j0.15 \times \left(\frac{11}{11}\right)^2 \times \frac{100}{30}$

$= j0.5$ pu

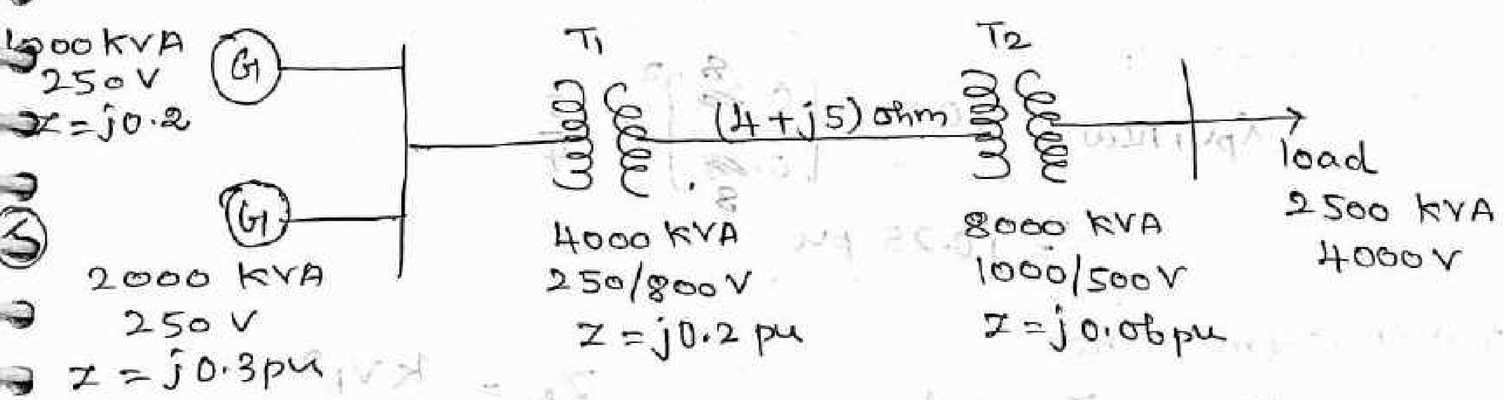
Motor:

$X_{pu,new} = j0.3 \times \left(\frac{11}{11}\right)^2 \times \frac{100}{50}$

$= j0.6$ pu



Sample power s/m is shown in figure. Redraw this system where the pu impedance of the components are represented on a common 5000 kVA base and common system base voltage of 250 V.



$MVA_{b,new} = 5$ MVA

Section 1: $kV_{b,new} = 0.25$ kV

Section 2: kV_b on HT side $T_1 = kV_b \times \frac{\text{LT rating}}{\text{HT rating}}$

$= \frac{0.8}{0.25} \times 0.25 = 0.8$ kV

Section 3

KV_b on LT section (T2) $= KV_b \times \frac{\text{LT rating}}{\text{HT rating}}$

$= \frac{0.5}{1} \times 0.8 = 0.4$ KV

Generator 1

$$X_{pu,new} = X_{pu,old} \times \left[\frac{KV_{b,old}}{KV_{b,new}}\right]^2 \times \frac{MVA_{b,new}}{MVA_{b,old}}$$

$$= j0.2 \times \left[\frac{0.25}{0.25}\right]^2 \times \left[\frac{5}{1}\right]$$

$$= j\,1 \text{ pu}$$

Generator 2:

$$X_{pu,new} = j\,0.3 \times \left[\frac{0.25}{0.25}\right]^2 \times \frac{5}{2}$$

$$= j0.75 \text{ pu}$$

Transformer T_1

$$X_{pu,new} = j\,0.2 \times \left[\frac{0.8}{0.8}\right]^2 \times \frac{5}{4}$$

$$= j\,0.25 \text{ pu}$$

Transmission line:

$$Z_{pu} = \frac{Z_{actual}}{Z_{base}}$$

$$= \frac{4 + 5j}{0.128}$$

$$= 31.25 + j\,39.0625$$

$$Z_b = \frac{KV_b^2}{MVA_b} = \frac{0.8^2}{5} = 0.128 \text{ ohm}$$

For transformer T_2

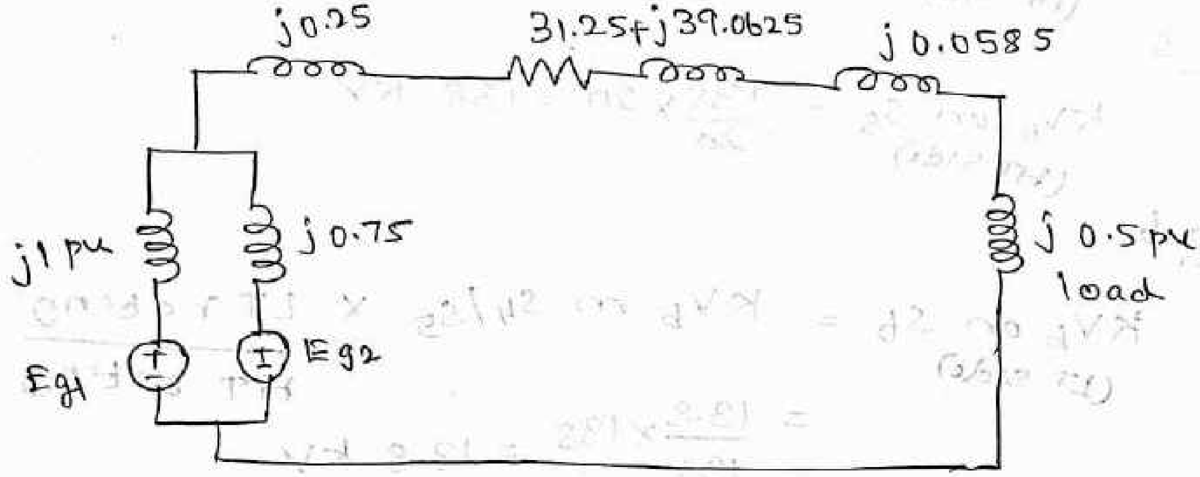
$$X_{pu,new} = j0.06 \times \left[\frac{1}{0.8}\right]^2 \times \frac{5}{8}$$

$$= j0.0585 \text{ pu}$$

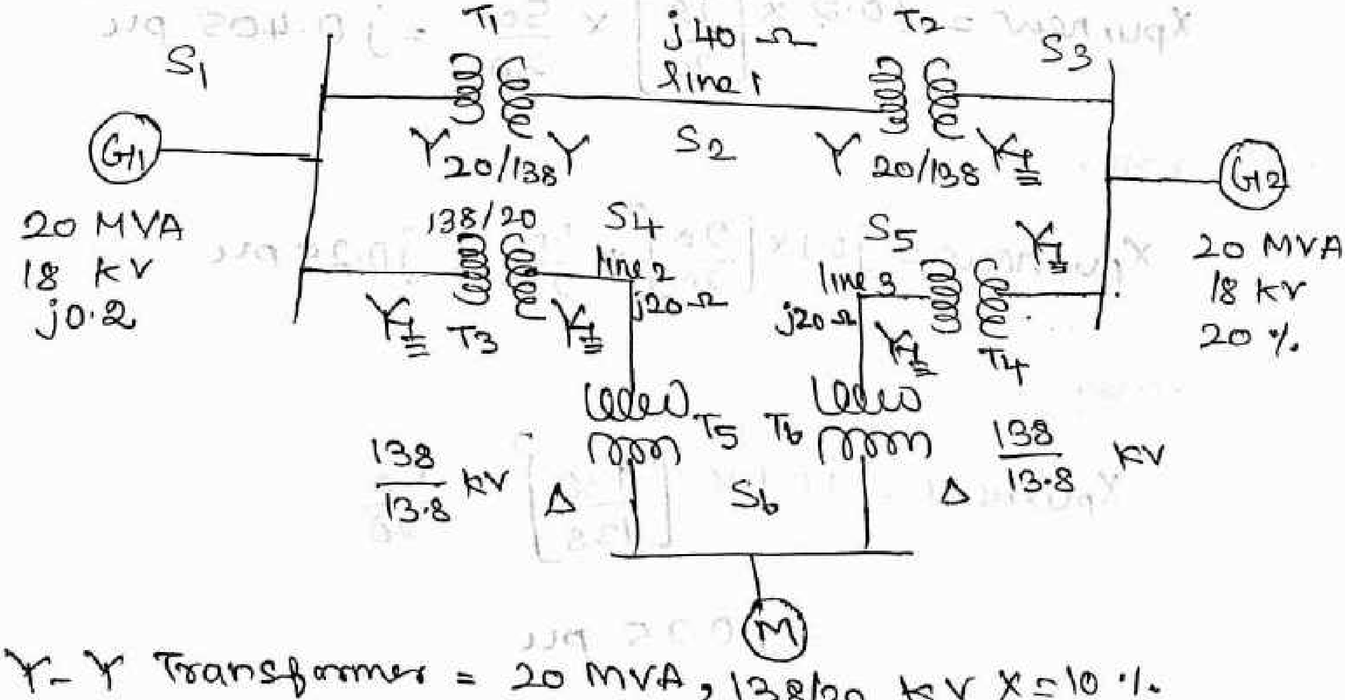
load:

$$Z_{pu,new} = \frac{2500}{5000} = j0.5 \text{ pu}$$

$$\frac{KV_b^2}{MVA_b} \quad \frac{4\times4}{5}$$



Draw the reactance diagram for the given system by using a base of 50 MVA, 138 KV in 40 ohm transmission line



Y-Y Transformer = 20 MVA, 138/20 KV X = 10 %.

Y-Δ Transformer = 15 MVA, 138/13.8 KV X = 10%.

Motor = 30 MVA, 13.8 KV, X = 20 %.

Section 2 : $KV_{b,new} = 138$ kV $MVA_{b,new} = 50$ MVA

Section 1 :

$$KV_b \text{ on } S_1 \text{ (LT side)} : KV_b(S_2) \times \frac{LT \text{ rating}}{HT \text{ rating}} = \frac{20}{138} \times 138 = 20 \text{ kV}$$

Section 3

$$KV_b \text{ on } S_3 \text{ (LT side)} = \frac{20}{138} \times 138 = 20 \text{ kV}$$

Section 4

$$KV_b \text{ on } S_4 \text{ (HT side)} = \frac{138}{20} \times 20 = 138 \text{ kV} \qquad \frac{KV_b (HT)}{LT} \times KV_b \text{ old}$$

Section 5

$$KV_b \text{ on } S_5 \text{ (HT side)} = \frac{138}{20} \times 20 = 138 \text{ kV}$$

Section 6

$$KV_b \text{ on } S_6 \text{ (LT side)} = KV_b \text{ on } S_4/S_5 \times \frac{LT \text{ rating}}{HT \text{ rating}}$$

$$= \frac{13.8}{138} \times 138 = 13.8 \text{ kV}$$

Generator 1

$$X_{pu,new} = j0.2 \times \left[\frac{18}{20}\right]^2 \times \frac{50}{20} = j0.405 \text{ pu}$$

Transformer 1

$$X_{pu,new} = j0.1 \times \left[\frac{20}{20}\right]^2 \times \frac{50}{20} = j0.25 \text{ pu}$$

Transformer 2

$$X_{pu,new} = j0.1 \times \left[\frac{138}{138}\right]^2 \times \frac{50}{20}$$

$$= j0.25 \text{ pu}$$

Transformer 3

$$X_{pu,new} = j0.1 \times \left[\frac{20}{20}\right]^2 \times \frac{50}{20}$$

$$= j0.25 \ pu$$

Transformer 4

$$X_{pu,new} = j0.1 \times \left(\frac{20}{20}\right)^2 \times \frac{50}{20} = j0.25 \ pu$$

Transformer 5

$$X_{pu,new} = j0.1 \times \left(\frac{138}{138}\right)^2 \times \frac{50}{15} = j0.3333 \ pu$$

Transformer 6

$$X_{pu,new} = j0.1 \times \left[\frac{138}{138}\right]^2 \times \frac{50}{15}$$

$$= j0.3333 \ pu$$

Transmission line 1

$$X_{pu} = \frac{X_{act}}{X_{base}} \qquad X_b = \frac{KV_b^2}{MVA_b}$$

$$= \frac{j40}{380} \qquad = \frac{138^2}{50}$$

$$= j0.1050 \ pu \qquad = 380 \ ohm$$

Transmission line 2, 3

$$X_{pu} = \frac{X_{act}}{X_{base}}$$

$$X_b = \frac{138^2}{50}$$

$$= 380 \ ohm$$

$$X_{pu} = \frac{j20}{380}$$

$$= j0.0526 \ pu$$

For Generator 2 :

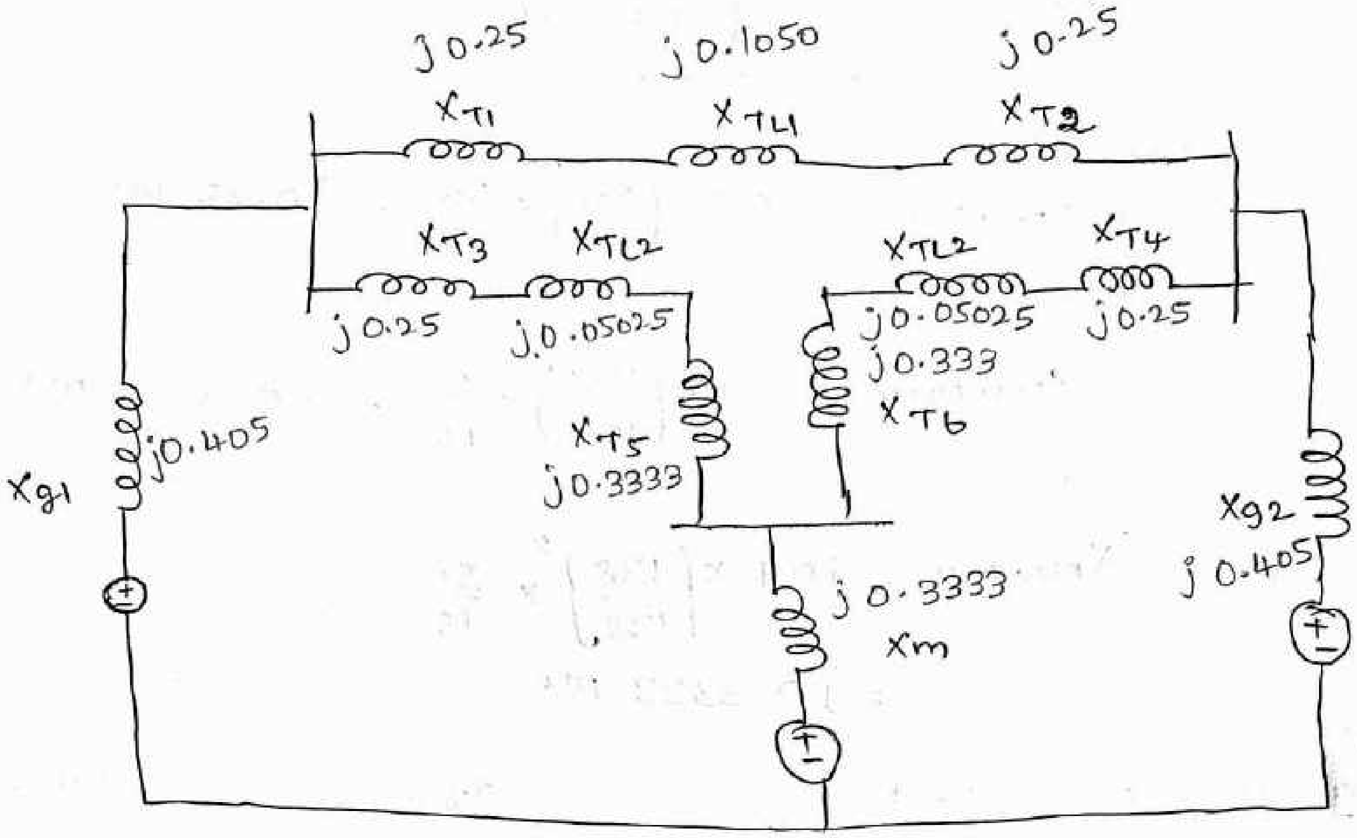
$$X_{pu,new} = j0.2 \times \left[\frac{18}{20}\right]^2 \times \frac{50}{20}$$

$$= j0.405 \ pu$$

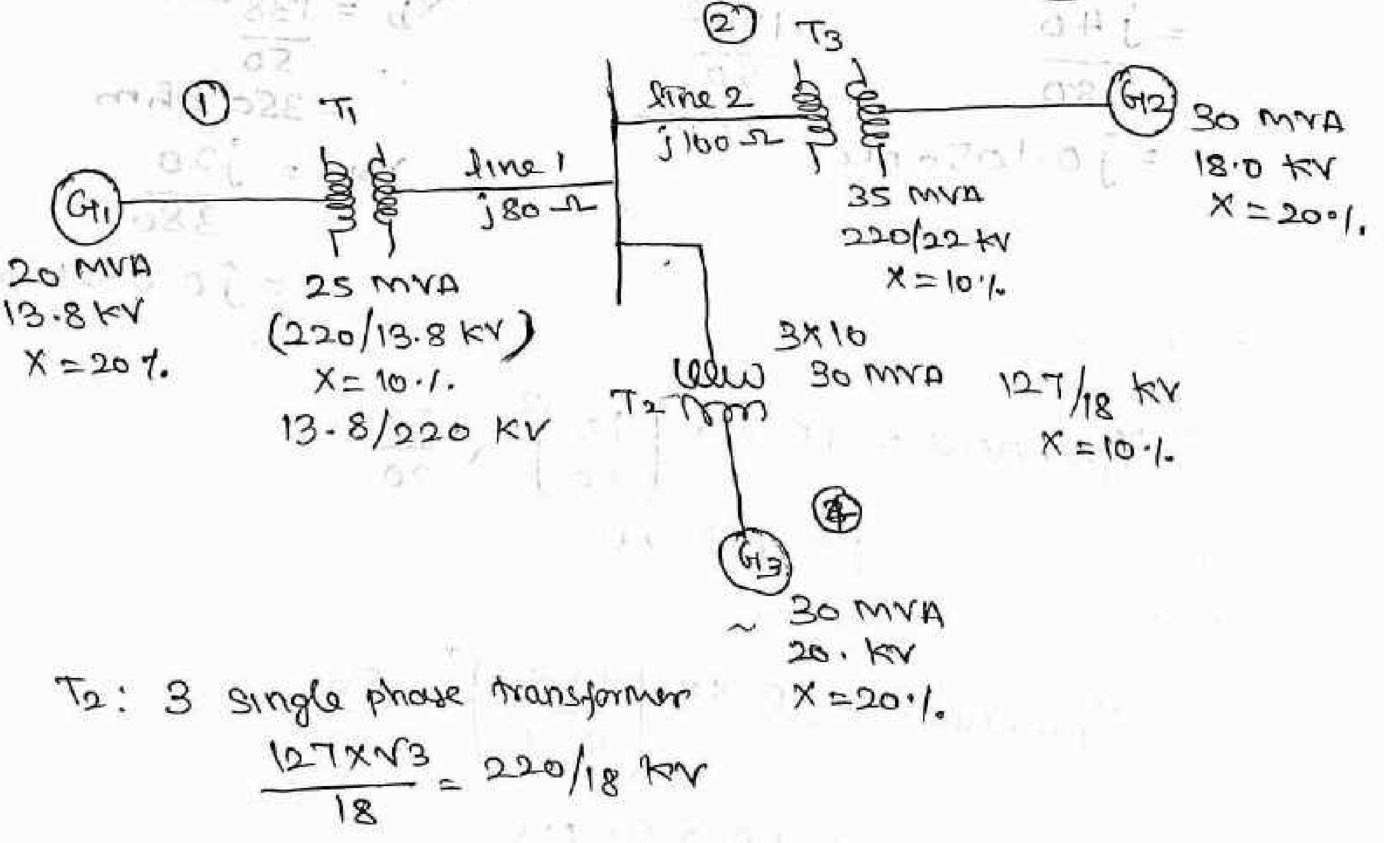
For motor :

$$X_{pu,new} = j0.3 \times \left[\frac{13.8}{13.8}\right]^2 \times \frac{50}{30}$$

$$= j0.333 \ pu$$



Draw the reactance diagram using a base of 50MVA, 13.8 kV on Generator G_1



T_2: 3 single phase transformer

$$\frac{127\times\sqrt{3}}{18} = 220/18 \text{ kV}$$

$MVA_{b,new} = 50\ MVA$

Section 1: $KV_{b,new} = 13.8\ KV$

Section 2:

KV_b on HT (T_1) : KV_b on HT section 1 $\times \frac{\text{HT Rating}}{\text{LT Rating}}$

$= \frac{220}{13.8} \times 13.8 = 220\ KV$

Section 4:

KV_b on LT side T_2 $= \frac{\text{LT rating}}{\text{HT rating}} \times KV_b$ on section 2

$= \frac{18}{220} \times 220 = 18\ KV$

Section 3:

KV_b on LT section T_2 $= KV_b$ on section 2 $\times \frac{\text{LT rating}}{\text{HT rating}}$

$= \frac{22}{220} \times 220 = 22\ KV$

Generator 1

$$X_{pu,new} = j0.2 \times \left[\frac{13.8}{13.8}\right]^2 \times \frac{50}{20}$$

$$= j0.5\ pu$$

Transformer 1:

$$X_{pu,new} = X_{pu,old} \times \left[\frac{KV_{b,old}}{KV_{b,new}}\right]^2 \times \frac{MVA_{b,new}}{MVA_{b,old}}$$

$$= j0.1 \times \left[\frac{13.8}{13.8}\right]^2 \times \frac{50}{25} = j0.2\ pu$$

Transformer 2:

$$X_{pu,new} = j0.1 \times \left[\frac{220}{220}\right]^2 \times \frac{50}{30}$$

$$= j0.1667\ pu$$

Transmission line 1 (j 80 Ω)

$$X_{pu} = \frac{X_{act}}{X_{base}} \qquad X_{base} = \frac{220^2}{50} = 968\ \Omega$$

$$X_{pu} = \frac{j80}{968} = j0.0826\ pu$$

Transmission line 2 (j 100 Ω)

$$X_{pu} = \frac{j100}{968}$$

$$= j0.1033\ pu$$

For Generator 2:

$$X_{pu,new} = j0.2 \times \left[\frac{18}{18}\right]^2 \times \frac{50}{30} = j0.333\ pu$$

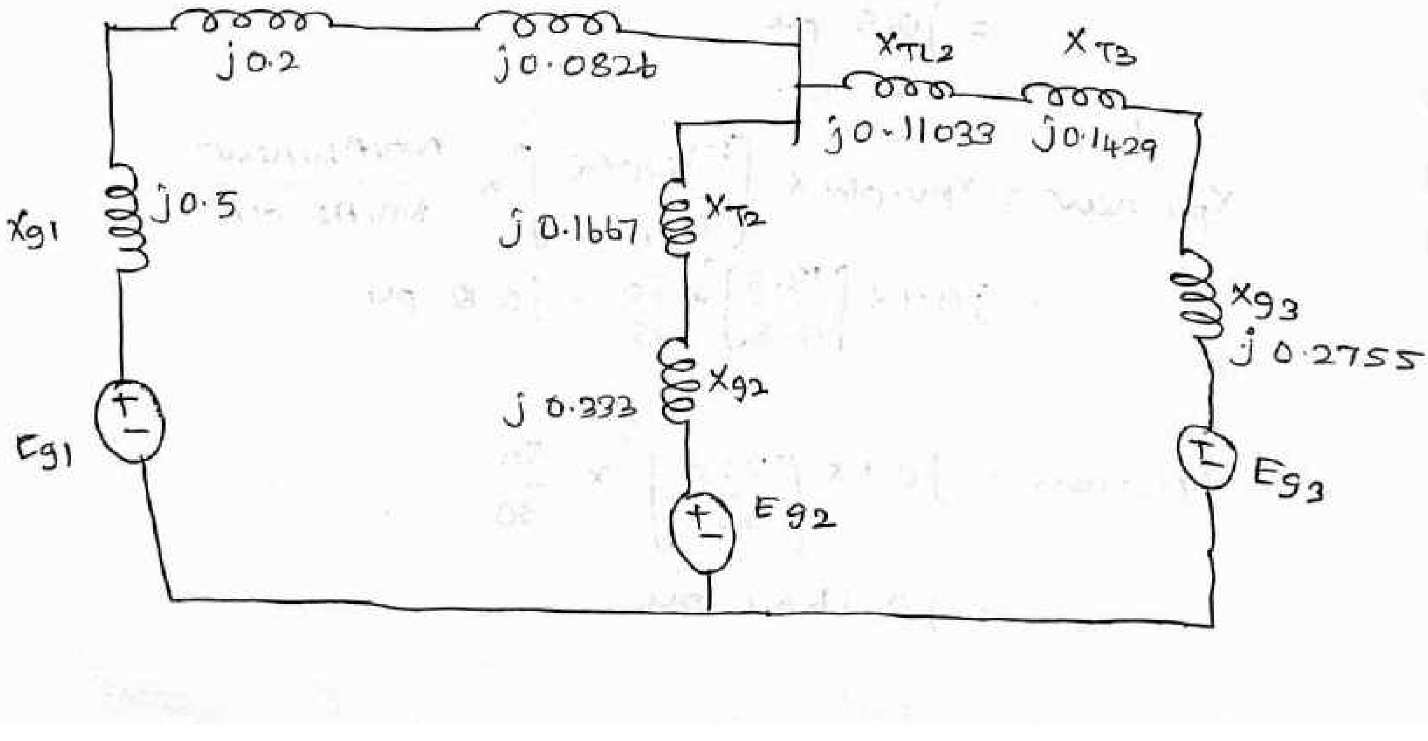
Transformer 3:

$$X_{pu,new} = j0.1 \times \left[\frac{220}{220}\right]^2 \times \frac{50}{30}$$

$$= j0.1429\ pu$$

Generator 3:

$$X_{pu,new} = j0.2 \times \left[\frac{20}{20}\right]^2 \times \frac{50}{30} = j0.2755\ pu$$



Bus Admittance Matrix (Y_{Bus})

* Matrix consisting of self and mutual admittance of the power system network

* Diagonal Elements of each node is sum of admittances connected to it. - self admittance
  driving point admittance

$$\boxed{Y_{ii} = \sum_{j=0}^{n} y_{ij} \ , \ j \neq i}$$

* Off-diagonal elements is equal to the negative of the admittance connected between the nodes.
  - mutual admittance
  - transfer admittance

$$\boxed{Y_{ij} = Y_{ji} = -y_{ij}}$$

Methods available for Y_{bus} formation

* Direct (or) Inspection method
* Singular transformation method

$$\text{Admittance } (Y) = \frac{1}{Z} = \frac{1}{R+jX}$$

$$= \frac{R-jX}{(R+jX)(R-jX)}$$

$$= \frac{R-jX}{R^2+X^2}$$

$$= \frac{R}{R^2+X^2} - \frac{jX}{R^2+X^2}$$

$$= G - jB$$

$G \rightarrow$ conductance

$B \rightarrow$ susceptance

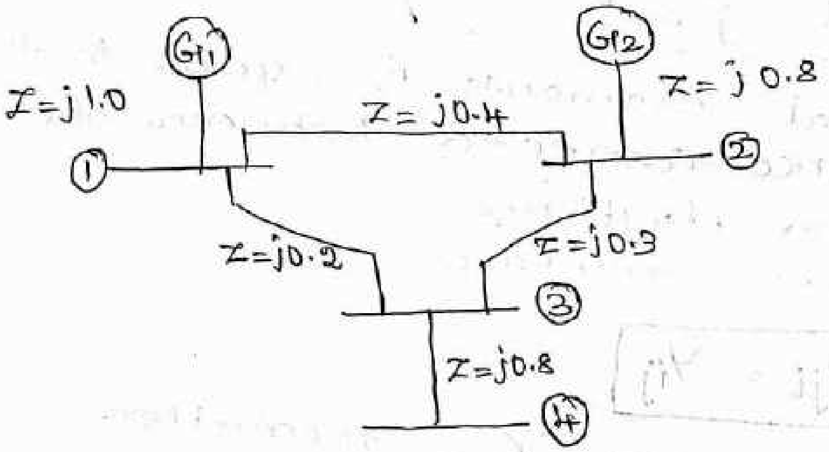
* Load flow analysis
* stability analysis
* optimal power flow analysis

] Application of Y_{bus} - matrix.

* reduce computer memory - sparse matrix

Formation of Y_{bus} matrix

01. Find out the Y_{bus} matrix of the sample power system shown in figure



$Y_{11} = Y_{10} + Y_{12} + Y_{13} = 1/j1.0 + 1/j0.4 + 1/j0.2$

$= -j1.0 - j2.5 - j5$

$= -j8.5$

$Y_{12} = Y_{21} = -Y_{12} = -(-2.5j) = 2.5j$ $\quad Y_{14} = 0$

$Y_{13} = Y_{31} = -Y_{13} = -(1/0.2j) = 5j$ $\quad Y_{24} = 0$

$Y_{22} = Y_{20} + Y_{21} + Y_{23} = -j\,7.08$

$Y_{23} = Y_{32} = -Y_{23} = 3.33j$

$Y_{33} = Y_{31} + Y_{32} + Y_{34} = -9.58j$

$Y_{34} = Y_{43} = -Y_{34} = +j\ 1.25$

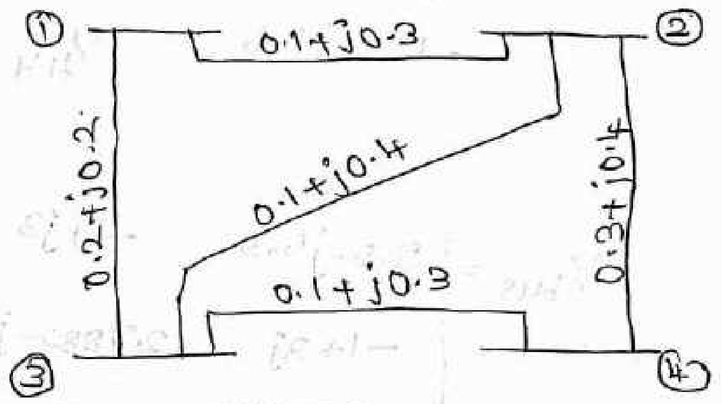
$= -1.25j$

$Y_{44} = +1/0.8j =$

$$Y_{bus} = \begin{bmatrix} -j8.5 & +j2.5 & j5.0 & 0 \\ +j2.5 & -j7.08 & j3.33 & 0 \\ j5.0 & j3.33 & -j9.58 & j1.25 \\ 0 & 0 & j1.25 & -j1.25 \end{bmatrix}$$

Q2. Form the bus admittance matrix.

line i – j	R (pu)	X (pu)
1–2	0.1	j0.3
1–3	0.2	j0.2
2–3	0.1	j0.4
2–4	0.3	j0.4
3–4	0.1	j0.3



$Y_{11} = Y_{12} + Y_{13}$

$= 3.5 - j5.5$

$Y_{12} =$

$= +Y_{12}$

$= \mp 1 \pm 3j$

$Y_{13} = +2.5 - j2.5$ $\quad Y_{14} = 0$

$Y_{21} = -Y_{12}$

$= -1 + 3j$

$Y_{22} = Y_{21} + Y_{23} + Y_{24}$

$= 1 - j3 + 0.5882 - j2.353 + 1.2 - j1.6$

$= 2.7882 - j6.953$

$Y_{23} = 0.588 - j2.3529$

$Y_{24} = 1.2 - j1.6$

$Y_{32} = -Y_{23}$

$= -0.588 + j2.353$

$Y_{13} = Y_{31}$

$= -Y_{13} = -2.5 + j2.5$

$Y_{34} = 1 - j3$

$Y_{33} = Y_{31} + Y_{32} + Y_{34}$

$= 4.0882 - j7.853$

$Y_{41} = -Y_{14}$

$= 0$

$Y_{43} = -Y_{34}$

$= -1 + 3j$

$Y_{42} = -Y_{24}$

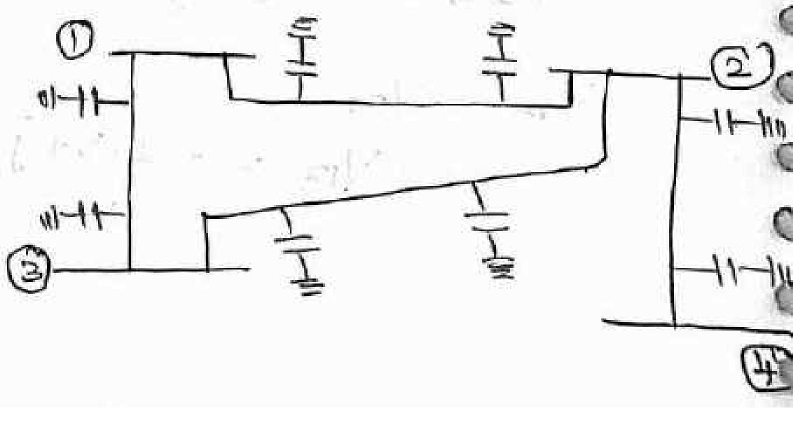
$= -1.2 + j1.6$

$Y_{44} = 2.2 - j4.6$

$$Y_{bus} = \begin{bmatrix} 3.5-j5.5 & -1+j3 & -2.5+2.5j & 0 \\ -1+3j & 2.7882-j6.953 & -0.5882+j2.353 & -1.2+j1.6 \\ -2.5+j2.5 & -0.5882+j2.353 & 4.0882-j7.853 & -1+j3 \\ 0 & -1.2+j1.6 & -1+j3 & 2.2-j4.6 \end{bmatrix}$$

The line data for 4-bus system is given below, Find the bus admittance matrix

line i-j	R	X	Half line charging admittance
1-2	0	j0.15	j0.05
1-3	0	j0.2	j0.01
2-3	0	j0.3	j0.02
2-4	0	j0.4	j0.05



Line charging admittance

$$y_{10} = j0.05 + j0.01$$
$$= +j0.06$$

$$y_{20} = j0.05 + j0.02 + j0.05 = j0.12$$

$$y_{30} = j0.01 + j0.02 = j0.03$$

$$y_{40} = j0.05$$

$$y_{12} = \frac{1}{Z_{12}} = -j6.667 \qquad y_{13} = -j5$$

$$y_{23} = -j3.333$$

$$y_{24} = -j2.5$$

$$Y_{11} = y_{12} + y_{13} + y_{10} \qquad Y_{22} = y_{20} + y_{21} + y_{23} + y_{24}$$
$$= -j11.61 \qquad\qquad = -j12.38$$

$$Y_{33} = y_{30} + y_{31} + y_{32} = -j8.3$$

$$Y_{44} = y_{42} + y_{40} = -j2.45$$

$$Y_{21} = -y_{12} = j6.667 \qquad Y_{14} = Y_{41} = 0$$

$$Y_{31} = -y_{13} = j5 \qquad Y_{34} = Y_{43} = 0$$

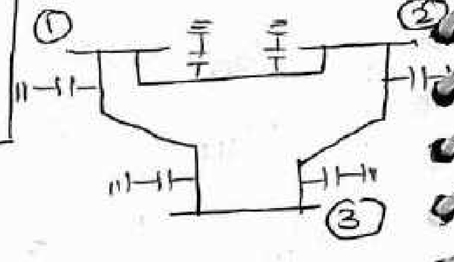
$$Y_{23} = Y_{32} = -y_{23}$$
$$= j3.33$$

$$Y_{42} = -y_{24} = j2.5$$

$$Y_{BUS} = \begin{bmatrix} -j11.61 & j6.667 & j5 & 0 \\ j6.667 & -j12.38 & j3.33 & j2.5 \\ j5 & j3.33 & -j8.3 & 0 \\ 0 & j2.5 & 0 & -j2.45 \end{bmatrix}$$

Compute Y_{bus} matrix of the sample power system.

Bus Code	Impedance	Half line Charging
1-2	0.02+j0.06	j0.03
1-3	0.08+j0.24	j0.025
2-3	0.06+j0.18	j0.020



$y_{10} = j0.025 + j0.03 = j0.055$

$y_{20} = j0.03 + j0.02 = j0.05$

$y_{30} = j0.025 + j0.020 = j0.045$

$y_{12} = \frac{1}{0.02+j0.06} = 15.82 \angle -71.56 = 5.004 - 15.007j$

$y_{13} = 3.955 \angle -71.56 = 1.249 - 3.747j$ $\quad Y_{23} = 5.273 \angle -71.56 = 1.668 - 5.002j$

$Y_{11} = y_{10} + y_{12} + y_{13} = 6.255 - j18.704$

$Y_{22} = 6.672 - j19.96 = y_{12} + y_{23} + y_{20}$

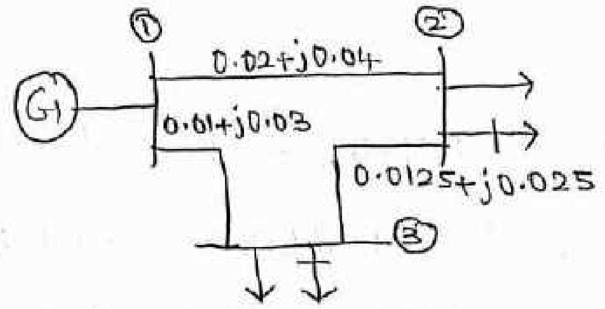
$Y_{33} = Y_{30} + Y_{13} + Y_{23} = 2.918 - j8.709$

$Y_{21} = -Y_{12} = -5 + 15j$ $\quad Y_{31} = -Y_{13} = -1.25 + 3.75j$

$Y_{32} = -Y_{23} = -1.667 + 5j$

$$Y_{bus} = \begin{bmatrix} 6.255 - j18.704 & -5+15j & -1.25+3.75j \\ -5+15j & 6.672-j19.96 & -1.667+j5 \\ -1.25+j3.75 & -1.25+j3.75 & 2.918-j8.709 \end{bmatrix}$$

Consider the system shown in figure. Compute Y_{Bus} matrix. It shown a transmission network with impedance of transmission lines all pu as shown.



$y_{12} = 10 - j20$ $\quad y_{13} = 10 - j30$ $\quad y_{32} = 16 - j32$

$$Y_{bus} = \begin{bmatrix} 20-j50 & -10+j20 & -10+j30 \\ -10+j20 & 26-j52 & -16-j52 \\ -10+j30 & -16+j32 & 26-j62 \end{bmatrix}$$

Form the Y_{bus} matrix by inspection method for a 3-bus system as shown in table

Bus code	Impedance	line charging admittance
1-2	0.15+j0.3	j0.05
1-3	0.02+j0.03	j0.04
2-3	0.05+j0.2	j0.06

Bus code	Half line charging admittance
1-2	j0.025
1-3	j0.02
2-3	j0.03

$y_{10} = j0.045$

$y_{20} = j0.55$

$y_{30} = j0.05$

$$Y_{bus} = \begin{bmatrix} 16.69 - j25.69 & -1.336 + j2.66 & -15.36 + j23.04 \\ -1.336 + j2.66 & 22.51 - j6.82 & -1.176 + j4.704 \\ -15.365 + j23.04 & -1.176 + j4704 & 16.542 - j27.702 \end{bmatrix}$$

The bus admittance matrix is given by $Y_{bus} = \begin{bmatrix} -j5 & j3 \\ j3 & -j8 \end{bmatrix}$ Determine bus impedance matrix.

$$Z_{bus} = [Y_{bus}]^{-1}$$

$$= \begin{bmatrix} -j5 & j3 \\ j3 & -j8 \end{bmatrix}^{-1}$$

$$Z_{Bus} = \begin{bmatrix} j0.258 & j0.097 \\ j0.097 & j0.161 \end{bmatrix}$$

Addition of a line	Removal of line
$Y_{ii,new} = Y_{ii,old} + y$	$Y_{ii,new} = Y_{ii,old} - y$
$Y_{ij,new} = Y_{ij,old} - y$	$Y_{ij,new} = Y_{ij,old} + y$
$Y_{ji,new} = Y_{ji,old} - y$	$Y_{ji,new} = Y_{ji,new} + y$
$Y_{jj,new} = Y_{jj,old} + y$	$Y_{jj,new} = Y_{jj,old} - y$

The Parameters of (4) bus system are as follows: 01.49

Bus code	line impedance	line charging admittance
1-2	0.2+j0.8	j0.02
2-3	0.3+j0.9	j0.03
2-4	0.25+j1.0	j0.04
3-4	0.2+j0.8	j0.02
1-3	0.1+j0.4	j0.01

Draw the network and find the admittance matrix.

$$Y_{bus} = \begin{bmatrix} 0.882-j3.514 & -0.294+j1.176 & -0.588+j2.353 & 0 \\ -0.294+j1.176 & 0.862-j3.072 & -0.333+j1 & -0.235+j0.941 \\ -0.588+j2.353 & -0.333+j1 & 1.215-j4.49 & -0.294+j1.176 \\ 0 & -0.235+j0.941 & -0.294+j1.176 & 0.529-j2.088 \end{bmatrix}$$

Remove the line between (2) and (4) for the above example and determine new Y-bus.

$$Y_{22,new} = Y_{22,old} - Y$$

$$= 0.862-j3.072 - \frac{1}{0.25+j1} - j0.02$$

$$= 0.627-j2.15$$

$$Y_{24,new} = Y_{42,new} = Y_{42,old} + Y$$

$$= -0.235+j0.941 + \frac{1}{0.25+j1} = 0$$

$$Y_{44,new} = Y_{44,old} - Y$$

$$= 0.529-j2.088 - \frac{1}{0.25+j1} - j0.02$$

$$= 0.294-j1.166$$

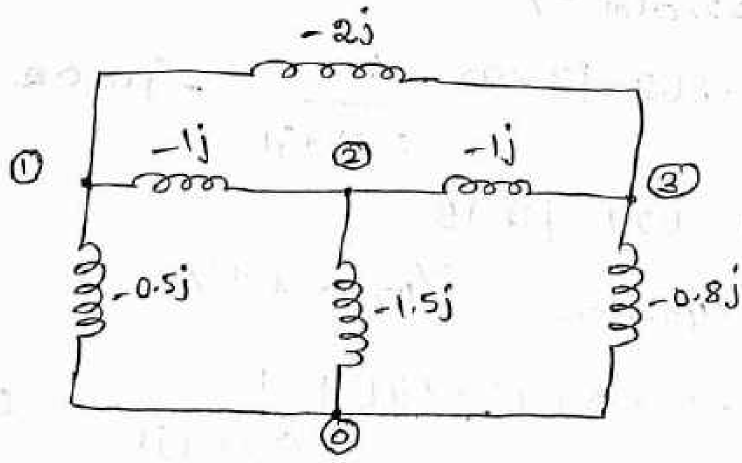
$$Y_{bus,new} = \begin{bmatrix} 0.882-j3.514 & -0.294+j1.176 & -0.588+j2.353 & 0 \\ -0.294+j1.176 & 0.627-j2.15 & -0.33+j1 & 0 \\ -0.588+j2.353 & -0.333+j1 & 1.215-j4.49 & -0.294+j1.176 \\ 0 & 0 & -0.294+j1.176 & 0.294-j1.16 \end{bmatrix}$$

Elimination of a node or Bus
(Gaussian Elimination or Kron reduction method)

$$Y_{ij,new} = Y_{ij,old} - \frac{Y_{in} \cdot Y_{nj}}{Y_{nn}}$$

where
$n \rightarrow$ node which is to be removed
$i = 1, 2, \ldots n \quad i \neq n$
$j = 1, 2 \ldots n \quad j \neq n$

Determine the bus admittance matrix of the system is shown in figure. Determine the reduced bus admittance matrix after eliminating node 3.



$$Y_{bus} = \begin{bmatrix} -0.5j-j1.0-j2.0 & j1.0 & j2.0 \\ j1.0 & -j1.0-j1.0-j1.5 & j1.0 \\ j2.0 & j1.0 & -j2-j1-j0.8 \end{bmatrix}$$

$$= \begin{bmatrix} -j3.5 & j1 & j2 \\ j1 & -j3.5 & j1.0 \\ j2 & j1.0 & -j3.8 \end{bmatrix}$$

$$Y_{11,new} = Y_{11,old} - \frac{Y_{13} \cdot Y_{31}}{Y_{33}} = -j2.45$$

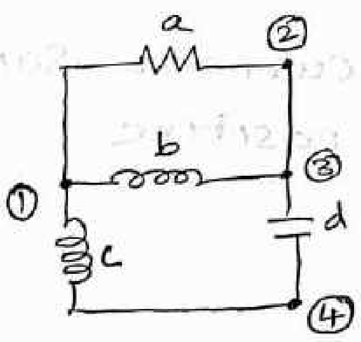
$$Y_{12,new} = Y_{12,old} - \frac{Y_{13} \cdot Y_{32}}{Y_{33}} = j1.526$$

$$Y_{22,new} = Y_{22,old} - \frac{Y_{23} \cdot Y_{32}}{Y_{33}} = -j3.237$$

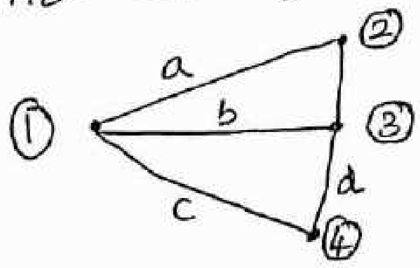
$$\text{Reduced } Y_{Bus} = \begin{bmatrix} -j2.45 & j1.526 \\ j1.526 & -j3.237 \end{bmatrix}$$

Formation of Y_{bus} by singular transformation

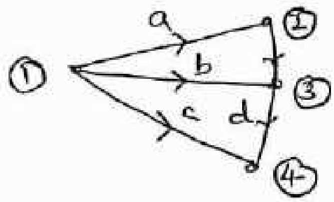
Network – interconnection of elements in various branches at different nodes.



Graph: representation of n/w obtained by replacing every element of n/w by a line segment and every junction point by a node.

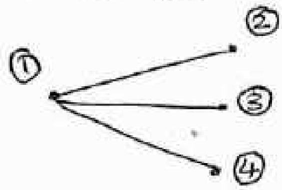


Oriented graph



every branch of a graph has direction

Tree or twig



subgraph of a network which interconnect all the nodes

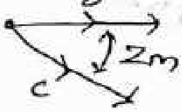
* Number of tree branches = Nodes − 1
* rank of tree = Rank of graph
* Evry connected graph has atleast one tree

Link (or) chord branches not be longing to the tree - links.

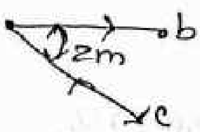
Primitive Impedance matrix (Zprimitive)

Matrix which contain info. about transmission line (impedance).

Assumptions



Direction of current same then Zm (mutual impedance) is positive



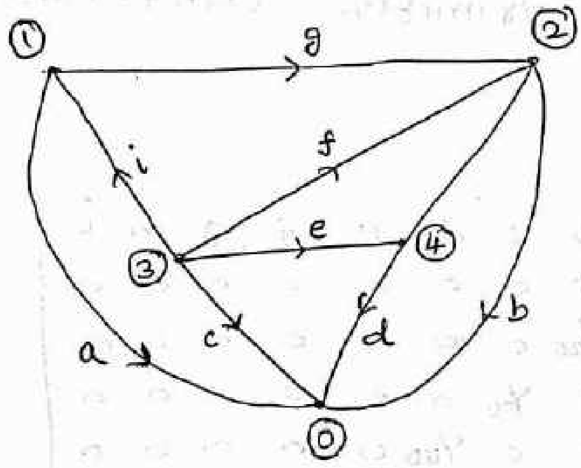
Direction of current different then Zm (mutual impedance) is negative

Primitive admittance Matrix ($Y_{primitive}$)

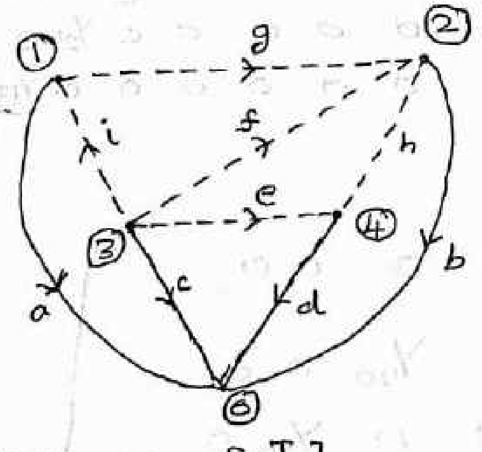
Matrix which contains info. about transmission line admittance

$$Y_{primitive} = [Z_{primitive}]^{-1}$$

Find Y_{Bus} using singular transformation for the system shown.



Step 1: Draw a tree



* Outward from node (+1)
* Toward the node (−1)

Step 2: Find [A] and [A^T]

Bus incidence Matrix [A] =

	a	b	c	d	e	f	g	h	i
(1)	1	0	0	0	0	0	+1	0	[illegible]
(2)	0	1	0	0	0	−1	−1	−1	0
(3)	0	0	1	0	1	1	0	0	[illegible]
(4)	0	0	0	1	−1	0	0	1	0

4×

$$
A^T = \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \\ f \\ g \\ h \\ i \end{array}
\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\
\left[\begin{array}{cccc}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 1 & -1 \\
0 & -1 & 1 & 0 \\
+1 & -1 & 0 & 0 \\
0 & -1 & 0 & 1 \\
-1 & 0 & 1 & 0
\end{array}\right]_{9\times 4}
\end{array}
$$

Step 3: Calculate primitive admittance matrix $[Y_{primitive}]$

$$
Y_{primitive} = \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \\ f \\ g \\ h \\ i \end{array}
\begin{array}{c} \begin{array}{ccccccccc} a & b & c & d & e & f & g & h & i \end{array} \\
\left[\begin{array}{ccccccccc}
Y_{10} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & Y_{20} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & Y_{30} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & Y_{40} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & Y_{34} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & Y_{23} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & Y_{12} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & Y_{24} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Y_{13}
\end{array}\right]_{9\times 9}
\end{array}
$$

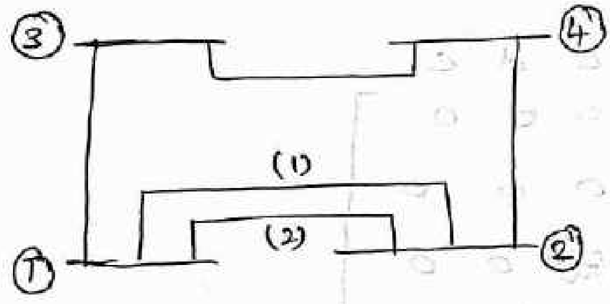
$$
[Y_{primitive}][A^T] = \left[\begin{array}{cccc}
Y_{10} & 0 & 0 & 0 \\
0 & Y_{20} & 0 & 0 \\
0 & 0 & Y_{30} & 0 \\
0 & 0 & 0 & Y_{40} \\
0 & 0 & Y_{34} & -Y_{34} \\
0 & -Y_{23} & Y_{23} & Y_{10} \\
Y_{12} & -Y_{12} & 0 & 0 \\
0 & -Y_{24} & 0 & Y_{24} \\
-Y_{13} & 0 & Y_{13} & 0
\end{array}\right]_{9\times 4}
$$

Step 4 : calculate bus admittance matrix Y_{Bus}

$$Y_{bus} = [A][Y_{primitive}][A^T]$$

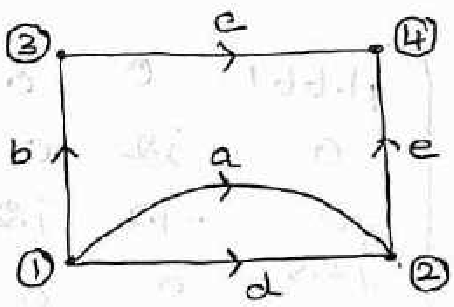
$$\therefore Y_{primitive} = \begin{bmatrix} Y_{10}+Y_{12}+Y_{13} & -Y_{12} & -Y_{13} & 0 \\ -Y_{12} & Y_{20}+Y_{12}+Y_{23}+Y_{24} & -Y_{23} & -Y_{24} \\ Y_{13} & -Y_{23} & Y_{30}+Y_{31}+Y_{23}+Y_{24} & -Y_{34} \\ 0 & -Y_{24} & -Y_{34} & Y_{40}+Y_{24}+Y_{34} \end{bmatrix}$$

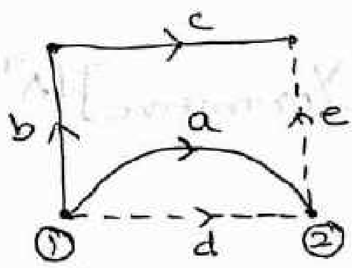
Form Y_{bus} by singular transformation for the n/w shown in figure. The impedance data is given in table. Take (1) as reference node.



Bus code	Self impedance
1-2 (1)	0.6
1-3	0.5
3-4	0.5
1-2 (2)	0.4
2-4	0.2

oriented graph :-





$$\text{Incidence matrix } [A] = \begin{array}{c} \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{bmatrix} -1 & 0 & 0 & -1 & 1 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & -1 \end{bmatrix} \end{array}$$

$$A^T = \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \end{array} \begin{array}{c} \begin{matrix} (2) & (3) & (4) \end{matrix} \\ \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 0 \\ 1 & 0 & -1 \end{bmatrix} \end{array}$$

$$Z_{primitive} = \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \end{array} \begin{array}{c} \begin{matrix} a & b & c & d & e \end{matrix} \\ \begin{bmatrix} j0.6 & 0 & 0 & 0 & 0 \\ 0 & j0.5 & 0 & 0 & 0 \\ 0 & 0 & j0.5 & 0 & 0 \\ 0 & 0 & 0 & j0.4 & 0 \\ 0 & 0 & 0 & 0 & j0.2 \end{bmatrix} \end{array}$$

$$Y_{primitive} = [Z_{primitive}]^{-1}$$

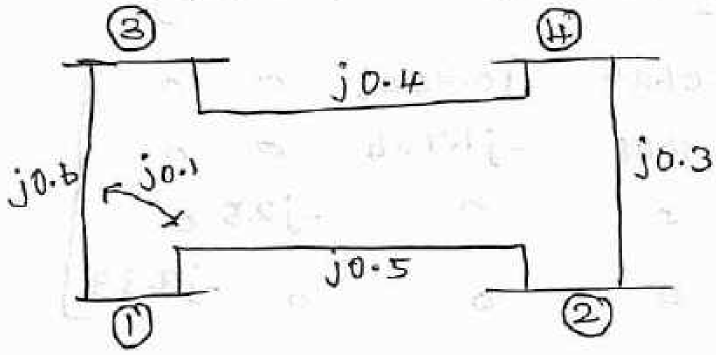
$$= \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \end{array} \begin{array}{c} \begin{matrix} a & b & c & d & e \end{matrix} \\ \begin{bmatrix} -j1.667 & 0 & 0 & 0 & 0 \\ 0 & -j2 & 0 & 0 & 0 \\ 0 & 0 & -j2 & 0 & 0 \\ 0 & 0 & 0 & -j2.5 & 0 \\ 0 & 0 & 0 & 0 & -j5 \end{bmatrix} \end{array}$$

$$[Y_{primitive}][A^T] = \begin{bmatrix} j1.667 & 0 & 0 \\ 0 & j2 & 0 \\ 0 & -j2 & j2 \\ j2.5 & 0 & 0 \\ -j5 & 0 & j5 \end{bmatrix}$$

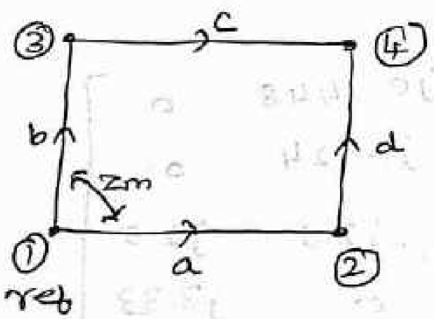
$$Y_{BUS} = [A][Y_{primitive}][A^T] = \begin{bmatrix} -j9.167 & 0 & j5 \\ 0 & -j4 & j2 \\ j5 & j2 & -j7 \end{bmatrix}$$

the admittance matrix Ybus of ... shown in figure. Take node 1 as reference

Bus code	Self impedance	Bus code	mutual impedance
1-2	0.5	1-2	0.1
1-3	0.6		
3-4	0.4		
2-4	0.3		



Oriented Graph:



$$\text{Incidence matrix } [A] = \begin{matrix} & (a) & (b) & (c) & (d) \\ (2) & -1 & 0 & 0 & 1 \\ (3) & 0 & -1 & 1 & 0 \\ (4) & 0 & 0 & -1 & -1 \end{matrix}$$

$$A^T = \begin{matrix} & (2) & (3) & (4) \\ (a) & -1 & 0 & 0 \\ (b) & 0 & -1 & 0 \\ (c) & 0 & 1 & -1 \\ (d) & 1 & 0 & -1 \end{matrix}$$

$$Z_{primitive} = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & j0.5 & j0.1 & 0 & 0 \\ b & j0.1 & j0.6 & 0 & 0 \\ c & 0 & 0 & j0.4 & 0 \\ d & 0 & 0 & 0 & j0.3 \end{array}$$

consider this matrix $\begin{bmatrix} j0.5 & j0.1 \\ j0.1 & j0.6 \end{bmatrix}^{-1} = \frac{1}{-0.29} \begin{bmatrix} j0.6 & -j0.1 \\ -j0.1 & j0.5 \end{bmatrix}$

$$= \begin{bmatrix} -j2.0689 & j0.3448 \\ j0.3448 & -j1.724 \end{bmatrix}$$

$$Y_{primitive} = \begin{bmatrix} -j2.0689 & j0.3448 & 0 & 0 \\ j0.3448 & -j1.724 & 0 & 0 \\ 0 & 0 & -j2.5 & 0 \\ 0 & 0 & 0 & -j3.33 \end{bmatrix}$$

$$Y_{bus} = [A][Y_{primitive}][A^T]$$

$$[Y_{primitive}][A^T] = \begin{bmatrix} +j2.0689 & -j0.3448 & 0 \\ -j0.3448 & j1.724 & 0 \\ 0 & -j2.5 & j2.5 \\ -j3.33 & 0 & j3.33 \end{bmatrix}$$

$$[A][Y_{primitive}][A^T] = \begin{bmatrix} -j5.4019 & j0.3448 & j3.33 \\ j0.3448 & -j4.224 & +j2.5 \\ j3.33 & j2.5 & -j5.833 \end{bmatrix}$$

[illegible]

[illegible] of the impedance matrix (Z_{bus}) using building algorithm.

[illegible]

Advantages: * less time

* easily computed

* modifications can be easily done.

Type 1 modification: Adding a new bus to reference bus through the impedance Z_b.

Type 2 modification: Adding a new bus to old bus through Z_b (adding of branch)

Type 3 modification: Adding an impedance Z_b between old and reference bus (adding of link)

Type 4 modification: Adding an impedance Z_b between two old buses (adding of link)

Formula: Type 1 modification

$$Z_{bus(new)} = \left[\begin{array}{c|c} Z_{bus\ old} & \begin{matrix} 0 \\ \vdots \\ 0 \end{matrix} \\ \hline 0 \cdots 0 & Z_b \end{array}\right]$$

Formula Type 2 Modification

$$Z_{Bus(new)} = \left[\begin{array}{c|c} Z_{Bus,old} & \begin{matrix} Z_{1i} \\ Z_{2i} \\ \vdots \\ Z_{ni} \end{matrix} \\ \hline (Z_{i1} \; Z_{i2} \cdots Z_{in}) & Z_{ii} + Z \end{array}\right]$$

$$Z_i^T$$

Formula Type 3 Modification

Step 1: Proceed with type 2 modification

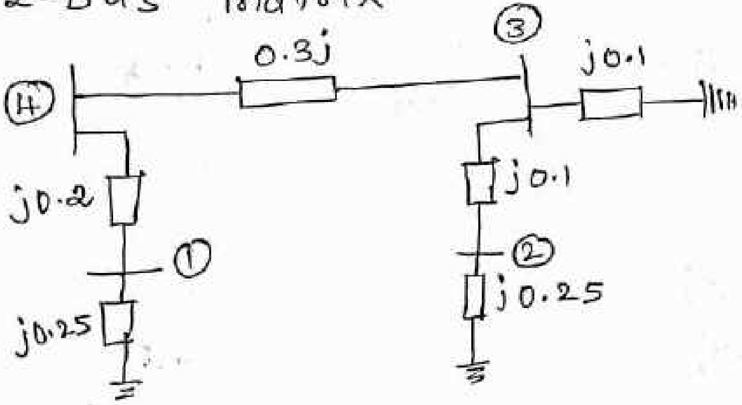
$$Z_{Bus(new)} = \left[\begin{array}{c|c} Z_{Bus,old} & \begin{matrix} Z_{1j} \\ Z_{2j} \\ \vdots \\ Z_{nj} \end{matrix} \\ \hline Z_{j1} \; Z_{j2} \cdots Z_{jn} & Z_{jj} + Z \end{array}\right]$$

Step 2: then Apply Kron's Reduction technique

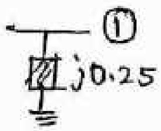
Formula Type 4 Modification

$$Z_{Bus,new} = \left[\begin{array}{c|c} Z_{Bus,old} & i^{th} - j^{th} \text{ column} \\ \hline i^{th} - j^{th} \text{ row} & Z_{ii} + Z_{jj} - 2Z_{ij} + Z \end{array}\right]$$

Obtain the Z-Bus matrix

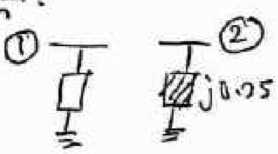


Step 1 :
Type 1



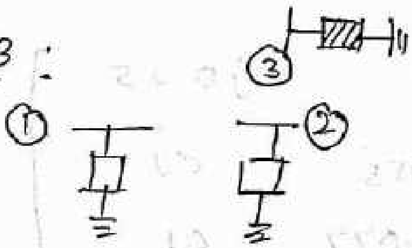
$$Z_{bus} = [j0.25]$$

Step 2 :
Type 1



$$Z_{bus} = \begin{bmatrix} j0.25 & 0 \\ 0 & j0.25 \end{bmatrix}$$

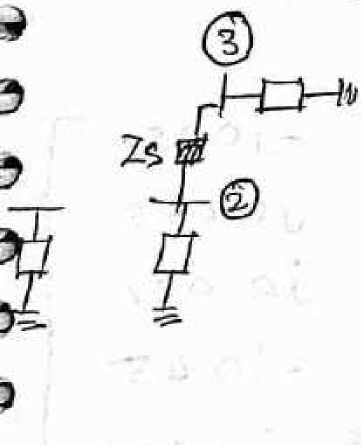
Step 3 :
Type 1



$$Z_{bus} = \begin{bmatrix} j0.25 & 0 & 0 \\ 0 & j0.25 & 0 \\ 0 & 0 & j0.1 \end{bmatrix}$$

Step 4 :
Type 4

$$Z_{bus} = \begin{bmatrix} j0.25 & 0 & 0 & 0 \\ 0 & j0.25 & 0 & j0.25 \\ 0 & 0 & j0.1 & j0.1 \\ 0 & j0.25 & j0.1 & j0.45 \end{bmatrix}$$



$$Z_{44} = Z_{22} + Z_{33} - 2Z_{23} + Z_s$$
$$= j0.25 + j0.1 - 2(0) + j0.1$$
$$= j0.45$$

Apply Kron's reduction (or) Gauss elimination technique

$$Z_{ij(new)} = Z_{ij\,old} - \frac{Z_{in} \cdot Z_{nj}}{Z_{nn}} \qquad n = 4$$

$Z_{11,new} = 0.25j$

$Z_{12,new} = Z_{21,new} = 0j$

$Z_{13,new} = Z_{31,new} = 0j$

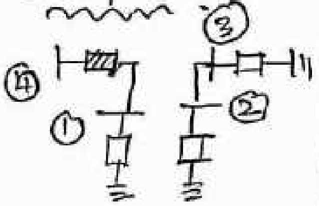
$Z_{22,new} = j0.111$

$Z_{23,new} = Z_{32,new} = j0.0555$

$Z_{33,new} = j0.0777$

$$Z_{bus} = \begin{bmatrix} j0.25 & 0j & 0j \\ 0j & j0.111 & j0.055 \\ 0j & j0.055 & j0.0777 \end{bmatrix}$$

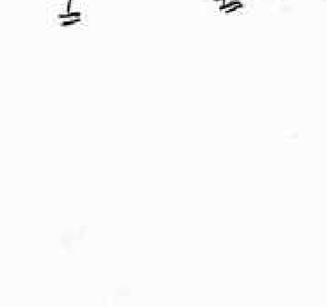
Type 2
Step 5:



$$Z_{bus} = \begin{bmatrix} j0.25 & 0j & 0j & j0.25 \\ 0j & j0.111 & j0.055 & 0j \\ 0 & j0.055 & j0.077 & 0j \\ j0.25 & 0 & 0 & j0.45 \end{bmatrix}$$

$$Z_{44} = Z_{11} + Z_s = 0.45j$$

Type 4.
Step 6.



$$Z_{bus} = \begin{bmatrix} j0.25 & 0j & 0j & j0.25 & -j0.25 \\ 0j & j0.111 & j0.055 & 0j & j0.055 \\ 0j & j0.055 & j0.077 & 0j & j0.077 \\ j0.25 & 0 & 0 & j0.45 & -j0.45 \\ j0.25 & j0.055 & j0.077 & j0.45 & j0.8277 \end{bmatrix}$$

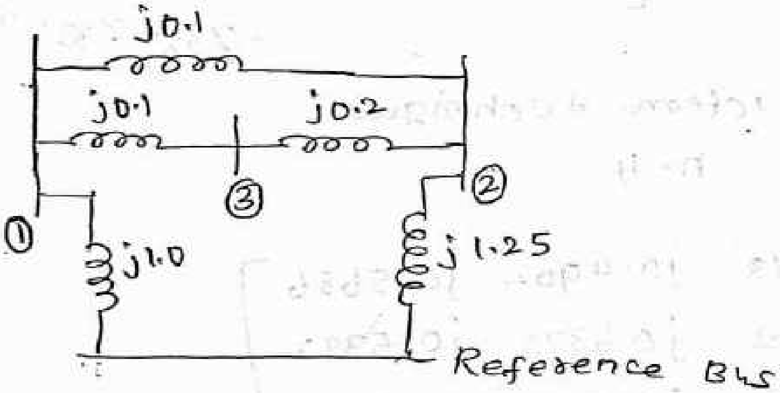
$$Z_{55} = Z_{33} + Z_{44} - 2Z_{34} + Z_s$$
$$= j0.8277$$

Apply Kron's reduction technique

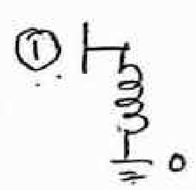
$n = 5 \quad Z_{ij,new} = Z_{ij,old} - \frac{Z_{in} \cdot Z_{ni}}{Z_{nn}}$

$$\therefore Z_{bus} = \begin{bmatrix} j0.1745 & j0.01676 & j0.02346 & j0.1141 \\ j0.01676 & j0.1073 & j0.0503 & j0.0302 \\ j0.02346 & j0.0503 & j0.0705 & j0.0423 \\ j0.1141 & j0.0302 & j0.0423 & j0.2053 \end{bmatrix}$$

Determine the Z_{Bus} for a 3-bus system as shown in figure, where impedances are shown and the values are in pu.

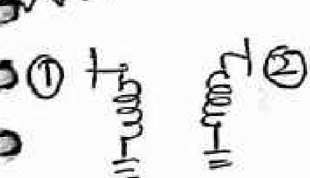


Step 1: Type 1



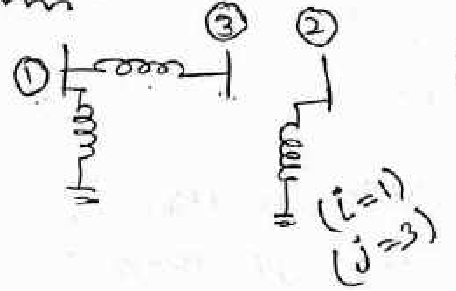
$Z_{Bus} = [j1.0]$

Step 2:



$$Z_{Bus} = \begin{bmatrix} j1.0 & 0 \\ 0 & j1.25 \end{bmatrix}$$

Step 3 : Type 2



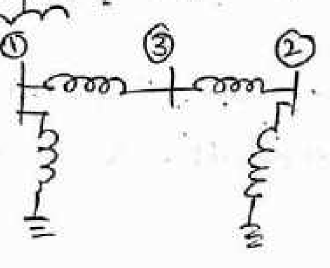
$$Z_{Bus} = \begin{bmatrix} j1.0 & 0 & j1.0 \\ 0 & j1.25 & 0 \\ j1.0 & 0 & j1.1 \end{bmatrix}$$

$= Z_{11} + Z_S$

$= j1.0 + j0.1$

$= j1.1$

Step 4 : Type 4



$$Z_{Bus} = \begin{bmatrix} j1.0 & 0 & j1.0 & j1.0 \\ 0 & j1.25 & 0 & -j1.25 \\ j1.0 & 0 & j1.1 & j1.1 \\ j1.0 & -j1.25 & j1.1 & j2.55 \end{bmatrix} \quad (i=3,\ j=2)$$

$= Z_{22} + Z_{33} - 2Z_{23} + Z_S$

Apply Kron's reduction technique

$n = 4$

$$Z_{Bus} = \begin{bmatrix} j0.6078 & j0.4902 & j0.5686 \\ j0.4902 & j0.6372 & j0.5392 \\ j0.5686 & j0.5392 & j0.6255 \end{bmatrix}$$

Step 5 Type 4

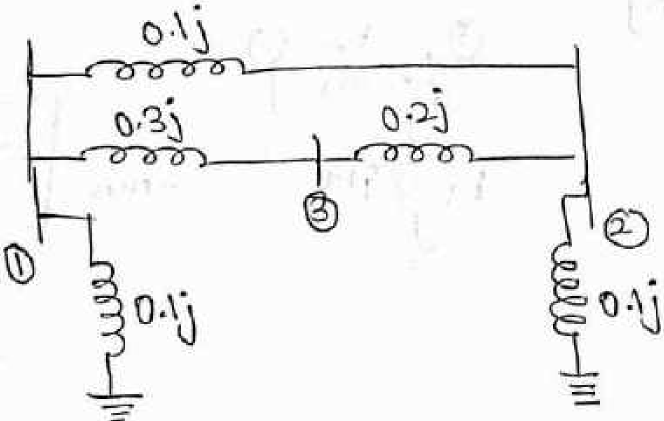
$$Z_{Bus} = \begin{bmatrix} j0.6078 & j0.4902 & j0.5686 & -j0.1176 \\ j0.4902 & j0.6372 & j0.5392 & j0.147 \\ j0.5686 & j0.5392 & j0.6255 & -j0.029 \\ -j0.1176 & j0.147 & -j0.0294 & j0.3646 \end{bmatrix} \quad i=2,\ j=1$$

$Z_{11} + Z_{22} \mp 2Z_{12} + Z_S$

Apply Kron's Reduction

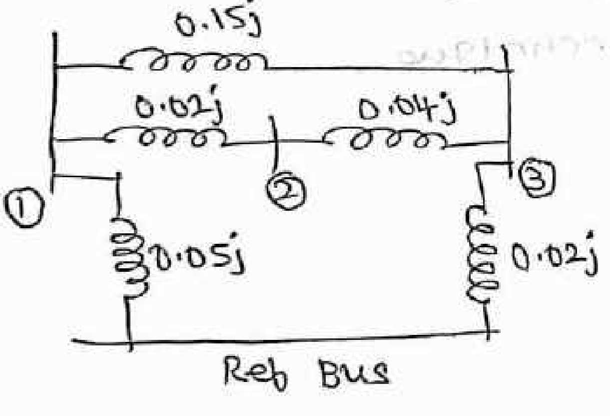
$$Z_{Bus} = \begin{bmatrix} j0.5699 & j0.5376 & j0.5591 \\ j0.5376 & j0.5779 & j0.5511 \\ j0.5591 & j0.5511 & j0.6231 \end{bmatrix}$$

Determine the Z-bus for a 3 Bus S/m shown in figure



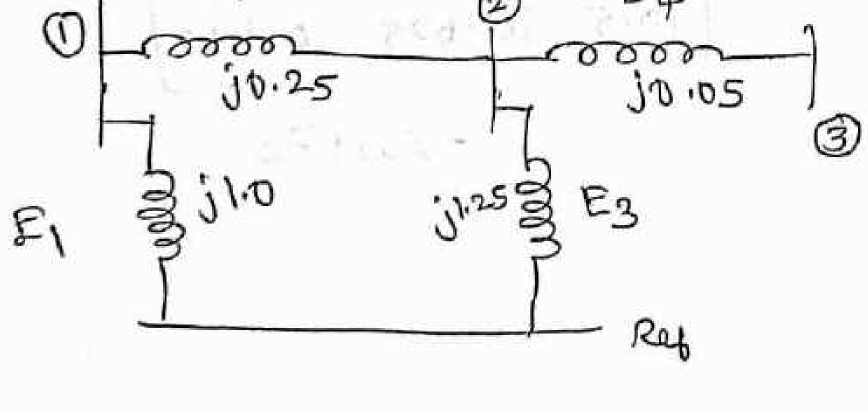
$$Z_{Bus} = \begin{bmatrix} j0.1422 & j0.1078 & j0.1164 \\ j0.1078 & j0.1422 & j0.1376 \\ j0.1164 & j0.1376 & j0.21402 \end{bmatrix}$$

Determine Z_{Bus} for the network shown in figure where the impedances are in per unit.



$$Z_{Bus} = \begin{bmatrix} j0.0279 & j0.02153 & j0.0088 \\ j0.02153 & j0.03145 & j0.0113 \\ j0.0088 & j0.0113 & j0.0164 \end{bmatrix}$$

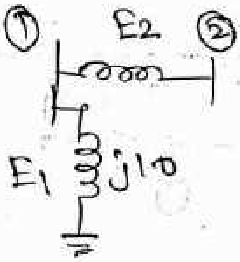
Determine Z_{Bus} using Building algorithm by adding the lines as per increasing element number.



Step 1 :

$Z_{Bus} = [j1.0]$

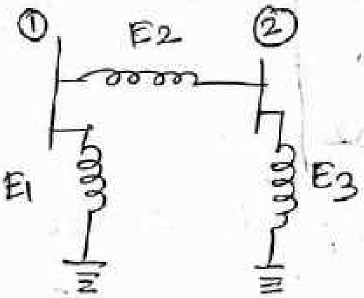
Step 2 Type 2



$$Z_{Bus} = \begin{bmatrix} j1.0 & j1.0 \\ j1.0 & j1.25 \end{bmatrix}$$

$= Z_{11} + Z_S$
$= j1.0 + j0.25$

Step 3 : Type 3



$$Z_{Bus} = \begin{bmatrix} j1.0 & j1.0 & j1.0 \\ j1.0 & j1.25 & j1.25 \\ j1.0 & j1.25 & j2.5 \end{bmatrix}$$

$= Z_{KK} + Z_S$
$= j1.25 + j1.25$

Apply Kron's reduction technique

$$Z_{Bus} = \begin{bmatrix} 0.6j & 0.5j \\ 0.5j & 0.625j \end{bmatrix}$$

Step 4 Type 2

$$Z_{Bus} = \begin{bmatrix} j0.6 & j0.5 & j0.5 \\ j0.5 & j0.625 & j0.625 \\ j0.5 & j0.625 & j0.675 \end{bmatrix}$$

$= Z_{22} + Z_S$

Z_{Bus} Building Algorithm

$$Z_{Bus(old)} \xrightarrow{Z_b = \text{Branch Impedance}} Z_{Bus(new)}$$

i, j = old buses

r = reference Bus

k = new Bus

Type 1 modification:

$$Z_{bus(new)} = \left[\begin{array}{c|c} Z_{bus(old)} & \begin{matrix} 0 \\ \vdots \\ 0 \end{matrix} \\ \hline 0 \cdots 0 & Z_b \end{array}\right]$$

Type 2 modification:

$$Z_{bus(new)} = \left[\begin{array}{c|c} Z_{bus(old)} & \begin{matrix} Z_{1j} \\ Z_{2j} \\ \vdots \\ Z_{nj} \end{matrix} \\ \hline Z_{ji}\, Z_{j2} \ldots Z_{jn} & Z_{jj} + Z_b \end{array}\right]$$

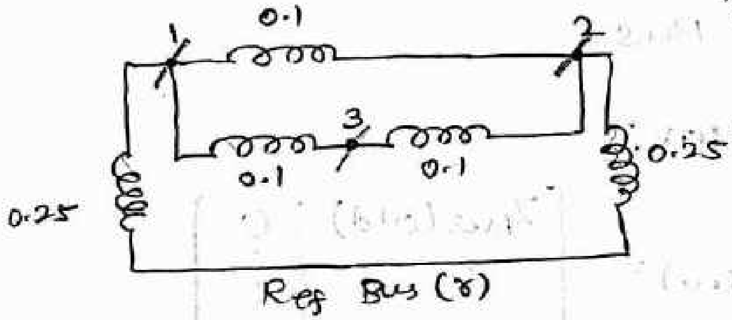
Type 3 modification:

$$Z_{Bus(new)} = Z_{bus\,old} - \frac{1}{Z_{jj} + Z_b} \begin{bmatrix} Z_{1j} \\ \vdots \\ Z_{nj} \end{bmatrix} \begin{bmatrix} Z_{j1} \ldots Z_{jn} \end{bmatrix}$$

Type 4 modification:

$$Z_{Bus(new)} = Z_{Bus(old)} - \frac{1}{Z_b + Z_{ii} + Z_{jj} - 2Z_{ij}} \begin{bmatrix} Z_{1i} \cdots Z_{1j} \\ \vdots \quad \vdots \\ Z_{ni} \cdots Z_{nj} \end{bmatrix} \left[[Z_{1i} \ldots Z_{j1}] \ldots [Z_{in} \cdots Z_{jn}] \right]$$

① For the 3-bus network shown in figure, build Bus impedance matrix



Add 'j' in all steps

Step 1: Add branch $Z_{1r} = 0.25$ (bus 1 (new) to bus r)

$$Z_{Bus} = j[0.25] \quad [\text{Type 1}]$$

Step 2: Add branch $Z_{21} = 0.1$ (from bus 2 (new) to bus 1 (old))

[Type 2]

$$Z_{bus} = j\begin{bmatrix} 0.25 & 0.25 \\ 0.25 & 0.35 \end{bmatrix} \quad 0.25 + 0.1$$

Step 3: Add branch $Z_{13} = 0.1$ (from bus (3) new to bus 1 (old))

$$Z_{bus} = j\begin{bmatrix} 0.25 & 0.25 & 0.25 \\ 0.25 & 0.35 & 0.25 \\ 0.25 & 0.25 & 0.35 \end{bmatrix} \quad [\text{Type 2}]$$

$0.35 + 0.1$

Step 4 : Add branch $Z_{2r} = 0.25$ (bus 2(old) to bus r)

2 Column   2 Row

$$Z_{bus} = \begin{bmatrix} 0.25 & 0.25 & 0.25 \\ 0.25 & 0.35 & 0.25 \\ 0.25 & 0.25 & 0.35 \end{bmatrix} - \frac{1}{0.35+0.25} \begin{bmatrix} 0.25 \\ 0.35 \\ 0.25 \end{bmatrix} \begin{bmatrix} 0.25 & 0.35 & 0.25 \end{bmatrix}$$

Type 3

$$= \begin{bmatrix} 0.1458 & 0.1042 & 0.1458 \\ 0.1042 & 0.1458 & 0.1042 \\ 0.1458 & 0.1042 & 0.2458 \end{bmatrix}$$

Step 5 Add branch $Z_{23} = 0.1$ (from bus 2(old) to bus 3(old))

$$Z_{Bus} = Z_{Bus,old} - \frac{1}{(0.1+0.1458+0.2458-2\times 0.1042)} \begin{bmatrix} -0.0416 \\ 0.0417 \\ -0.0417 \end{bmatrix} \begin{bmatrix} -0.0416 & 0.0417 & -0.0417 \end{bmatrix}$$

Type 4

$$Z_{Bus} = \begin{bmatrix} 0.1397 & 0.1103 & 0.1250 \\ 0.1103 & 0.1397 & 0.1250 \\ 0.1250 & 0.1250 & 0.1750 \end{bmatrix}$$

POWER SYSTEM ANALYSIS - UNIVERSITY SOLVED PROBLEMS

Unit-1

1. Draw the impedance diagram for the electric power system shown in figure showing all impedances in perunit on a 100 MVA base. Choose 20KV as the voltage base for generator. The three phase power and line-line ratings are given below (April/May 2011)

G_1 : 90MVA 20KV X=9%

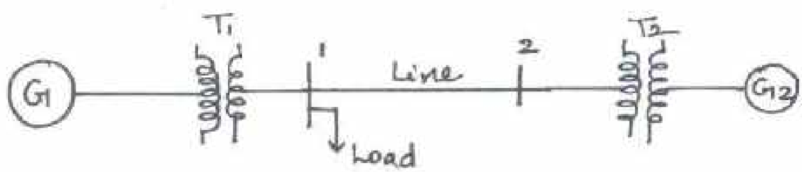
T_1 : 80MVA 20/200KV X=16%

T_2 : 80MVA 200/20KV X=20%

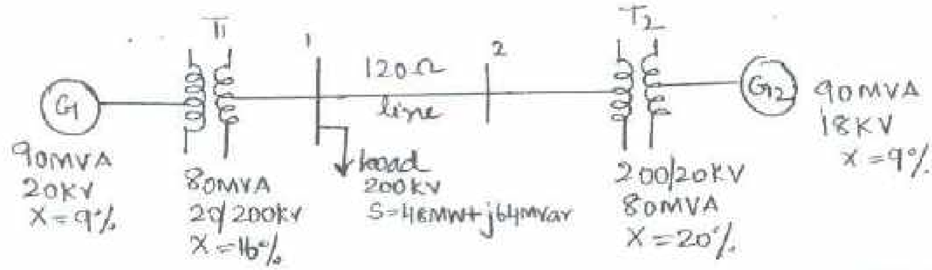
G_2 : 90MVA 18KV X=9%

Line : 200KV X=120Ω

Load : 200 KV, S=48MW+j64MVar



Soln:-



Impedance of G_1

$KV_{bnew} = 20KV$ $MVA_{bnew} = 100MVA$

$$Z_{pu,G1} = Z_{puold} \times \left(\frac{KV_{bold}}{KV_{bnew}}\right)^2 \times \left(\frac{MVA_{bnew}}{MVA_{bold}}\right)$$

$Z_{pu\,old} = \frac{9}{100} = j0.09pu$

$KV_{bold} = 20KV$

$KV_{bnew} = 20KV$

$MVA_{bnew} = 100MVA$

$MVA_{bold} = 90MVA$

$$Z_{pu,G1} = j0.09 \times \left(\frac{20}{20}\right)^2 \times \left(\frac{100}{90}\right)$$

$$\boxed{Z_{pu,G1} = j0.1pu}$$

Impedance of T_1 (with referred to primary)

$KV_{bold} = 20KV$

$KV_{bnew} = 20KV$

$MVA_{bnew} = 100 MVA$

$MVA_{bold} = 80 MVA$

$Z_{puold} = \frac{16}{100} = j0.16 pu$

$$Z_{pu.new T_1} = Z_{pu.old} \times \left(\frac{KV_{bold}}{KV_{bnew}}\right)^2 \times \left(\frac{MVA_{bnew}}{MVA_{bold}}\right)$$

$$= j0.16 \times \left(\frac{20}{20}\right)^2 \times \left(\frac{100}{80}\right)$$

$$\boxed{Z_{pu new T_1} = j0.2 pu}$$

Impedance of transmission line

$$KV_{bnew} \text{ HT side of } T_1 = KV_{bnew} \text{ on LT side of } T_1 \times \frac{\text{HT voltage rating}}{\text{LT voltage rating}}$$

$$KV_{bnew} \text{ on HT of } T_1 = \frac{20 \times 200}{20} = 200KV$$

now $KV_{bnew} = 200KV$

$MVA_{bnew} = 100 MVA$

$Z_{actual} = j120\Omega$

$$\text{Base impedance } Z_b = \frac{(KV_{bnew})^2}{MVA_{bnew}} = \frac{(200)^2}{100}$$

$$Z_b = 400\Omega$$

$$Z_{pu} \text{ of line} = \frac{\text{Actual value}}{\text{Base impedance}} = \frac{j120}{400}$$

$$\boxed{Z_{pu} \text{ of line} = j0.3 pu}$$

Impedance of load

$S = 48MW + j64Mvar \Rightarrow S^* = 48 - j64$

$V_{LL} = 200KV$

Actual load impedance $Z_L = \frac{V_{LL}^2}{S^*} = \frac{V_{LL}^2}{P - jQ}$

$$Z_L = \frac{200^2}{48 - j64} = 300 + 400j = 500\angle 53.13^\circ \Omega$$

Base impedance $Z_b = \frac{(KV_{bnew})^2}{MVA_{bnew}} = \frac{(200)^2}{100} = 400\Omega$

Z_{pu} of load $= \frac{\text{Actual value}}{\text{Base impedance}}$

Z_{pu} of load $= \frac{300 + 400j}{400} = 0.75 + j1.0pu$

$$\boxed{Z_{pu} \text{ of load} = 0.75 + j1.0pu}$$

Impedance of T_2 referred to primary

$KV_{bold} = 200KV$

$KV_{bnew} = 200KV$

$MVA_{bold} = 80MVA$

$MVA_{bnew} = 100MVA$

Z_{pu} old $= \frac{20}{100} = j0.2pu$

$Z_{pu,new}$ $T_2 = j0.2 \times \left(\frac{200}{200}\right)^2 \times \left(\frac{100}{80}\right)$

$$\boxed{Z_{pu new} \text{ of } T_2 = j0.25pu}$$

Impedance of G_{12}

$$KV_{b\,new} \text{ on LT side of } T_2 = KV_{b\,new} \text{ on HT side of } T_2 \times \frac{\text{LT voltage rating}}{\text{HT voltage rating}}$$

$$KV_{b\,new} \text{ on LT side of } T_2 = 200 \times \frac{20}{200} = 20\,KV$$

now, $KV_{b\,new} = 20KV$ $\quad Z_{pu\,old} = \frac{9}{100}\ j0.09\,pu$

$KV_{b\,old} = 18\,KV$

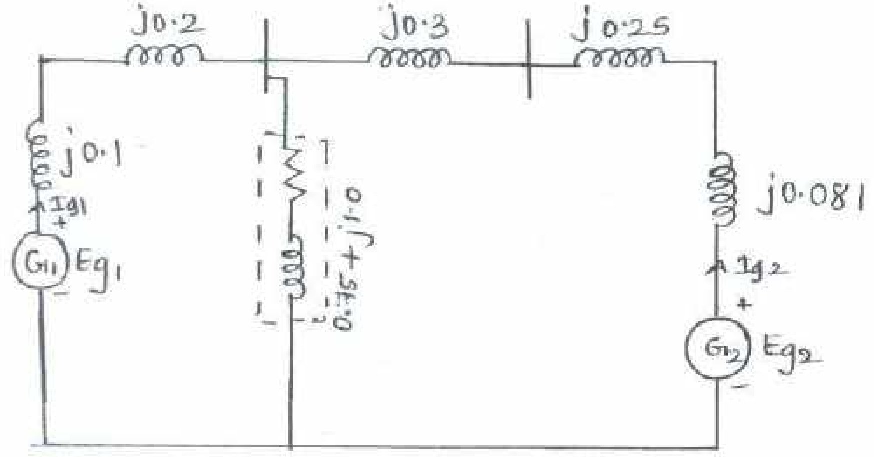
$MVA_{b\,old} = 90\,MVA$

$MVA_{b\,new} = 100\,MVA$

$$Z_{pu} \text{ of } G_{12} = j0.09 \times \left[\frac{18}{20}\right]^2 \times \left(\frac{100}{90}\right)$$

$$\boxed{Z_{pu} \text{ of } G_{12} = j0.081\,pu}$$

Impedance Diagram



2. Draw the reactance diagram for the power system shown. Neglect resistance and use a base of 100MVA, 220KV in 50Ω line. The ratings of the generator, motor and transformer are given below

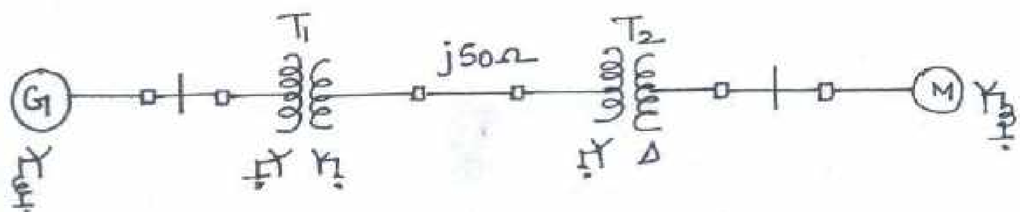
Generator : 40MVA, 25 KV, $X'' = 20\%$

Synchronous motor : 50MVA, 11KV, $X'' = 30\%$

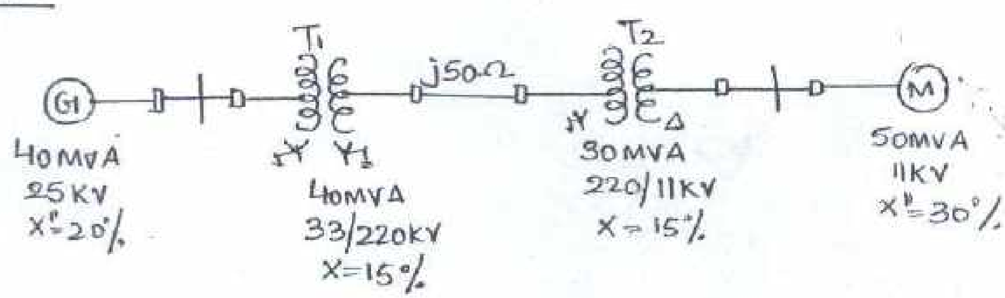
Y-Y Transformer : 40MVA, 33/220 KV, X = 15%

Y-Δ Transformer : 30 MVA, 11/220 KV (Δ/Y), X = 15%

(May/June 2012)



Soln:-



$MVA_{bnew} = 100MVA$

$KV_{bnew} = 220KV$ (on 50Ω line)

Reactance of 50Ω line

$$X_{pu} \text{ of line} = \frac{\text{Actual value}}{\text{Base reactance}}$$

$$\text{Base reactance } X_b = \frac{(KV_{bnew})^2}{MVA_{bnew}} = \frac{(220KV)^2}{100} = 484\Omega$$

Actual value $= j50\,\Omega$

X_{pu} of line $= \frac{j50}{484}$

X_{pu} new of line $= j0.1033$ pu

Reactance of T_1 referred to primary

KV_b new on LT side of T_1 = KV_b new on HT side $\times \frac{\text{LT voltage rating}}{\text{HT voltage rating}}$

$= 220 \times \frac{33}{220} = 33$ KV

now $KV_b new = 33$ KV

$KV_b old = 33$ KV

$MVA_b old = 40$ MVA

$MVA_b new = 100$ MVA

$X_{pu} old = \frac{15}{100} = j0.15$ pu

$$X_{pu} new = X_{pu} old \times \left(\frac{KV_b old}{KV_b new}\right)^2 \times \left(\frac{MVA_b new}{MVA_b old}\right)$$

$$X_{pu} new\ T_1 = j0.15 \times \left(\frac{33}{33}\right)^2 \times \left(\frac{100}{40}\right)$$

$X_{pu} new\ T_1 = j0.375$ pu

Reactance of G

$KV_b old = 25$ KV

$KV_b new = 33$ KV

$MVA_b new = 100$ MVA

$MVA_b old = 40$ MVA

$X_{pu} old = \frac{20}{100} = j0.2$ pu

T

$$X_{pu}\text{ new of G1} = j0.2 \times \left(\frac{25}{33}\right)^2 \times \left(\frac{100}{40}\right)$$

$$\boxed{X_{pu}\text{ new of G1} = j0.286\,pu}$$

Reactance of T_2 referred to secondary

$$KV_b\text{new on LT side of } T_2 = KV\text{new on HT side of } T_2 \times \frac{\text{LT rating}}{\text{HT rating}}$$

$$KV_b\text{ new on LT of } T_2 = 220 \times \frac{11}{220} = 11KV$$

now $KV_b\text{new} = 11KV$

$KV_b\text{old} = 11KV$

$MVA_b\text{new} = 100MVA$

$MVA_b\text{old} = 30MVA$

$X_{pu}\text{ old} = \frac{15}{100} = j0.15pu$

$$X_{pu}\text{ new of } T_2 = j0.15 \times \left(\frac{11}{11}\right)^2 \times \left(\frac{100}{30}\right)$$

$$\boxed{X_{pu}\text{new of } T_2 = j0.5\,pu}$$

Reactance of Motor

$KV_b\text{new} = 11KV$

$KV_b\text{old} = 11KV$

$MVA_b\text{old} = 50MVA$

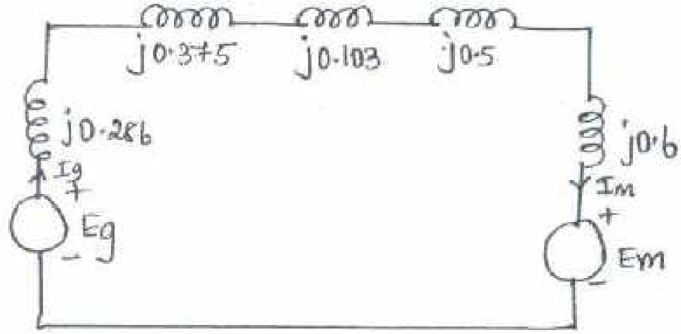
$MVA_b\text{new} = 100MVA$

$X_{pu}\text{ old} = \frac{30}{100} = j0.3pu$

$$X_{pu} \text{ new of } M = j0.3 \times \left(\frac{11}{11}\right)^2 \times \left(\frac{100}{50}\right)$$

$$\boxed{X_{pu} \text{ new of } M = j0.6\,pu}$$

Reactance Diagram



3. The one-line diagram of a power system is shown in fig. The three phase power and line-line ratings are given below

G1 : 80MVA 22 KV X=9%

T1 : 50MVA 22/220KV X=10%

T2 : 40MVA 220/22KV X=6.0%

T3, T4 : 40MVA 22/110 KV X=6.4%

Line1 : 220KV X=121Ω

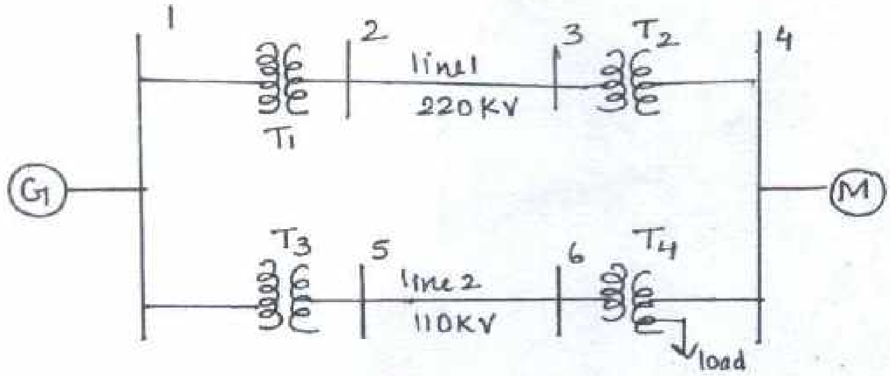
Line2 : 110KV X=42.35Ω

M : 68.85MVA 20KV X=22.5%

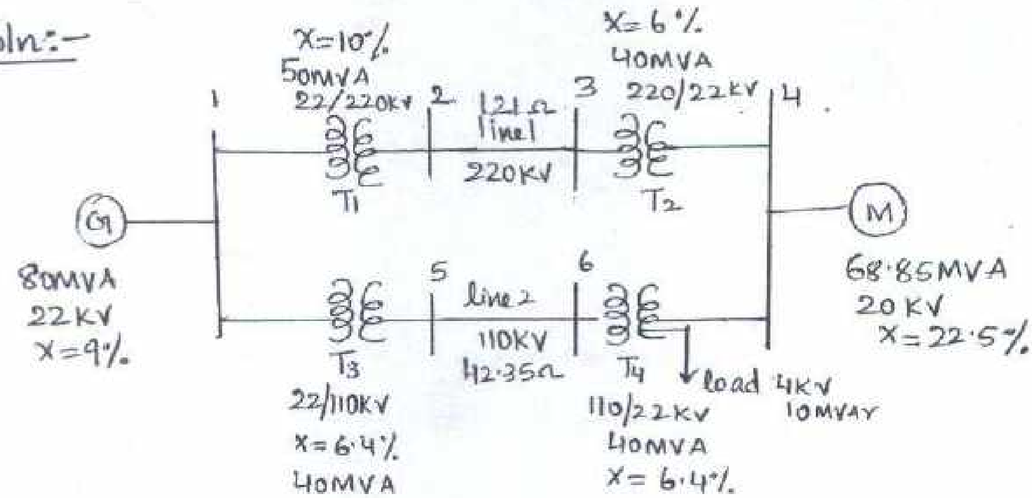
Load : 10MVAr 4KV Δ-connected capacitor.

Draw an impedance diagram showing all impedances in per unit on a 100MVA base. Choose 22KV as the voltage base for generator. (Nov/Dec 2012)

LECTURE NOTES PAPER



Soln:-



$MVA_{b\,new} = 100MVA$

$KV_{b\,new} = 22KV$

Impedance of G1

$KV_{b\,new} = 22KV$

$MVA_{b\,new} = 100MVA$

$MVA_{b\,old} = 80MVA$

$KV_{b\,old} = 22KV$

$$Z_{pu\,old} = \frac{9}{100} = j0.09\,pu$$

$$Z_{pu\,new} = Z_{pu\,old} \times \left(\frac{KV_{b\,old}}{KV_{b\,new}}\right)^2 \left(\frac{MVA_{b\,new}}{MVA_{b\,old}}\right)$$

$$Z_{pu\,new}\,G_1 = j0.09 \times \left(\frac{22}{22}\right)^2 \left(\frac{100}{80}\right)$$

$$\boxed{Z_{pu}\text{ new of }G_1 = j0.1125\,pu}$$

Impedance of T_1 referred to primary

$KV_{b\,old} = 22\,KV$

$KV_{b\,new} = 22\,KV$

$MVA_{b\,new} = 100\,MVA$

$MVA_{b\,old} = 50\,MVA$

$$Z_{pu\,old} = \frac{10}{100} = j.10\,pu$$

$$Z_{pu\,new}\,T_1 = j0.10 \times \left(\frac{22}{22}\right)^2 \times \left(\frac{100}{50}\right)$$

$$\boxed{Z_{pu\,new}\,T_1 = j0.2\,pu}$$

Impedance of line 1 (220KV line)

$$KV_{b\,new} \text{ on HT side of } T_1 = KV_{b\,new} \text{ on LT side} \times \frac{\text{HT rating}}{\text{LT rating}}$$

$$= \frac{22 \times 220}{22} = 220\,KV$$

now $KV_{b\,new} = 22$

$MVA_{b\,new} = 100\,MVA$

$$\text{Base impedance} = \frac{(KV_{b\,new})^2}{MVA_{b\,new}}$$

$$Z_b = \frac{(220)}{100} = 484\,\Omega$$

Actual value $= j121\,\Omega$

$$Z_{pu\,new} \text{ of Line 1} = \frac{\text{Actual value}}{\text{Base value}}$$

$$Z_{pu\,new} \text{ Line 1} = \frac{j121}{484}$$

$$\boxed{Z_{pu\,new} \text{ line} = j0.25\,pu}$$

Impedance of T_2 with referred to secondary

KV_bnew on LT side of T_2 = KV_bnew on HT side × $\frac{\text{LT rating}}{\text{HT rating}}$

$$= \frac{220 \times 22}{220} \quad 22KV$$

now $KV_b new = 22KV$

$KV_b old = 22KV$

$MVA_b new = 100MVA$

$MVA_b old = 40MVA$

$$Z_{pu} old = \frac{6}{100} = j0.06\ pu$$

$$Z_{pu}\ new\ T_2 = j0.06 \times \left(\frac{22}{22}\right)^2 \times \left(\frac{100}{40}\right)$$

$$\boxed{Z_{pu}\ new\ T_2 = j0.15pu}$$

Impedance of M

$KV_b new = 22KV$

$KV_b old = 20KV$

$MVA_b old = 68.85\ MVA$

$MVA_b new = 100MVA$

$$Z_{pu}\ old = \frac{22.5}{100} = j0.225pu$$

$$Z_{pu}\ new\ of\ M = j0.225 \times \left(\frac{20}{22}\right)^2 \times \left(\frac{100}{68.85}\right)$$

$$\boxed{Z_{pu}\ new\ of\ M = j0.2700pu}$$

Impedance of T_3 referred to primary

$KV_{b\,old} = 22KV$

$KV_{b\,new} = 22KV$

$MVA_{b\,old} = 40MVA$

$MVA_{b\,new} = 100MVA$

$$Z_{pu\,old} = \frac{6.4}{100} = j0.064\,pu$$

$$Z_{pu\,new}\,T_3 = j0.064 \times \left(\frac{22}{22}\right)^2 \times \left(\frac{100}{40}\right)$$

$$\boxed{Z_{pu\,new}\,T_3 = j0.16\,pu}$$

Impedance of line 2 (110KV line)

$$KV_{b\,new} \text{ on HT side of } T_3 = KV_{b\,new} \text{ on LT side } T_3 \times \frac{\text{HT rating}}{\text{LT rating}}$$

$$= \frac{22 \times 110}{22} = 110KV$$

now $KV_{b\,new} = 110KV$

$MVA_{b\,new} = 100MVA$

$$\text{Base impedance} = \frac{(KV_{b\,new})^2}{MVA_{b\,new}} = \frac{(110)^2}{100} = 121\Omega$$

$Z_{actual} = j42.35\Omega$

$$Z_{pu\,new} \text{ of line 2} = \frac{\text{Actual Value}}{\text{Base impedance}} = \frac{j42.35}{121}$$

$$\boxed{Z_{pu\,new}\text{ line 2} = j0.35\,pu}$$

Impedance of T_4 referred to primary

$KV_{bnew} = 110KV$

$KV_{bold} = 110KV$

$MVA_{bold} = 40MVA$

$MVA_{bnew} = 100MVA$

$Z_{pu\,old} = \frac{6.4}{100} = j0.064pu$

$$Z_{pu\,new}\ T_4 = j0.064 \times \left(\frac{110}{110}\right)^2 \times \left(\frac{100}{40}\right)$$

$$\boxed{Z_{pu\,new}\text{ of }T_4 = j0.16pu}$$

Impedance of load

$Q = 10MVar$

$V_{LL} = 4KV$ Since Δ load $P = 0$

$$Z_L = \frac{3V_{LL}^2}{P - jQ}$$

$$= \frac{3 \times 4^2}{0 - j10} = 4.8j$$

$$\text{Base impedance} = \frac{(KV_{bnew})^2}{MVA_{bnew}}$$

$$KV_{bnew}\text{ on LT side of }T_4 = KV_b\text{ on HT side} \times \frac{\text{LT rating}}{\text{HT rating}}$$

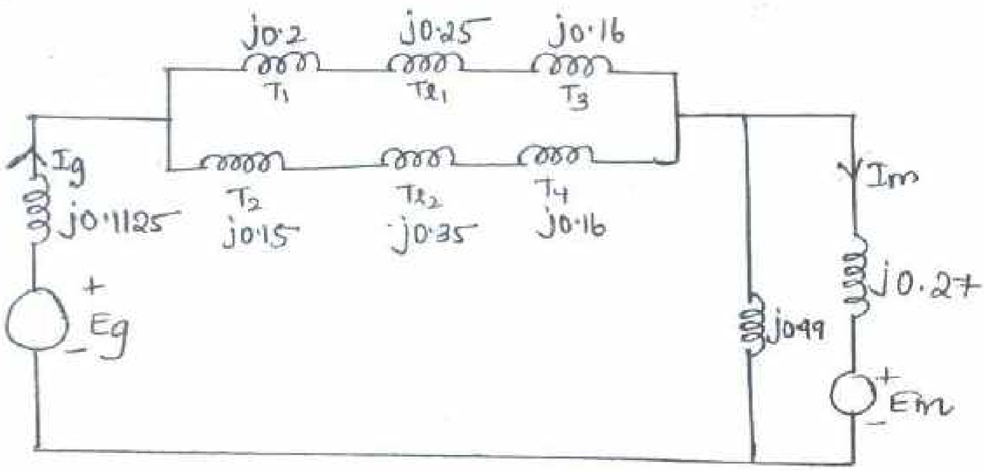
$$= 110 \times \frac{22}{110} = 22KV$$

now $KV_{bnew} = 22KV$

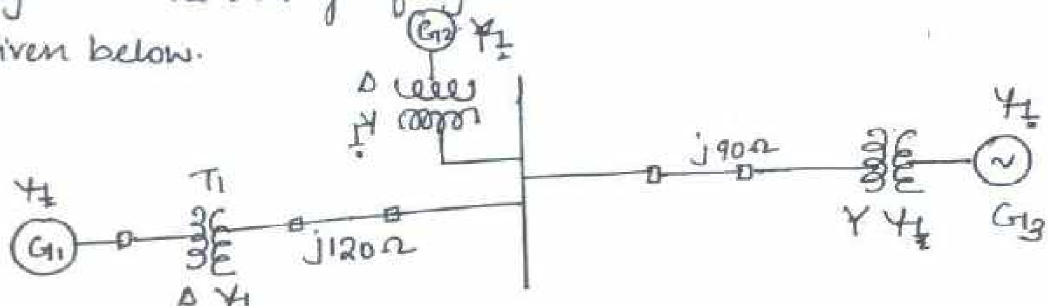
$$Z_b = \frac{(22)^2}{100} = 4.84j$$

$$Z_{pu\,new}\text{ of load} = \frac{4.8j}{4.84} = \boxed{j0.991pu}$$

Impedance Diagram:-



4. The single line diagram of unloaded three generator power system with interconnection between the generators by means of three transformers and a transmission line with two sections with their impedances marked on the diagram. The ratings of generators and transformers are given below.



(Nov/Dec 2013)

Generator	MVA	KV	Reactance in pu
1	25	6.6	0.2
2	15	6.6	0.15
3	30	13.2	0.15

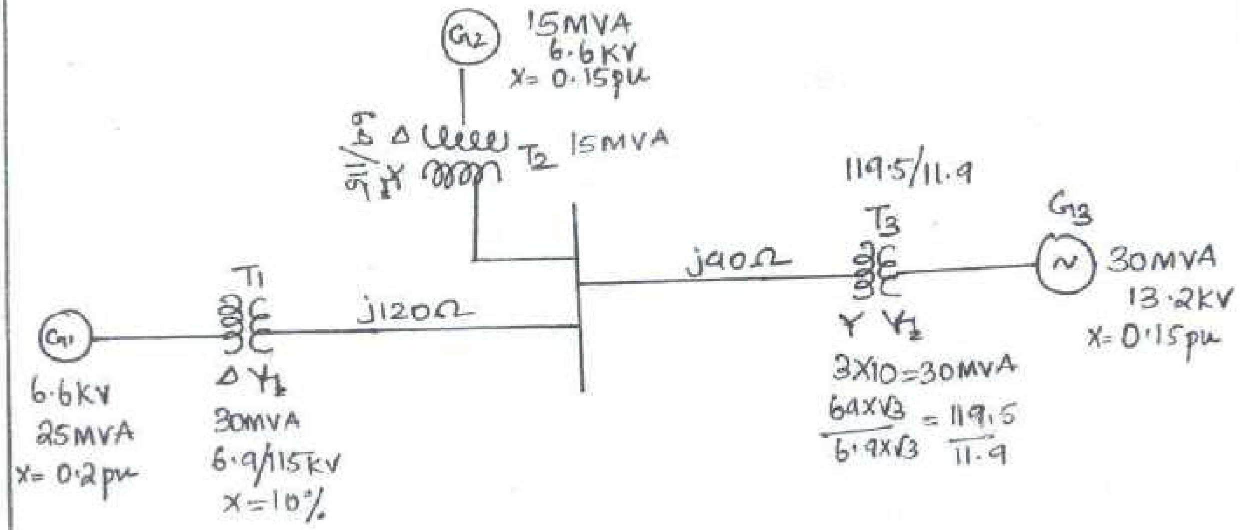
Transformer 1: 30 MVA, 6.9Δ – 115 Y KV, X = 10%

Transformer 2: 15 MVA, 6.9Δ – 115 Y KV, X = 10%

Transformer 3 : Single phase units each rated 10 MVA, 6.9/69 KV, X = 10%

Draw an impedance diagram. Choosing a base 30 MVA, 6.6 KV in G1 circuit

Soln:-



$MVA_{b\,new} = 30\ MVA$

$KV_{b\,new} = 6.6\ KV$

Impedance of G_1

$$Z_{pu\,new} = Z_{pu\,old} \times \left(\frac{KV_{b\,old}}{KV_{b\,new}}\right)^2 \left(\frac{MVA_{b\,new}}{MVA_{b\,old}}\right)$$

$$Z_{pu\,new}\ G_1 = j0.2 \times \left(\frac{6.6}{6.6}\right)^2 \times \left(\frac{30}{25}\right)$$

$KV_{b\,new} = 6.6KV$
$KV_{b\,old} = 6.6KV$
$MVA_{b\,old} = 25MVA$
$MVA_{b\,new} = 30MVA$
$Z_{pu\,old} = j0.2pu$

$$\boxed{Z_{pu\,new}\ G_1 = j0.24pu}$$

Impedance of T_1 referred to Secondary

$$KV_{b\,new}\text{ on HT side of } T_1 = KV_{b\,new}\text{ on LT of } T_1 \times \frac{\text{HT rating}}{\text{LT rating}}$$

$$= 6.6 \times \frac{115}{6.9} = 110KV$$

now $KV_{b\,new} = 110KV$; $KV_{b\,old} = 115KV$

$MVA_{b\,new} = 30MVA$; $MVA_{b\,old} = 30MVA$

$Z_{pu\,old} = \frac{10}{100} = j0.1pu$

$$Z_{pu\,new}\ T_1 = j0.1 \times \left(\frac{115}{110}\right)^2 \times \left(\frac{30}{30}\right)$$

$$\boxed{Z_{pu\,new} \text{ of } T_1 = j0.1092\,pu}$$

Impedance of line 1 (j120Ω line)

$KV_{b\,new} = 110\,KV$

$MVA_{b\,new} = 30\,MVA$

Base impedance $Z_b = \dfrac{(KV_{b\,new})^2}{MVA_{b\,new}} = \dfrac{(110)^2}{30} = 403.33\,\Omega$

Actual value $= j120\,\Omega$

$$Z_{pu_{new}} \text{ of line 1} = \frac{\text{Actual value}}{\text{Base value}} = \frac{j120}{403.33}$$

$$\boxed{Z_{pu\,new} \text{ of Line 1} = j0.297\,pu}$$

Impedance of line 2 (j90Ω line)

$KV_{b\,new} = 110\,KV$

$MVA_{b\,new} = 30\,MVA$

$$Z_b = \frac{(110)^2}{30} = 403.33\,\Omega$$

Actual value $= j90\,\Omega$

$$Z_{pu\,new} \text{ of line 2} = \frac{j90}{403.33}$$

$$\boxed{Z_{pu\,new} \text{ of line 2} = j0.2231\,pu}$$

Impedance of T_2

$KV_{b\,new} = 110\,KV$; $KV_{b\,old} = 115\,KV$

$MVA_{b\,new} = 30\,MVA$; $MVA_{b\,old} = 15\,MVA$

$$Z_{pu\,old} = \frac{10}{100} = j0.1\,pu$$

$$Z_{pu} \text{ new of } T_2 = j0.1 \times \left(\frac{115}{110}\right)^2 \times \left(\frac{30}{15}\right)$$

$$\boxed{Z_{pu} \text{ new of } T_2 = j0.2185 \text{ pu}}$$

Impedance of G_{12}

KV_b new on LT side of T_2 = KV_b new HT side of T_2 × $\frac{\text{LT rating}}{\text{HT rating}}$

$$= 110 \times \frac{6.9}{115} = 6.6 \text{ KV}$$

now KV_b new = 6.6 KV

KV_b old = 6.6 KV

MVA_b old = 15 MVA

MVA_b new = 30 MVA

Z_{pu} old = j0.15 pu

$$Z_{pu} \text{ new of } G_{12} = j0.15 \times \left(\frac{6.6}{6.6}\right)^2 \times \left(\frac{30}{15}\right)$$

$$\boxed{Z_{pu} \text{ new of } G_{12} = j0.3 \text{ pu}}$$

Impedance of T_3 referred to secondary

Since it is formed 3, 1ϕ transformer, voltag rating of 3ϕ transformer

$$= \frac{\sqrt{3} \times 69}{\sqrt{3} \times 6.9} = 119.5/11.9 \text{ KV}$$

MVA rating of 3ϕ transformer = 3×10 = 30 MVA

KV_b new on LT side of T_3 = KV_b new on HT side × $\frac{\text{LT rating}}{\text{HT rating}}$

$$= 110 \times \frac{11.9}{119.5} = 10.9 \simeq 11 \text{ KV}$$

now KV_b new = 11 KV ; MVA_b new = 30 MVA

KV_b old = 11.9 KV MVA_b old = 30 MVA

$$Z_{pu\,old} = \frac{10}{100} = j0.1\,pu$$

$$Z_{pu\,new} \text{ of } T_3 = j0.1 \times \left(\frac{11.9}{11}\right)^2 \times \left(\frac{30}{30}\right)$$

$$\boxed{Z_{pu\,new} \text{ of } T_3 = j0.117\,pu}$$

Impedance of G_3

$KV_{b\,old} = 13.2KV$; $KV_{b\,new} = 11KV$

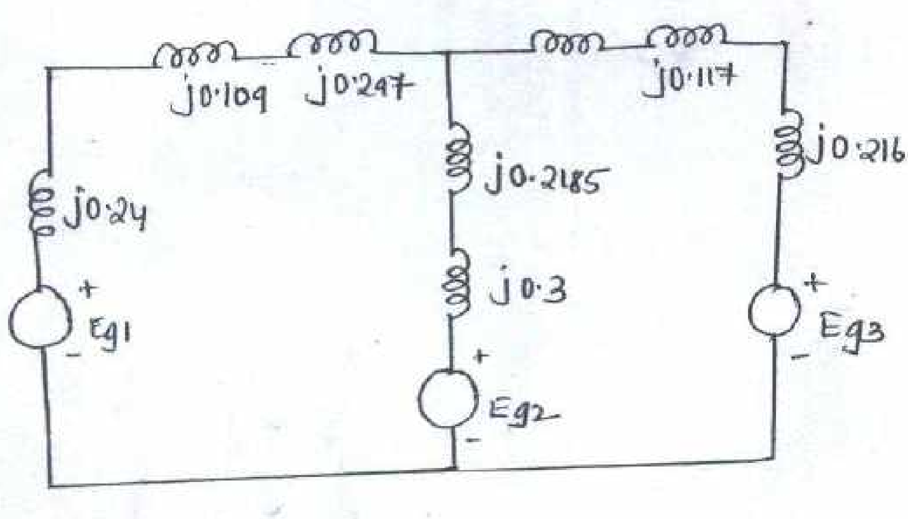
$MVA_{b\,old} = 30MVA$; $MVA_{b\,new} = 30MVA$

$Z_{pu\,old} = j0.15pu$

$$Z_{pu\,new} \text{ of } G_3 = j0.15 \times \left(\frac{13.2}{11}\right)^2 \times \left(\frac{30}{30}\right)$$

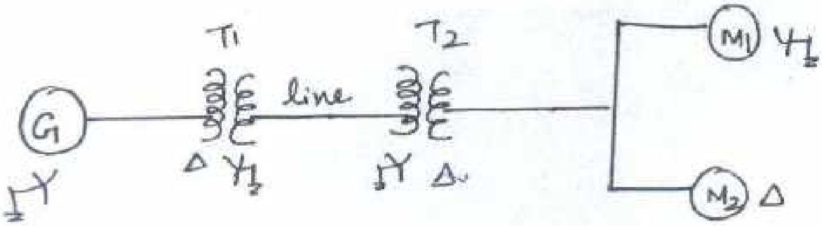
$$\boxed{Z_{pu\,new} \text{ of } G_3 = j0.216\,pu}$$

Impedance Diagram :-

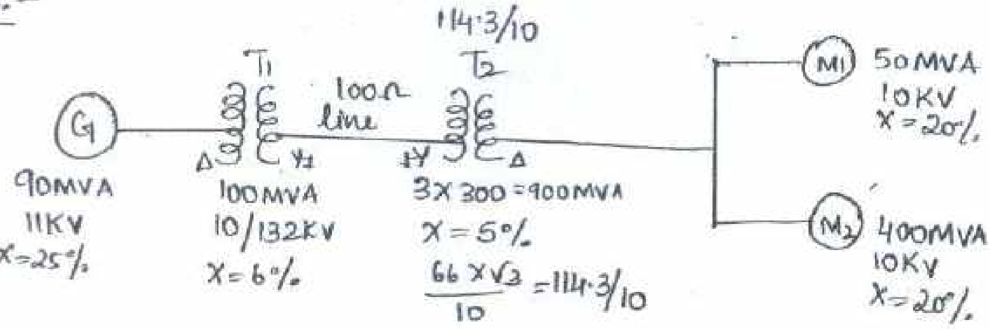


5) A 90MVA 11KV 3 phase generator has a reactance of 25%. The generator supplies two motors through transformer and transmission line as shown in fig. The transformer T_1 is a 3 phase transformer, 100MVA, 10/132KV, 6% reactance. The transformer T_2 is composed of 3 single phase units each rated 300MVA, 66/10KV with 5% reactance. The connection of T_1 and T_2 are shown. The motors are rated at 50MVA and 400MVA both 10KV and 20% reactance. Taking the generator rating as base, draw reactance diagram & indicate the reactance in p.u. The reactance of line is 100Ω

(Nov/Dec 2013)



Soln:-



KV_b new = 11KV

MVA_b new = 90MVA

Reactance of G

$KV_{b new} = 11KV$; $KV_{b old} = 11KV$; $MVA_{b new} = 90MVA$

$MVA_{b old} = 90MV$

$X_{pu old} = \frac{25}{100} = j0.25pu$

$X_{pu new}$ of $G = j0.25 \times \left(\frac{11}{11}\right)^2 \times \left(\frac{90}{90}\right)$

$$\boxed{X_{pu new} \text{ of } G = j0.25pu}$$

Reactance of T_1 referred to primary

$KV_{b new} = 11KV$; $KV_{b old} = 10KV$; $MVA_{b old} = 100MVA$; $MVA_{b new} = 90MVA$

$X_{pu old} = \frac{6}{100} = j0.06pu$

$X_{pu new}$ of $T_1 = j0.06 \times \left(\frac{10}{11}\right)^2 \times \left(\frac{90}{100}\right)$

$$\boxed{X_{pu new} \text{ of } T_1 = j0.0446pu}$$

Reactance of line (100Ω line)

$KV_{b new}$ on HT side of $T_1 = KV_{b new}$ on LT side of $T_1 \times \frac{\text{HT rating}}{\text{LT rating}}$

$$= 11 \times \frac{132}{10} = 145.2KV$$

now $KV_{b new} = 145.2KV$

Base reactance $X_b = \frac{(KV_{b new})^2}{MVA_{b new}}$

$$= \frac{(145.2)^2}{90} = 234.25\Omega$$

Actual value $= 100\Omega$

$$X_{pu} \text{ of line} = \frac{\text{Actual value}}{\text{Base value}} = \frac{100}{234.25}$$

$$\boxed{X_{pu} \text{ new of line} = j0.426 \text{ pu}}$$

Reactance of T_2 referred to secondary

Since T_2 is formed by 3, 1ϕ transformers.

$$\text{voltag rating of } T_2 = \frac{66 \times \sqrt{3}}{10} = 114.3/10$$

MVA rating of 3ϕ transformer $= 3 \times 300 = 900$ MVA

$$KV_{bnew} \text{ on LT of } T_2 = KV_{bnew} \text{ on HT side} \times \frac{\text{LT rating}}{\text{HT rating}}$$

$$= 145.2 \times \frac{10}{114.3} = 12.7 \text{ KV}$$

now $KV_{bnew} = 12.7$ KV

$KV_{bold} = 10$ KV

$MVA_{bnew} = 90$ MVA

$MVA_{bold} = 900$ MVA

$$X_{pu\,old} = \frac{5}{100} = j0.05 \text{ pu}$$

$$X_{pu\,new} \text{ of } T_2 = j0.05 \times \left(\frac{10}{12.7}\right)^2 \times \left(\frac{90}{900}\right)$$

$$\boxed{X_{pu} \text{ new of } T_2 = j0.0031 \text{ pu}}$$

Reactance of M_1

$KV_{bnew} = 12.7$ KV ; $KV_{bold} = 10$ KV

$MVA_{bnew} = 90$ MVA ; $MVA_{bold} = 50$ MVA

$$X_{pu\,old} = \frac{20}{100} = j0.2 \text{ pu}$$

$$X_{pu}\text{ new of } M_1 = j0.2 \times \left(\frac{10}{12.7}\right)^2 \times \left(\frac{90}{50}\right)$$

$$\boxed{X_{pu}\text{ new of } M_1 = j0.223\text{ pu}}$$

Reactance of M_2:-

$KV_{b\,new} = 12.7\,KV$; $KV_{b\,old} = 10\,KV$

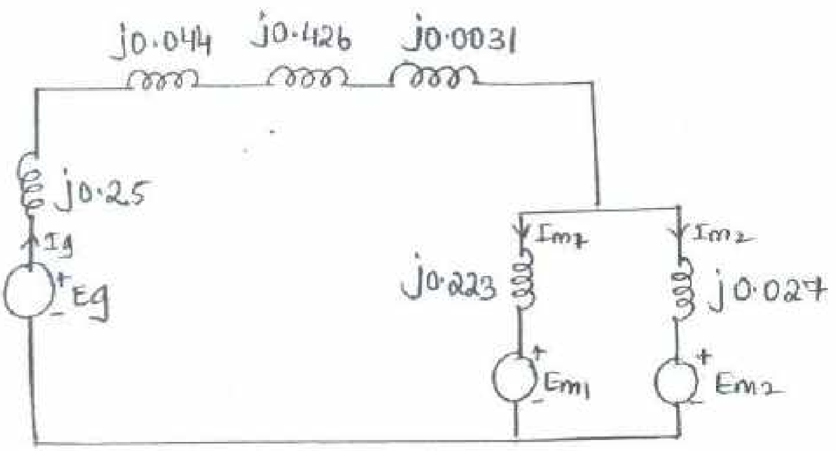
$MVA_{b\,new} = 90\,MVA$; $MVA_{b\,old} = 400\,MVA$

$$X_{pu\,old} = \frac{20}{100} = j0.2\text{ pu}$$

$$X_{pu}\text{ new of } M_2 = j0.2 \times \left(\frac{10}{12.7}\right)^2 \times \left(\frac{90}{400}\right)$$

$$\boxed{X_{pu}\text{ new of } M_2 = j0.027\text{ pu}}$$

Reactance Diagram:-



6. Prepare a per phase schematic of the system shown in figure showing all the impedance in perunit on a 100MVA, 132 KV base in the transmission line circuit. The necessary data are given as follows,

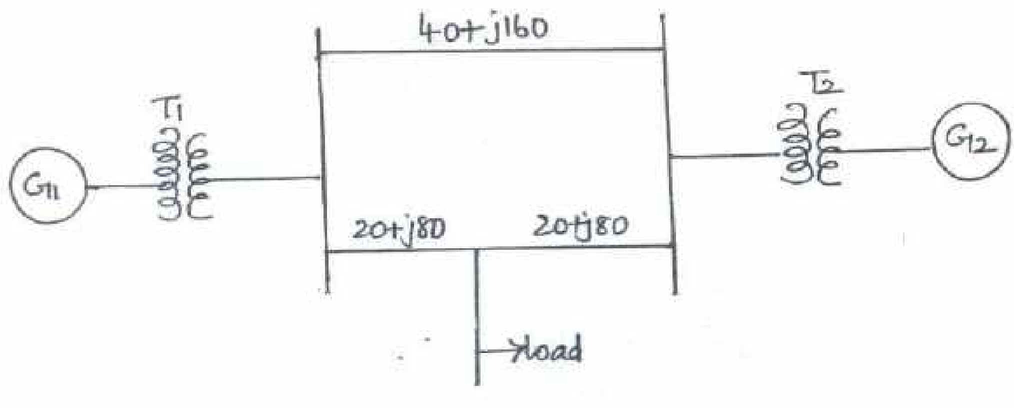
G_1 : 50MVA, 12.2KV, X=0.15pu

G_2 : 20MVA, 13.8KV, X=0.15pu

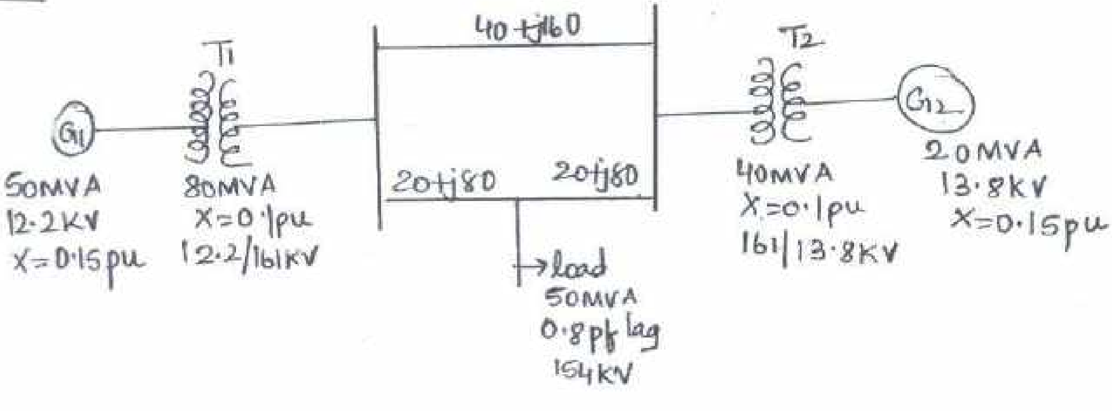
T_1 : 80MVA, 12.2/161KV, X=0.1pu

T_2 : 40MVA, 13.8/161KV, X=0.1pu

Load : 50MVA, 0.8 pf lag operating at 154KV. Determine the pu impedance of the load. [May/June 2009)



Soln:-



$MVA_{bnew} = 100\,MVA$; $KV_{bnew} = 132\,KV$ (on transmission line)

Impedance of (40+j160) transmission line

$KV_{bnew} = 132\,KV$

$MVA_{bnew} = 100\,MVA$

$$\text{Base impedance} = \frac{(KV_{bnew})^2}{MVA_{bnew}} = \frac{(132)^2}{100} = 174.24\,\Omega$$

Actual value $= 40+j160\,\Omega$

$$Z_{pu\,new} \text{ of transmission line} = \frac{\text{Actual value}}{\text{Base values}} = \frac{40+j160}{174.24}$$

$$\boxed{Z_{pu} \text{ of } (40+j160) \text{ line} = 0.229 + j0.918\ pu}$$

Impedance of (20+j80)Ω transmission line

$KV_{bnew} = 132\,KV$; $MVA_{bnew} = 100\,MVA$

Base impedance $= 174.24\,\Omega$

Actual value $= 20+j80\,\Omega$

$$Z_{pu} \text{ of } (20+j80)\Omega \text{ line} = \frac{(20+j80)}{174.24}$$

$$\boxed{Z_{pu} \text{ of } (20+j80)\Omega \text{ line} = 0.1147 + j0.459\,pu}$$

Impedance of load

$MVA = 50\,MVA = S$

$pf = \cos\phi = 0.8 \Rightarrow \phi = \cos^{-1}(0.8) = 36.86^\circ$

$V_L = 154\,KV$

$S_{L(3\phi)} = S\angle\phi = 50\angle 36.87$ $\quad S_L^* = 50\angle -36.87^\circ$

$$\text{Base impedance} = \frac{KV_{bnew}^2}{\ldots} = \frac{(132)^2}{\ldots} = 174.24\,\Omega$$

$$Z_L = \frac{V_{LL}^2}{S_L^*} = \frac{154^2}{50\angle -36.87} = 474.32\angle 36.87\ \Omega$$

$$= 379.45 + j284.59$$

$$Z_{pu} \text{ of load} = \frac{\text{Actual value}}{\text{Base impedance}} = \frac{Z_L}{Z_b}$$

$$= \frac{379.45 + j284.59}{174.24}$$

$$\boxed{Z_{pu} \text{ of load} = 2.177 + j1.633\ pu}$$

Impedance of T_1 (referred to primary)

$$KV_{bnew} \text{ on LT side of } T_1 = KV_{bnew} \text{ on HT side} \times \frac{\text{LT voltage rating}}{\text{HT voltage rating}}$$

$$= 132 \times \frac{12.2}{161} = 10KV$$

now $KV_{bnew} = 10KV$

$KV_{bold} = 12.2KV$

$MVA_{bnew} = 100MVA$

$MVA_{bold} = 80MVA$

$\therefore Z_{puold} = j0.1pu$

$$Z_{pu} \text{ new of } T_1 = Z_{puold} \times \left(\frac{KV_{bold}}{KV_{bnew}}\right)^2 \times \left(\frac{MVA_{bnew}}{MVA_{bold}}\right)$$

$$= j0.1 \times \left(\frac{12.2}{10}\right)^2 \times \left(\frac{100}{80}\right)$$

$$\boxed{Z_{punew} \text{ of } T_1 = j0.186pu}$$

Impedance of G_1

$KV_{b\,new} = 10KV$

$KV_{b\,old} = 12.2KV$

$MVA_{b\,old} = 50MVA$

$MVA_{b\,new} = 100MVA$

$Z_{pu\,old} = j0.15pu$

$$Z_{pu\,new} \text{ of } G_1 = j0.15 \times \left(\frac{12.2}{10}\right)^2 \times \left(\frac{100}{50}\right)$$

$$\boxed{Z_{pu\,new} \text{ of } G_1 = j0.446pu}$$

Impedance of T_2 referred to secondary

$$KV_{b\,new} \text{ on LT side of } T_2 = KV_{b\,new} \text{ on HT side of } T_2 \times \frac{\text{LT rating}}{\text{HT rating}}$$

$$= 132 \times \frac{13.8}{161} = 11.31KV$$

now $KV_{b\,new} = 11.31KV$

$KV_{b\,old} = 13.8KV$; $MVA_{b\,new} = 100MVA$; $MVA_{b\,old} = 40MVA$

$Z_{pu\,old} = j0.1pu$

$$Z_{pu\,new} \text{ of } T_2 = j0.1 \times \left(\frac{13.8}{11.31}\right)^2 \times \left(\frac{100}{40}\right)$$

$$\boxed{Z_{pu\,new} \text{ of } T_2 = j0.372pu}$$

Impedance of G_2

$KV_{b\,new} = 11.31KV$; $KV_{b\,old} = 13.8KV$

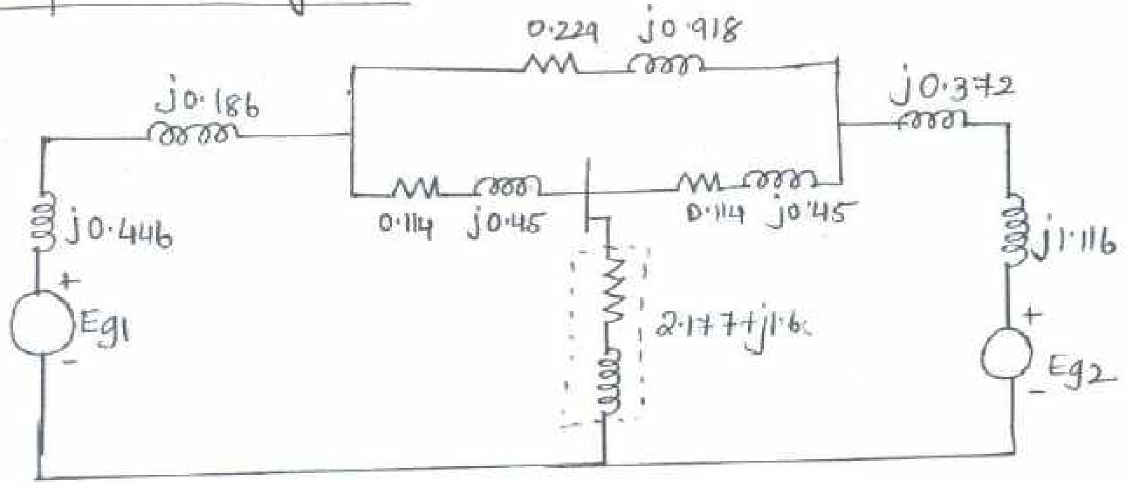
$MVA_{b\,old} = 20MVA$; $MVA_{b\,new} = 100MVA$

$Z_{pu\,old} = j0.15pu$

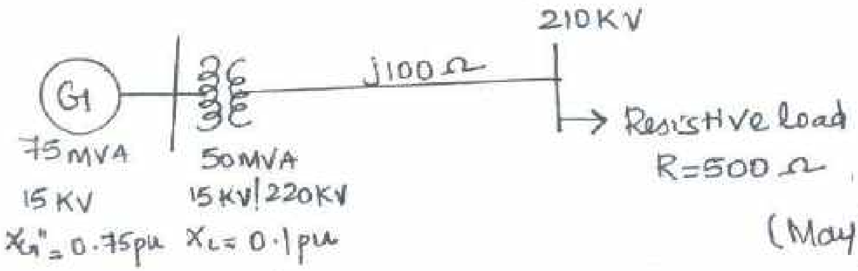
$$Z_{pu\,new} \text{ of } G_2 = j0.15 \times \left(\frac{13.8}{11.31}\right)^2 \times \left(\frac{100}{20}\right)$$

$$\boxed{Z_{pu\,new} \text{ of } G_2 = j1.118pu}$$

Impedance Diagram



7. For the system shown in figure (a) determine the generator voltage. Take a base of 100MVA and 210KV in the transmission line.



(May/June 2013)

Soln:-

$MVA_{b new} = 100MVA$

$KV_{b new} = 210KV$ (on transmission line)

Impedance of transmission line

$Z_{actual} = j100\,\Omega$

$$\text{Base impedance} = \frac{(KV_b\, new)^2}{MVA_b\, new} = \frac{(210)^2}{100} = 441\,\Omega$$

$$Z_{pu} \text{ of transmission line} = \frac{Z_{actual}}{Z_{base}} = \frac{j100}{441}$$

$$\boxed{Z_{pu} \text{ of line} = j0.226\, pu}$$

Impedance of transformer (referred to primary)

KV_b new on LT side of T = KV_b new on HT of T $\times$ $\frac{\text{LT voltage rating}}{\text{HT voltage rating}}$

$$= 210 \times \frac{15}{220} = 14.318\,KV$$

now $KV_{b\,new} = 14.318\,KV$

$KV_{b\,old} = 15\,KV$

$MVA_{b\,new} = 100\,MVA$

$MVA_{b\,old} = 50\,MVA$

$Z_{pu\,old} = j0.1\,pu$

$$Z_{pu}\text{ new of } T = j0.1 \times \left(\frac{15}{14.3}\right)^2 \times \left(\frac{100}{50}\right)$$

$$\boxed{Z_{pu}\text{ new of } T = j0.220\,pu}$$

Impedance of Generator

$KV_{b\,new} = 14.31$ $\qquad$ $MVA_{b\,new} = 100\,MVA$

$KV_{b\,old} = 15\,KV$ $\qquad$ $MVA_{b\,old} = 75\,MVA$

$Z_{pu\,old} = j0.75\,pu$

$$Z_{pu}\text{ new of } G_1 = j0.75 \times \left(\frac{15}{14.31}\right)^2 \times \left(\frac{100}{75}\right)$$

$$\boxed{Z_{pu}\text{ new of } G_1 = j1.098\,pu}$$

Impedance of load:-

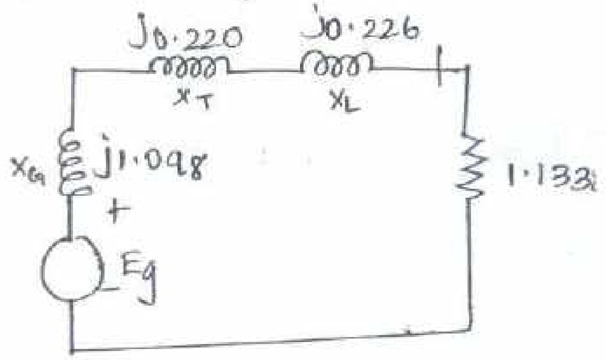
$KV_{b\,new} = 210\,KV$ $\qquad$ $MVA_{b\,new} = 100\,MVA$

$$\therefore Z_b = \frac{(KV_{b\,new})^2}{MVA_{b\,new}} = \frac{(210)^2}{100} = 441\,\Omega$$

$Z_{actual} = 500\,\Omega$

$$Z_{pu}\text{ of R load} = \frac{500}{441} \Rightarrow \boxed{Z_{pu}\text{ of R load} = 1.133\,pu}$$

Impedance diagram:-



At the load bus

$$V_{pu} = 1 pu$$

$$R_{pu} = 1.1338$$

$$\therefore I_{pu} = \frac{V_{pu}}{R_{pu}} = \frac{1}{1.133} = 0.882 pu$$

let Ipu taken as the reference phasor

$$I_{pu} = 0.882\angle 0^\circ = 0.882 + j0$$

By KVL, the voltage drop in the network

$$V_{pu} = I_{pu}[R_{pu} + j(X_G + X_T + X_L)]$$

$$= 0.882[1.133 + j(1.098 + 0.220 + 0.226)]$$

$$= 0.882[1.133 + j1.544]$$

$$= 0.882 \times 1.915\angle 53.72$$

$$V_{pu} = 1.689 pu$$

Actual generator terminal voltage,

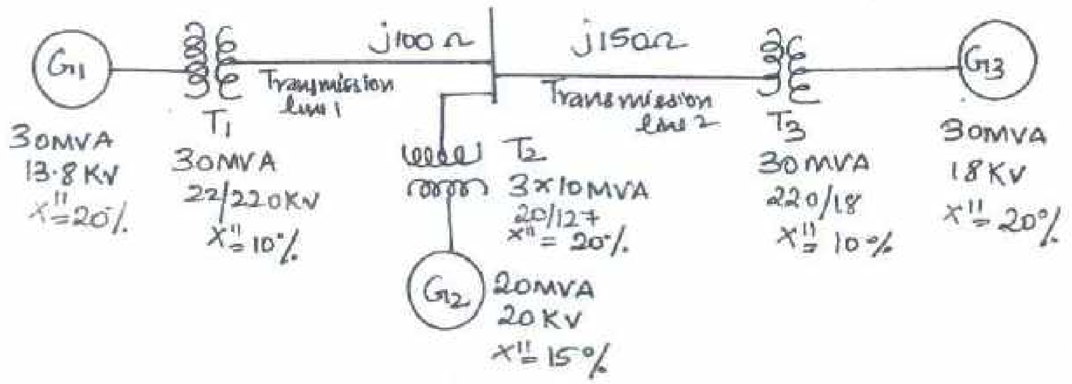
V_G = V_{pu} x Base KV at generator terminals

V_G = $V_{pu} \times KV_b$ new at generator

$= 1.689 \times 14.31$

$$V_G = 24.16 KV$$

8. The Single line diagram of a power system is shown in fig along with the components data. Determine the new per unit value and draw the reactance diagram. Assume 25MVA and 20KV as new base on generator G_1



Soln:-

$KV_{bnew} = 20KV$

$MVA_{bnew} = 25MVA$

Reactance of G_1

$KV_{bnew} = 20KV$

$KV_{bold} = 13.8KV$

$MVA_{bold} = 30MVA$

$MVA_{bnew} = 25MVA$

$$X_{pu\,old} = \frac{20}{100} = j0.2pu$$

$$X_{pu\,new} \text{ of } G_1 = X_{pu\,old} \times \left(\frac{KV_{bold}}{KV_{bnew}}\right)^2 \times \left(\frac{MVA_{bnew}}{MVA_{bold}}\right)$$

$$= j0.2 \times \left(\frac{13.8}{20}\right)^2 \times \left(\frac{25}{30}\right)$$

$$\boxed{X_{pu\,new} \text{ of } G_1 = j0.0793pu}$$

Reactance of T_1 referred to secondary

KV_b new on HT side of T_1 = KV_b new on LT side of $T_1 \times \frac{\text{HT rating}}{\text{LT rating}}$

$$= 20 \times \frac{220}{22} = 200\,KV$$

Now $KV_b new = 200KV$

$KV_{b old} = 220KV$

$MVA_{b old} = 30MVA$

$MVA_{b new} = 25MVA$

$X_{pu\,old} = \frac{10}{100} = 0.1j$

$$X_{pu\,new} \text{ of } T_1 = j0.1 \times \left(\frac{220}{200}\right)^2 \times \left(\frac{25}{30}\right)$$

$$\boxed{X_{pu\,new} \text{ of } T_1 = j0.1\,pu}$$

Reactance of transmission line 1 :-

$KV_b new = 200KV$

$MVA_b new = 25MVA$

$X_{actual} = j100\Omega$

Base reactance $X_b = \frac{(KV_b new)^2}{MVA_b new} = \frac{(200)^2}{25} = 1600\,\Omega$

$$X_{pu} \text{ of line } 1 = \frac{\text{Actual value}}{\text{Base value}} = \frac{X_{actual}}{X_b} = \frac{j100}{1600}$$

$$\boxed{X_{pu} \text{ of line } 1 = j0.0625\,pu}$$

Reactance of transmission line 2

$KV_b new = 200KV$; $MVA_b new = 25MVA$; $X_{actual} = j150$

$X_b = \frac{(200)^2}{25} = 1600\Omega$

X_{pu} of line $2 = \frac{j150}{1600}$

$$\boxed{X_{pu} \text{ of line } 2 = j0.093\,pu}$$

Reactance of T_3 referred to secondary

KV_{bnew} on LT side of T_3 = KV_{bnew} on HT side of T_3 × $\frac{\text{LT rating}}{\text{HT rating}}$

$= 200 \times \frac{18}{220} = 16.36\,KV$

now $KV_{bnew} = 16.36\,KV$

$KV_{bold} = 18\,KV$

$MVA_{bnew} = 25\,MVA$

$MVA_{bold} = 30\,MVA$

$X_{pu\,old} = \frac{10}{100} = j0.10\,pu$

$X_{pu\,new}$ of $T_3 = j0.1 \times \left(\frac{18}{16.36}\right)^2 \times \left(\frac{25}{30}\right)$

$X_{pu\,new}$ of $T_3 = j0.1\,pu$

Reactance of G_3

$KV_{bnew} = 16.36\,KV$; $KV_{bold} = 18\,KV$

$MVA_{bnew} = 25\,MVA$; $MVA_{bold} = 30\,MVA$

$X_{puold} = \frac{20}{100} = j0.2\,pu$

$X_{pu\,new}$ of $G_3 = j0.2 \times \left(\frac{18}{16.36}\right)^2 \times \left(\frac{25}{30}\right)$

$X_{pu\,new}$ of $G_3 = j0.2017\,pu$

Reactance of T_2 referred to Secondary

Since it is formed by 3 single phase transformer

voltage rating of 3 phase transformer $= \frac{127 \times \sqrt{3}}{20} = \frac{220}{20}$

MVA rating of 3 phase transformer $= 3 \times 10 = 30\,MVA$

KV_{bnew} on LT side of T_2 = $KV_{b\,new}$ on HT side × $\frac{\text{LT rating}}{\text{HT rating}}$

$= 200 \times \frac{20}{220} = 18.18\,KV$

new $KV_{bnew} = 18.18\,KV$

$MVA_{bold} = 30\,MVA$

$MVA_{bnew} = 25\,MVA$

$KV_{bold} = 20\,KV$

$X_{pu\,old} = \frac{20}{100} = j0.2\,pu$

$X_{pu\,new}$ of $T_2 = j0.2 \times \left(\frac{20}{18.18}\right)^2 \times \left(\frac{25}{30}\right)$

$X_{pu\,new}$ of $T_2 = j0.2017\,pu$

Reactance of G_2

$KV_{bnew} = 18.18\,KV$ $MVA_{bnew} = 25\,MVA$

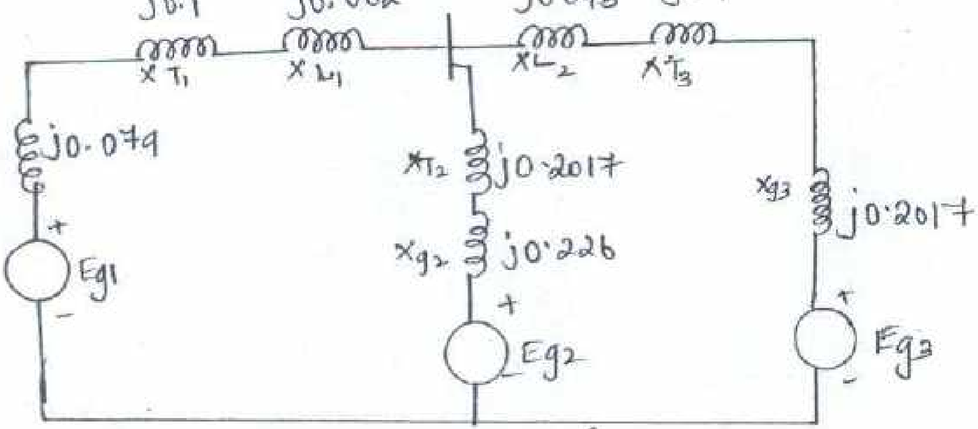
$KV_{bold} = 20\,KV$ $MVA_{bold} = 20\,MVA$

$X_{pu\,old} = \frac{15}{100} = j0.15\,pu$

$X_{pu\,new}$ of $G_2 = j0.15 \times \left(\frac{20}{18.18}\right)^2 \times \left(\frac{25}{20}\right)$

$X_{pu\,new}$ of $G_2 = j0.226\,pu$

Reactance diagram



9. Obtain per unit impedance diagram of the power system of fig shown below

Generator No 1 : 30 MVA, 10.5KV, $X'' = 1.6$ ohms

Generator No 2 : 15 MVA, 6.6KV, $X'' = 1.2$ ohms

Generator No : 3 : 25 MVA, 6.6KV, $X'' = 0.56$ ohms

Transformer T_1 (3 phase) : 15MVA, 33/11 KV, X = 15.2 ohm per phase on high tension side

Transformer T_2 (3 phase) : 15MVA, 33/6.2 KV, X = 16 ohms per phase on high tension side

Transmission line : 20.5 ohms/phase

Load A : 15MW, 11KV, 0.9 lagging power factor

Load B : 40MW, 6.6KV, 0.85 lagging power factor

Soln:-

High tension of T_1 and T_2 is specified, so start from transmission line

$MVA_{b\,new} = 30\ MVA$ (High rating of machine)

$KV_{b\,new} = 10.5 KV$

Impedance of Generators

$MVA_{b\,new} = 30 MVA$

$KV_{b\,new} = 10.5 KV$

Actual impedance = 1.6 ohms.

Base Impedance $= \dfrac{(KV_{bnew})^2}{MVA_{bnew}} = \dfrac{(10.5)^2}{30} = 3.675\,\Omega$

$Z_{punew} = \dfrac{\text{Actual impedance}}{\text{Base impedance}}$

Z_{punew} of $G_1 = \dfrac{j1.6}{3.675}$

Z_{punew} of $G_1 = j0.435$ pu

Impedance of T_1 (referred to Secondary)

KV_{bnew} on HT of T_1 = KV_{bnew} on LT of $T_1 \times \dfrac{\text{HT rating}}{\text{LT rating}}$

$= 10.5 \times \dfrac{33}{11} = 31.5$ KV

now $KV_{bnew} = 31.5$ KV

Actual impedance of T_1 on HT $= 15.2\,\Omega/ph$

Base impedance $= \dfrac{(KV_{bnew})^2}{MVA_{bnew}} = \dfrac{(31.5)^2}{30} = 33.07$

Z_{pu} of $T_1 = \dfrac{\text{Actual value}}{\text{Base value}} = \dfrac{j15.2}{33.07}$

Z_{pu} of $T_1 = j0.4595$ pu

Impedance of Transmission line

$KV_{bnew} = 31.5$ KV

MVA_{b} new $= 30$ MVA

Actual impedance $= 20.5\,\Omega/ph$

Base impedance $Z_b = \dfrac{(31.5)^2}{30} = 33.07\,\Omega$

Z_{pu} of $TL = \dfrac{j20.5}{33.07}$

Z_{pu} of $TL = j0.6198$

Impedance of T_2 referred to primary

$KV_{b\,new} = 31.5\,KV$

$MVA_{b\,new} = 30\,MVA$

Z_{actual} on HT Side $= 16\,\Omega/ph$

$$Z_b = \frac{(KV_{b\,new})^2}{MVA_{b\,new}} = \frac{(31.5)^2}{30} = 33.07\,\Omega$$

$$Z_{pu} \text{ of } T_2 = \frac{Z_{actual}}{Z_b} = \frac{j16}{33.07}$$

$$\boxed{Z_{pu} \text{ of } T_2 = j0.4838\,pu}$$

Impedance of G_{12}

$$KV_{b\,new} \text{ on LT side of } T_2 = KV_{b\,new} \text{ on HT side of } T_2 \times \frac{\text{LT rating}}{\text{HT rating}}$$

$$= 31.5 \times \frac{6.2}{33} = 6\,KV$$

now $KV_{b\,new} = 6\,KV$

$MVA_{b\,new} = 30\,MVA$

$Z_{actual} = 1.2\,\Omega$

$$Z_b = \frac{(KV_{b\,new})^2}{MVA_{b\,new}} = \frac{6^2}{30} = 1.2\,\Omega$$

$$Z_{pu} \text{ of } G_{12} = \frac{Z_{actual}}{Z_b} = \frac{j1.2}{1.2} = j1\,pu$$

$$\boxed{Z_{pu} \text{ of } G_{12} = j1\,pu}$$

Impedance of G13

$KV_{bnew} = 6KV$

$MVA_{bnew} = 30MVA$

$Z_{actual} = 0.56\Omega$

Base impedance $Z_b = \frac{(KV_{bnew})^2}{MVA_{bnew}} = \frac{6^2}{30} = 1.2\Omega$

Z_{pu} of $G_3 = \frac{Z_{actual}}{Z_b} = \frac{j0.56}{1.2} = j0.466pu$

$\boxed{Z_{pu} \text{ of } G_3 = j0.466pu}$

Impedance of load A

$KV_{bnew} = 10.5KV$

$MVA_{bnew} = 30MVA$

$P = 15MW$

$\cos\phi = 0.9 \quad \phi = \cos^{-1}(0.9) = 25.84$

$P_L = S\cos\phi$

$S = \frac{P_L}{\cos\phi}$ — (1)

$Q_L = S\cos\phi$

$S = \frac{Q_L}{\sin\phi}$ — (2)

from (1) & (2)

$\frac{P_L}{\cos\phi} = \frac{Q_L}{\sin\phi}$

$Q_L = \frac{P_L \times \sin\phi}{\cos\phi} = \frac{P_L \times \sin(25.84)}{0.8}$

$= \frac{15 \times \sin(25.84)}{0.8}$

$Q_L = 8.17 MVar$

$$Z_L = \frac{V_L^2}{P_L - jQ_L} = \frac{11^2}{15 - j8.17} = 6.22 + j3.388$$

$$Z_b = \frac{KV_{b\,new}^2}{MVA_{b\,new}} = \frac{(10.5)^2}{30} = 3.675$$

$$Z_{pu} \text{ of load A} = \frac{Z_L}{Z_b} = \frac{6.22 + j3.338}{3.675}$$

$$\boxed{Z_{pu} \text{ of load A} = 1.69 + j0.90 \text{ pu}}$$

Impedance of load B.

$KV_{b\,new} = 6KV$ $\quad MVA_{b\,new} = 30$

$$Z_b = \frac{6^2}{30} = 1.2$$

$$Z_L = \frac{V_L^2}{P_L - jQ_L}$$

$P_L = 40MW$ $\quad \cos\phi = 0.85 \Rightarrow \phi = \cos^{-1}(0.85) = 31.7$

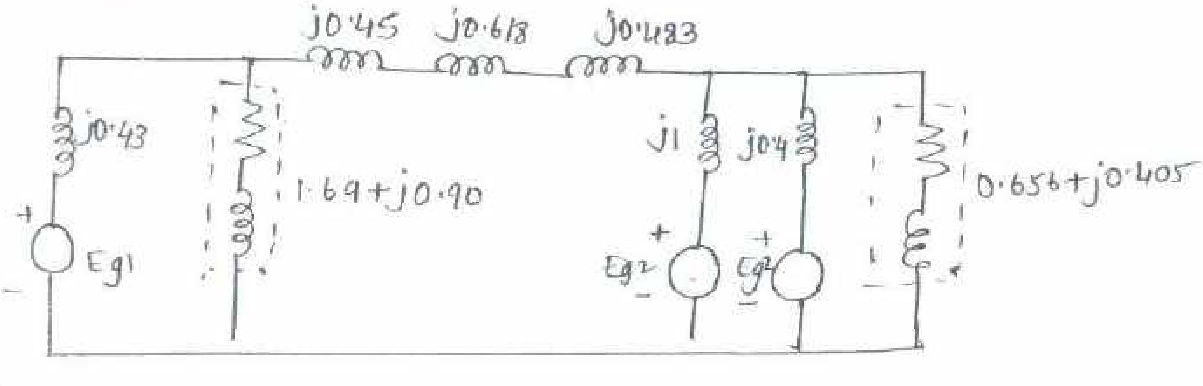
$$Q_L = \frac{P_L \times \sin\phi}{\cos\phi} = \frac{40 \times \sin(31.7)}{0.85} = 24.72 MVar$$

$$Z_L = \frac{(6.6)^2}{40 - j24.72} = 0.788 + j0.487$$

$$Z_{pu} \text{ of load B} = \frac{Z_L}{Z_b} = \frac{0.788 + j0.487}{1.2}$$

$$\boxed{Z_{pu} \text{ of load B} = 0.656 + j0.405 \text{ pu}}$$

Impedance Diagram



10) The subtransient reactance of a 500MVA, 18 KV generator is 0.25 pu on its ratings. It is connected to a network through a 20/400 KV transformer. Find out the subtranseint reactance of a generator on a base of 100MVA and 20KV

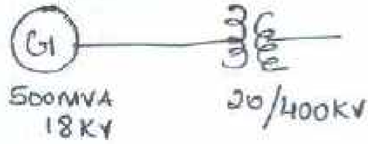
(April/May 2010)

Soln:-

given $KV_{b\,old} = 18\,KV$ $KV_{b\,new} = 20\,KV$

$MVA_{b\,old} = 500\,MVA$, $MVA_{b\,new} = 100\,MV$

$X_{pu\,old} = j0.25\,pu$



$$X_{pu\,new} = X_{pu\,old} \times \left(\frac{KV_{b\,old}}{KV_{b\,new}}\right)^2 \times \left(\frac{MVA_{b\,new}}{MVA_{b\,old}}\right)$$

$$X_{pu\,new}\text{ of }G_1 = j0.25 \times \left(\frac{18}{20}\right)^2 \times \left(\frac{100}{500}\right)$$

$$\boxed{X_{pu}\text{ of }G_1 = j0.0405}$$

11) A three phase, Δ-Y transformer with rating 100KVA, 11KV/400V has its primary and secondary leakage reactance as 12 Ω/phase and 0.05 Ω/phase respectively. Calculate the p.u reactance of transformer

(Nov/Dec 2011)

Case(i)

The HV side (primary) rating are chosen as base values

$KV_{b1} = 11\,KV$

$KVA_{b1} = 100\,KVA$

Base reactance $X_b = \frac{(KV_{b1})^2}{MVA_{b1}} = \frac{(11)^2}{100 \times 10^{-3}} = 1210\,\Omega$

transformer line voltage ratio $K = \frac{KV_{b2}}{KV_{b1}} = \frac{400}{11000} = 0.0364$

total leakage reactance referred to primary } $X_{01} = X_1 + X_2' = X_1 + \frac{X_2}{K^2}$

$= 12 + \frac{0.05}{(0.0364)^2}$

$X_{01} = 49.737\ \Omega/\text{phase}$

$X_{pu} = \frac{\text{Total leakage reactance}}{\text{Base reactance}} = \frac{X_{01}}{X_b}$

$X_{pu} = \frac{49.737}{1210} \Rightarrow \boxed{X_{pu} = 0.0411 pu}$

Case(ii) The LV side (secondary) rating are chose as base values

$KV_{b2} = 400V = 0.4KV$

$KVA_{b2} = 100 KVA$

Base reactance $X_b = \frac{(KV_{b2})^2}{MVA_{b2}} = \frac{(0.4)^2}{100 \times 10^{-3}} = 1.6\Omega$

$K = 0.0364$

total leakage reactance referred to secondary } $X_{02} = X_1' + X_2 = X_1K^2 + X_2$

$= (0.0364)^2 12 + 0.05$

$= 0.0659\ \Omega/\text{phase}$

$X_{pu} = \frac{\text{total leakage reactance}}{\text{Base reactance}} = \frac{X_{02}}{X_b}$

$= \frac{0.0659}{1.6}$

$\boxed{X_{pu} = 0.0411 pu}$

12) Draw the p.u reactance diagram of a three winding transformer whose three phase ratings are primary (P), wye grounded, 15MVA, 66 KV; secondary (S), wye-grounded, 10MVA, 13.2KV; tertiary (t), delta connected, 5MVA, 2.3KV. Mark the appropriate values of the impedances. The measured data for impedances are Zps = 7% on 15MVA & 66 KV; ZPT = 9% on 15MVA and 66KV; ZST = 8% on 10 MVA and 13.2KV (April/May 2010)

Soln:-

Choose $KV_b = 66KV$

$MVA_b = 15MVA$

Pu value of $Z_{ps} = \frac{7}{100} = j0.07pu$

Pu value of $Z_{PT} = \frac{9}{100} = j0.09pu$

Pu value of $Z_{st} = \frac{8}{100} = j0.08pu$

The pu value of Zst on a base of 10MVA, 13.2KV can be change to base of 15MVA, 66 KV by using

$$Z_{pu,new} = Z_{pu\,old} \times \left(\frac{KV_{b\,old}}{KV_{b\,new}}\right)^2 \times \left(\frac{MVA_{b\,new}}{MVA_{b\,old}}\right)$$

$$KV_{b\,new} = 13.2 \times \frac{66}{13.2} = 66KV$$

$$Z_{st}' = Z_{pu\,old} \times \left(\frac{KV_{b\,old}}{KV_{b\,new}}\right)^2 \times \left(\frac{MVA_{b\,new}}{MVA_{b\,old}}\right)$$

$$= j0.08 \times \left(\frac{66}{66}\right)^2 \times \left(\frac{15}{10}\right) = j0.12pu$$

$$Z_p = \frac{1}{2}\left[Z_{ps} + Z_{pt} - Z_{st'}\right] = \frac{1}{2}\left[j0.07 + j0.09 - j0.12\right]$$

$$\boxed{Z_p = j0.02pu}$$

$$Z_s' = \frac{1}{2}\left[Z_{ps} + Z_{st'} - Z_{pt}\right] = \frac{1}{2}\left[j0.07 + j0.12 - j0.09\right]$$

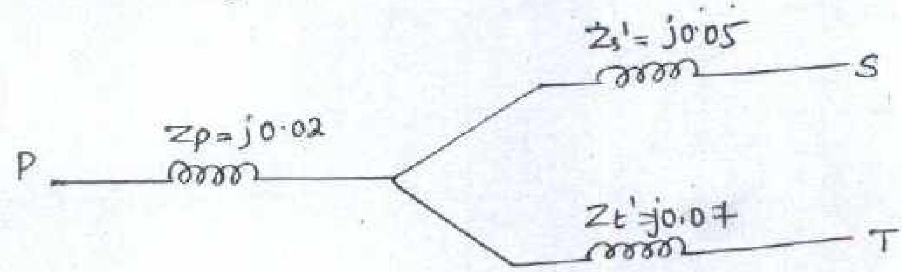
$$\boxed{Z_s' = j0.05pu}$$

$$Z_t' = \frac{1}{2}\left[Z_{pt} + Z_{st'} - Z_{ps}\right]$$

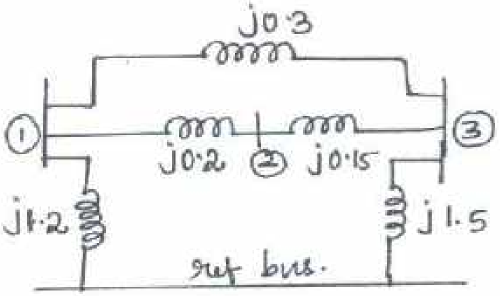
$$= \frac{1}{2}\left[j0.09 + j0.12 - j0.07\right]$$

$$\boxed{Z_t' = j0.07pu}$$

Reactance diagram:-



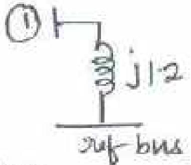
13) Determine Z_{bus} for system whose reactance diagram is shown below. Where the impedance is given in p.u Preserve all the nodes (Nov/Dec 2011)



Soln:-

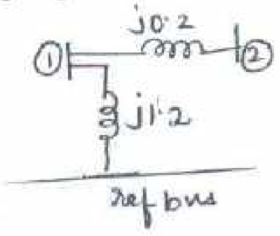
Step 1:- Add bus (1) to ref bus through j1·2

$$Z_{bus} = [j1{\cdot}2]$$



Step 2:- Connect bus (2) to bus (1) through j0·2

$$Z_{bus} = \begin{bmatrix} j1{\cdot}2 & j1{\cdot}2 \\ j1{\cdot}2 & j1{\cdot}2 + j0{\cdot}2 \end{bmatrix}$$

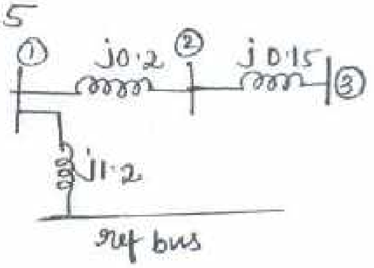


$$Z_{bus} = \begin{bmatrix} j1{\cdot}2 & j1{\cdot}2 \\ j1{\cdot}2 & j1{\cdot}4 \end{bmatrix}$$

Step 3:-

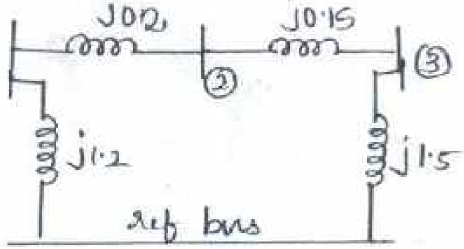
Connect bus ③ to bus ② through j0·15

$$Z_{bus} = \begin{bmatrix} j1{\cdot}2 & j1{\cdot}2 & j1{\cdot}2 \\ j1{\cdot}2 & j1{\cdot}4 & j1{\cdot}4 \\ j1{\cdot}2 & j1{\cdot}4 & j1{\cdot}4 + j0{\cdot}15 \end{bmatrix}$$



$$Z_{bus} = \begin{bmatrix} j1{\cdot}2 & j1{\cdot}2 & j1{\cdot}2 \\ j1{\cdot}2 & j1{\cdot}4 & j1{\cdot}4 \\ j1{\cdot}2 & j1{\cdot}4 & j1{\cdot}55 \end{bmatrix}$$

<u>Step 4</u>:- Connect bus (3) to ref bus through j1.5



$$Z_{bus} = \begin{bmatrix} j1.2 & j1.2 & j1.2 & j1.2 \\ j1.2 & j1.4 & j1.4 & j1.4 \\ j1.2 & j1.4 & j1.65 & j1.55 \\ j1.2 & j1.4 & j1.55 & j1.55+j1.5 \end{bmatrix}$$

$$Z_{bus} = \begin{bmatrix} j1.2 & j1.2 & j1.2 & j1.2 \\ j1.2 & j1.4 & j1.4 & j1.4 \\ j1.2 & j1.24 & j1.55 & j1.55 \\ j1.2 & j1.4 & j1.55 & j3.05 \end{bmatrix}$$

Reduce the Z_{bus} to $n \times n$ (ie 3×3) by kron reduction technique

$$Z_{jk\,new} = Z_{jk\,old} - \frac{Z_{j(n+1)} Z_{(n+1)k}}{Z_{(n+1)(n+1)}} \qquad \begin{array}{l} n=3 \\ j=1,2,3 \\ K=1,2,3 \end{array}$$

$$Z_{11\,new} = Z_{11} - \frac{Z_{14} Z_{41}}{Z_{44}} = j1.2 - \frac{j1.2 \times j1.2}{j3.05} = j0.728$$

$\boxed{Z_{11} = j0.728}$

$$Z_{12\,new} = Z_{12} - \frac{Z_{14} Z_{42}}{Z_{44}}$$

$$= j1.2 - \frac{j1.2 \times j1.4}{j3.05} = j0.649$$

$$Z_{13\,new} = Z_{13} - \frac{Z_{14} Z_{43}}{Z_{44}} = j1.2 - \frac{j1.2 \times j1.55}{j3.05} = j0.590$$

$$Z_{21\,new} = Z_{12\,new} = j0.649$$

$$Z_{22\,new} = Z_{22} - \frac{Z_{24} Z_{42}}{Z_{44}} = j1.4 - \frac{j1.4 \times j1.4}{j3.05} = j0.757$$

LECTURE NOTES PAPER

$$Z_{23new} = Z_{23} - \frac{Z_{24}Z_{43}}{Z_{44}} = j1.4 - \frac{j1.4\, j1.55}{j3.05} = j0.689$$

$Z_{31new} = Z_{13new}$

$Z_{32new} = Z_{23new}$

$$Z_{33new} = Z_{33} - \frac{Z_{34}Z_{43}}{Z_{44}} = j1.55 - \frac{j1.55 \times j1.55}{j3.05} = j0.762$$

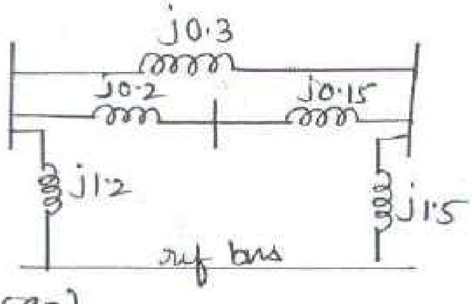
$$\therefore Z_{bus} = \begin{bmatrix} j0.728 & j0.649 & j0.590 \\ j0.649 & j0.757 & j0.689 \\ j0.590 & j0.689 & j0.762 \end{bmatrix}$$

Step 5:-

Connect j0.3 between bus ① and bus ③

Column ① − column ③ = 4th column

row ① − row ③ = 4th row



$$Z_{44} = Z_b + Z_{11} + Z_{33} - 2Z_{13}$$

$$= j0.3 + j0.728 + j0.762 - 2(j0.590)$$

$Z_{44} = j0.61$

$$Z_{bus} = \begin{bmatrix} j0.728 & j0.649 & j0.590 & j0.138 \\ j0.649 & j0.757 & j0.689 & -j0.04 \\ j0.59 & j0.689 & j0.762 & -j0.172 \\ j0.138 & -j0.04 & -j0.172 & j0.61 \end{bmatrix}$$

Reduce the matrix to 3×3 by Kron reduction technique.

$$Z_{11new} = Z_{11} - \frac{Z_{14}Z_{41}}{Z_{44}} = j0.728 - \frac{j0.138 \times j0.138}{j0.61} = j0.697$$

$$Z_{12new} = Z_{12} - \frac{Z_{14}Z_{42}}{Z_{44}} = j0.649 - \frac{j0.138 \times (-j0.04)}{j0.61} = j0.658$$

$$Z_{13new} = Z_{13} - \frac{Z_{14}Z_{43}}{Z_{44}} = j0.59 - \frac{j0.138 \times (-j0.172)}{j0.61} = j0.629$$

$$Z_{21new} = Z_{12new} = j0.658$$

$$Z_{22new} = Z_{22} - \frac{Z_{24}Z_{42}}{Z_{44}} = j0.757 - \frac{(-j0.04)(-j0.04)}{j0.61} = j0.754$$

$$Z_{23new} = Z_{23} - \frac{Z_{24}Z_{43}}{Z_{44}} = j0.689 - \frac{(-j0.04)(-j0.172)}{j0.61} = j0.678$$

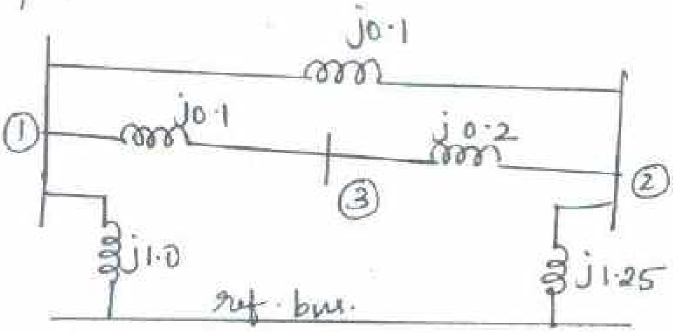
$$Z_{31new} = Z_{13new} = j0.629$$

$$Z_{32new} = Z_{23new} = j0.678$$

$$Z_{33new} = Z_{33} - \frac{Z_{34}Z_{43}}{Z_{44}} = j0.762 - \frac{(-j0.172)(-j0.172)}{j0.61} = j0.714$$

$$Z_{bus} = \begin{bmatrix} j0.697 & j0.658 & j0.629 \\ j0.658 & j0.754 & j0.678 \\ j0.629 & j0.678 & j0.714 \end{bmatrix}$$

14) Determine the Z_{bus} for a 3 bus system as shown in fig. where the impedances are shown and the values are in pu



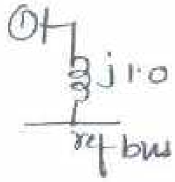
(May/June 2009)

Soln:-

Step 1:-

Add j0·1 between bus ① and reference bus

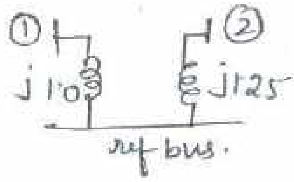
$$Z_{bus} = [j1{\cdot}0]$$



Step 2:-

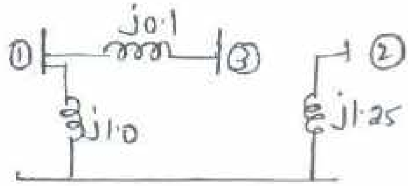
Add bus (2) to ref. bus through j1·25

$$Z_{bus} = \begin{bmatrix} j1{\cdot}0 & 0 \\ 0 & j1{\cdot}25 \end{bmatrix}$$



Step 3:-

Adding j0·1 between ① and ③

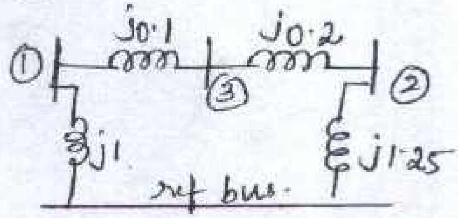


$$Z_{bus} = \begin{bmatrix} j1{\cdot}0 & 0 & j1{\cdot}0 \\ 0 & j1{\cdot}25 & 0 \\ j1{\cdot}0 & 0 & j1+j0{\cdot}1 \end{bmatrix}$$

$$Z_{bus} = \begin{bmatrix} j1 & 0 & j1 \\ 0 & j1{\cdot}25 & 0 \\ j1 & 0 & j1{\cdot}1 \end{bmatrix}$$

Step 4:—

Adding j0.2 between buses ② and ③



$$Z_{bus} = \begin{bmatrix} j1.0 & 0 & j1 & j1-0 \\ 0 & j1.25 & 0 & 0-j1.25 \\ j1.0 & 0 & j1.1 & j1.1-0 \\ j1-0 & 0-j1.25 & j1.1-0 & Z_{44} \end{bmatrix}$$

column 4 = column 3 − column 2

row 4 = row 3 − row 2

$$Z_{44} = j0.2 + j1.25 + j1.1 - 2\times 0 = j2.55$$

$$Z_{bus} = \begin{bmatrix} j1 & 0 & j1 & j1 \\ 0 & j1.25 & 0 & -j1.25 \\ j1 & 0 & j1.1 & j1.1 \\ j1 & -j1.25 & j1.1 & j2.55 \end{bmatrix}$$

By kron reduction reduce the matrix to n×n (ie 3×3)

$$Z_{jk\,new} = Z_{jk\,actual} - \frac{Z_{j(n+1)}\,Z_{(n+1)k}}{Z_{(n+1)(n+1)}}$$

$n = 3$

$j, k = 1, 2, 3$

$$Z_{11\,new} = Z_{11} - \frac{Z_{14}Z_{41}}{Z_{44}} = j1 - \frac{j1 \times j1}{j2.55} = j0.608$$

$$Z_{12\,new} = Z_{12} - \frac{Z_{14}Z_{42}}{Z_{44}} = 0 - \frac{j1(-j1.25)}{j2.55} = j0.4902$$

$$Z_{13\,new} = Z_{13} - \frac{Z_{14}Z_{43}}{Z_{44}} = j1 - \frac{j1(j1.1)}{j2.55} = j0.5686$$

$$Z_{21\,new} = Z_{12\,new} = j0.4902$$

$$Z_{22\,new} = Z_{22} - \frac{Z_{24}Z_{42}}{Z_{44}} = j1.25 - \frac{(-j1.25)(-j1.25)}{j2.55} = j0.6372$$

$$Z_{23\,new} = Z_{23} - \frac{Z_{24}Z_{43}}{Z_{44}} = 0 - \frac{(-j1.25)(j1.1)}{j2.55} = j0.5392$$

$$Z_{31\,new} = Z_{13\,new} = j0.5686$$

$$Z_{32\,new} = Z_{23\,new} = j0.5392$$

$$Z_{33\,new} = Z_{33} - \frac{Z_{34}Z_{43}}{Z_{44}} = j1.1 - \frac{(j1.1)(j1.1)}{j2.55} = j0.6255$$

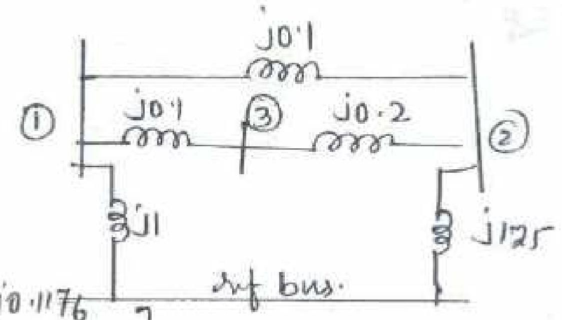
$$Z_{bus} = \begin{bmatrix} j0.6078 & j0.4902 & j0.5686 \\ j0.4902 & j0.6372 & j0.5392 \\ j0.5686 & j0.5392 & j0.6255 \end{bmatrix}$$

Step 5:-

Adding j0.1 between buses ① and ②

Column 4 = Column ② − column ①

row 4 = row ② − row ①



$$Z_{bus} = \begin{bmatrix} j0.6078 & j0.4902 & j0.5686 & -j0.1176 \\ j0.4902 & j0.6372 & j0.5392 & j0.147 \\ j0.5686 & j0.5392 & j0.6255 & -j0.0294 \\ -j0.1176 & j0.147 & -j0.0294 & j0.3646 \end{bmatrix}$$

$$Z_{44} = j0.1 + j0.6078 + j0.6372 - 2(j0.4902)$$

$$Z_{44} = j0.3646$$

reduce the matrix to 3x3 by Kron reduction

$Z_{11new} = Z_{11} - \frac{Z_{14}Z_{41}}{Z_{44}} = j0.6078 - \frac{(-j0.1176 \times -j0.1176)}{j0.3646} = j0.5699$

$Z_{12new} = Z_{12} - \frac{Z_{14}Z_{42}}{Z_{44}} = j0.4902 - \frac{(-j0.1176 \times j0.147)}{j0.3646} = j0.5376$

$Z_{13new} = Z_{13} - \frac{Z_{14}Z_{43}}{Z_{44}} = j0.5686 - \frac{(-j0.1176 \times -j0.0294)}{j0.3646} = j0.5591$

$Z_{21new} = Z_{12new} = j0.5376$

$Z_{22new} = Z_{22} - \frac{Z_{24}Z_{42}}{Z_{44}} = j\,0.6372 - \frac{(j0.147 \times j0.147)}{j0.3646} = j0.5779$

$Z_{23new} = Z_{23} - \frac{Z_{24}Z_{43}}{Z_{44}} = j0.5392 - \frac{(j0.147 \times -j0.0294)}{j0.3646} = j0.5511$

$Z_{13new} = Z_{31new} = j0.5591$

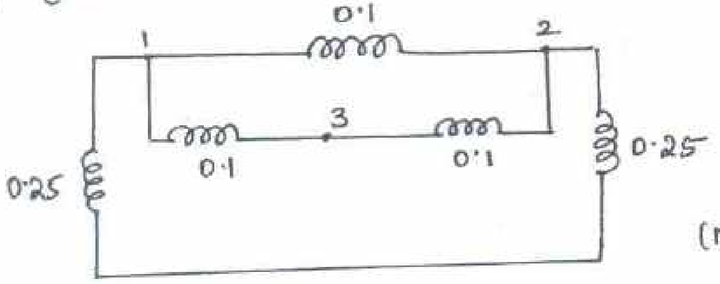
$Z_{32new} = Z_{23new} = j0.5511$

$Z_{33new} = Z_{33} - \frac{Z_{34}Z_{43}}{Z_{44}} = j0.6255 - \frac{(-j0.0294 \times -j0.0294)}{j0.3646}$

$= j0.6231$

$$Z_{bus} = \begin{bmatrix} j0.5699 & j0.5376 & j0.5591 \\ j0.5376 & j0.5779 & j0.5511 \\ j0.559 & j0.5511 & j0.6231 \end{bmatrix}$$

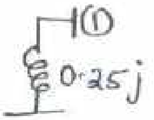
(5) For the 3-bus network shown below obtain Zbus by building algorithm



(Nov|Dec 2014)

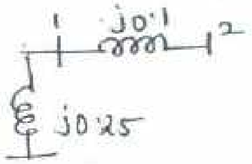
Soln:-

Step 1:- Add bus ① to reference bus



$$Z_{bus} = [j0.25]$$

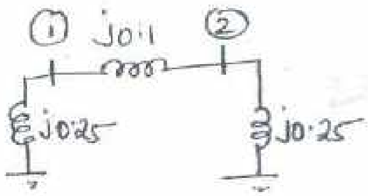
Step 2:- Add bus ② to bus ① through j0.1



$$Z_{bus} = \begin{bmatrix} j0.25 & j0.25 \\ j0.25 & j0.25+j0.1 \end{bmatrix}$$

$$Z_{bus} = \begin{bmatrix} j0.25 & j0.25 \\ j0.25 & j0.35 \end{bmatrix}$$

Step 3:- Add bus ② to ref. bus



$$Z_{bus} = \begin{bmatrix} j0.25 & j0.25 & j0.25 \\ j0.25 & j0.35 & j0.35 \\ j0.25 & j0.35 & j0.35+j0.25 \end{bmatrix}$$

$$Z_{bus} = \begin{bmatrix} j0.25 & j0.25 & j0.25 \\ j0.25 & j0.35 & j0.35 \\ j0.25 & j0.35 & j0.6 \end{bmatrix}$$

Reduce the matrix to 2x2 by Kron reduction

$$Z_{jk\,new} = Z_{jk} - \frac{Z_{j(n+1)} Z_{(n+1)k}}{Z_{(n+1)(n+1)}}$$

$$Z_{11new} = Z_{11} - \frac{Z_{13}Z_{31}}{Z_{33}} = j0.25 - \frac{(j0.25)(j0.25)}{j0.6} = j0.1458$$

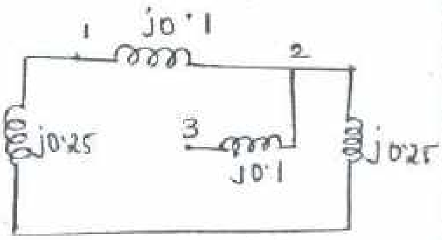
$$Z_{12new} = Z_{12} - \frac{Z_{13}Z_{32}}{Z_{33}} = j0.25 - \frac{(j0.25)(j0.35)}{j0.6} = j0.10416$$

$$Z_{22new} = Z_{22} - \frac{Z_{23}Z_{32}}{Z_{33}} = j0.35 - \frac{(j0.35)(j0.35)}{j0.6} = j0.1458$$

$$Z_{bus} = \begin{bmatrix} 0.1458j & j0.10416 \\ j0.10416 & j0.1458 \end{bmatrix}$$

Step 4:-

Add bus (3) to bus (2) through j0.1

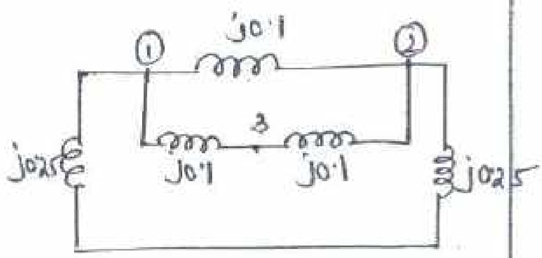


$$Z_{bus} = \begin{bmatrix} j0.1458 & j0.10416 & j0.10416 \\ j0.10416 & j0.1458 & j0.1458 \\ j0.10416 & j0.1458 & j0.1458+j0.1 \end{bmatrix}$$

$$Z_{bus} = \begin{bmatrix} j0.1458 & j0.10416 & j0.10416 \\ j0.10416 & j0.1458 & j0.1458 \\ j0.10416 & j0.1458 & j0.2458 \end{bmatrix}$$

Step 5:-

Add buses (1) and (3) through j0.1



$$Z_{bus} = \begin{bmatrix} j0.1458 & j0.10416 & j0.10416 & -j0.0416 \\ j0.10416 & j0.1458 & j0.1458 & j0.0416 \\ j0.10416 & j0.1458 & j0.2458 & j0.1416 \\ -j0.0416 & j0.0416 & j0.1416 & j0.2832 \end{bmatrix}$$

Column 4 = Column 3 − column 1
row 4 = row 3 − row 1

$$Z_{44} = j0.1 + j0.1458 + j0.2458 - 2(j0.10416)$$

Z_{44} = j0.283[illegible]

Reduce the matrix to nxn (ie 3x3) by kron reduction technique.

$$Z_{jk\,new} = Z_{jk} - \frac{Z_{j(n+1)} Z_{(n+1)k}}{Z_{(n+1)(n+1)}}$$

$$Z_{11new} = Z_{11} - \frac{Z_{14} Z_{41}}{Z_{44}} = j0.1458 - \frac{(-j0.0416)(-j0.0416)}{j0.2832} = j0.1396$$

$$Z_{12new} = Z_{12} - \frac{Z_{14} Z_{42}}{Z_{44}} = j0.10416 - \frac{(-j0.0416)(j0.0416)}{j0.2832} = j0.1102$$

$$Z_{13new} = Z_{13} - \frac{Z_{14} Z_{43}}{Z_{44}} = j0.10416 - \frac{(-j0.0416)(j0.1416)}{j0.2832} = j0.12496.$$

$$Z_{21new} = Z_{12new} = j0.1102$$

$$Z_{22new} = Z_{22} - \frac{Z_{24} Z_{42}}{Z_{44}} = j0.1458 - \frac{(j0.0416)(j0.0416)}{j0.2832} = j0.13968$$

$$Z_{23new} = Z_{23} - \frac{Z_{24} Z_{43}}{Z_{44}} = j0.1458 - \frac{(j0.0416)(j0.1416)}{j0.2832} = j0.125$$

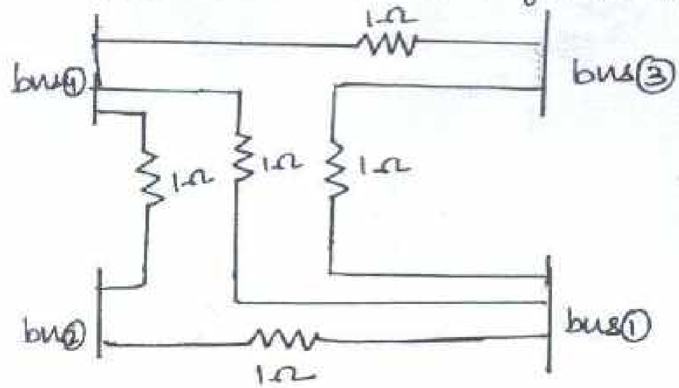
$$Z_{31new} = Z_{13new} = j0.12496$$

$$Z_{32new} = Z_{23new} = j0.125$$

$$Z_{33new} = Z_{33} - \frac{Z_{34} Z_{43}}{Z_{44}} = j0.2458 - \frac{(j0.1416)(j0.1416)}{j0.2832} = j0.175$$

$$Z_{bus} = \begin{bmatrix} j0.1396 & j0.1102 & j0.1249 \\ j0.1102 & j0.1396 & j0.125 \\ j0.1249 & j0.125 & j0.175 \end{bmatrix}$$

16) Find the bus impedance matrix for the 4 bus system shown in figure. Consider bus 4 as the reference bus.

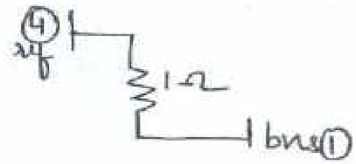


(May/June 2012)

Soln:-

Step 1:- Add an element between ref bus (ie bus 4) and bus ①

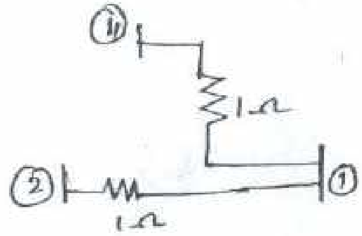
$$Z_{bus} = [1]$$



Step 2:- Add 1Ω between bus ② and bus ①

$$Z_{bus} = \begin{bmatrix} 1 & 1 \\ 1 & 1+1 \end{bmatrix}$$

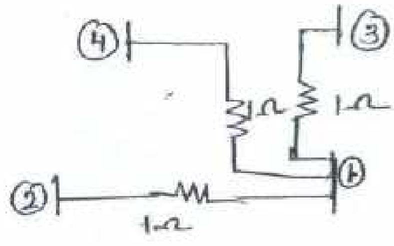
$$Z_{bus} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$$



Step 3:-

Add 1Ω between bus ③ and bus ①

$$Z_{bus} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 1+1 \end{bmatrix}$$

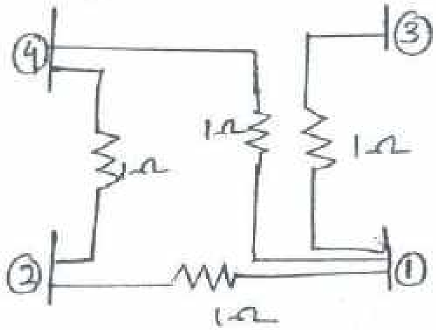


$$Z_{bus} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

Step 4:-

Add 1Ω between bus ② and bus(4) (i.e ref bus)

$$Z_{bus} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & 2 \\ 1 & 1 & 2 & 1 \\ 1 & 2 & 1 & 2+1 \end{bmatrix}$$



$$Z_{bus} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & 2 \\ 1 & 1 & 2 & 1 \\ 1 & 2 & 1 & 3 \end{bmatrix}$$

Reduce the matrix to 3x3 by Kron reduction

$$Z_{11new} = Z_{11} - \frac{Z_{14} Z_{41}}{Z_{44}} = 1 - \frac{1 \times 1}{3} = 0.66$$

$$Z_{12new} = Z_{12} - \frac{Z_{14} Z_{42}}{Z_{44}} = 1 - \frac{1 \times 2}{3} = 0.33$$

$$Z_{13new} = Z_{13} - \frac{Z_{14} Z_{43}}{Z_{44}} = 1 - \frac{1 \times 1}{3} = 0.66$$

$$Z_{21new} = Z_{12new} = 0.33$$

$$Z_{22new} = Z_{22} - \frac{Z_{24} Z_{42}}{Z_{44}} = 2 - \frac{2 \times 2}{3} = 0.66.$$

$$Z_{23new} = Z_{23} - \frac{Z_{24} Z_{43}}{Z_{44}} = 1 - \frac{2 \times 1}{3} = 0.33$$

$$Z_{31new} = Z_{13new} = 0.66$$

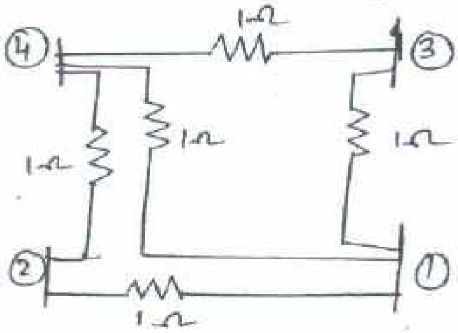
$$Z_{32new} = Z_{23new} = 0.33$$

$$Z_{33new} = Z_{33} - \frac{Z_{34} Z_{43}}{Z_{44}} = 2 - \frac{1 \times 1}{3} = 1.66$$

$$Z_{bus} = \begin{bmatrix} 0.66 & 0.33 & 0.66 \\ 0.33 & 0.66 & 0.33 \\ 0.66 & 0.33 & 1.66 \end{bmatrix}$$

Step 5:-

Adding an element between ③ and ref bus ④



$$Z_{bus} = \begin{bmatrix} 0.66 & 0.33 & 0.66 & 0.66 \\ 0.33 & 0.66 & 0.33 & 0.33 \\ 0.66 & 0.33 & 1.66 & 1.66 \\ 0.66 & 0.33 & 1.66 & 1.66+1 \end{bmatrix}$$

$$Z_{bus} = \begin{bmatrix} 0.66 & 0.33 & 0.66 & 0.66 \\ 0.33 & 0.66 & 0.33 & 0.33 \\ 0.66 & 0.33 & 1.66 & 1.66 \\ 0.66 & 0.33 & 1.66 & 2.66 \end{bmatrix}$$

Reduce the matrix to 3X3 by Kron reduction.

$$Z_{jk(new)} = Z_{jk} - \frac{Z_{j(n+1)} Z_{(n+1)k}}{Z_{(n+1)(n+1)}}$$

$$Z_{11(new)} = Z_{11} - \frac{Z_{14} Z_{41}}{Z_{44}} = 0.66 - \frac{0.66 \times 0.66}{2.66} = 0.496$$

$$Z_{12(new)} = Z_{12} - \frac{Z_{14} Z_{42}}{Z_{44}} = 0.33 - \frac{0.66 \times 0.33}{2.66} = 0.248$$

$$Z_{13(new)} = Z_{13} - \frac{Z_{14} Z_{43}}{Z_{44}} = 0.66 - \frac{0.66 \times 1.66}{2.66} = 0.248$$

$$Z_{21(new)} = Z_{12(new)} = 0.243$$

$$Z_{22(new)} = Z_{22} - \frac{Z_{24} Z_{42}}{Z_{22}} = 0.66 - \frac{0.33 \times 0.33}{2.66} = 0.619$$

$$Z_{23(new)} = Z_{23} - \frac{Z_{24} Z_{43}}{Z_{44}} = 0.33 - \frac{0.33 \times 1.66}{2.66} = 0.124$$

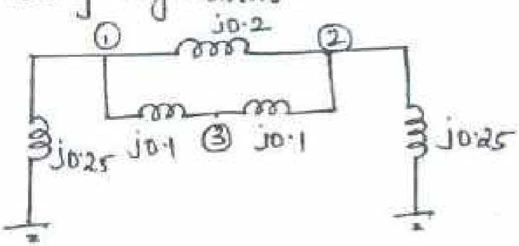
$$Z_{31(new)} = Z_{13(new)} = 0.248$$

$$Z_{32(new)} = Z_{23(new)} = 0.124$$

$$Z_{33(new)} = Z_{33} - \frac{Z_{34} Z_{43}}{Z_{44}} = 1.66 - \frac{1.66 \times 1.66}{2.66} = 0.6240$$

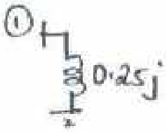
$$Z_{bus} = \begin{bmatrix} 0.496 & 0.248 & 0.248 \\ 0.248 & 0.619 & 0.124 \\ 0.248 & 0.124 & 0.624 \end{bmatrix}$$

17) Form the bus impedance matrix for the network shown in fig by bus building algorithms. (May/June 2013)



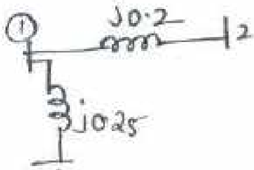
Soln:-

Step 1:- Add bus ① to ref bus.



$$Z_{bus} = [0.25j]$$

Step 2:- Adding bus ② to bus ① through j0.2

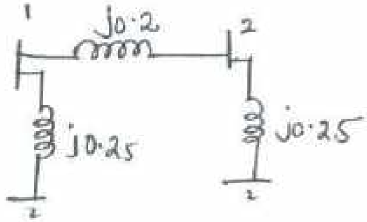


$$Z_{bus} = \begin{bmatrix} 0.25j & 0.25j \\ 0.25j & 0.25j + 0.2j \end{bmatrix}$$

$$Z_{bus} = \begin{bmatrix} 0.25j & 0.25j \\ 0.25j & 0.45j \end{bmatrix}$$

Step 3:-

Adding bus (2) to ref bus through j0.25



$$Z_{bus} = \begin{bmatrix} j0.2 & j0.25 & j0.25 \\ j0.25 & j0.45 & j0.45 \\ j0.25 & j0.45 & j0.45+j0.25 \end{bmatrix}$$

$$Z_{bus} = \begin{bmatrix} j0.2 & j0.25 & j0.25 \\ j0.25 & j0.45 & j0.45 \\ j0.25 & j0.45 & j0.7 \end{bmatrix}$$

reduce the matrix to 2x2 by Kron reduction

$$Z_{11} = Z_{11} - \frac{Z_{13}Z_{31}}{Z_{33}} = j0.25 - \frac{j0.25 \times j0.25}{j0.7} = j0.1607$$

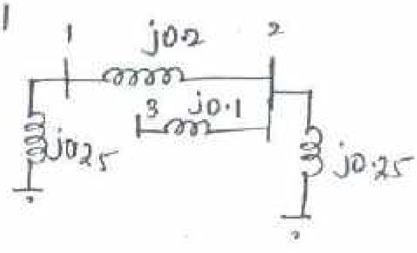
$$Z_{12} = Z_{12} - \frac{Z_{13}Z_{23}}{Z_{33}} - j0.25 - \frac{j0.25 \times j0.45}{j0.7} = j0.0892$$

$$Z_{22} = Z_{22} - \frac{Z_{23}Z_{32}}{Z_{33}} = j0.45 - \frac{j0.45 \times j0.45}{j0.7} = j0.1607$$

$$Z_{bus} = \begin{bmatrix} 0.1607j & 0.0842j \\ 0.0892j & 0.1607j \end{bmatrix}$$

Step 4:-

Adding bus (3) to bus (2) through j0.1

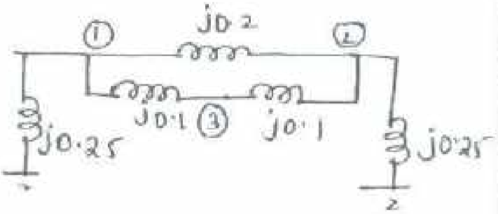


$$Z_{bus} = \begin{bmatrix} 0.1607j & 0.0892j & 0.0892j \\ 0.0892j & 0.1607j & 0.1607j \\ 0.0892j & 0.1607j & 0.1607j+j0.1 \end{bmatrix}$$

$$Z_{bus} = \begin{bmatrix} 0.1607j & 0.0892j & 0.0892j \\ 0.0892j & 0.1607j & 0.1607j \\ 0.0892j & 0.1607j & 0.2607j \end{bmatrix}$$

Step 5:-

Add bus (3) to bus (1) through j0·1



$$Z_{bus} = \begin{bmatrix} 0.1607j & 0.0892j & 0.0892j & 0.0715j \\ 0.0892j & 0.1607j & 0.1607j & -0.0715j \\ 0.0892j & 0.1607j & 0.2607j & -0.175j \\ 0.0715j & -0.0715j & -0.1715j & 0.343j \end{bmatrix}$$

Column 4 = Column 1 – Column 2

row 4 = row 1 – row 4

$$Z_{44} = Z_{11} + Z_{33} + Z_b - 2Z_{13}$$

$$= 0.1607j + 0.2607j + j0.1 - 2 \times 0.0892j = 0.343j$$

reduce the matrix to 3X3 by Kron reduction

$$Z_{11\,new} = Z_{11} - \frac{Z_{14}Z_{41}}{Z_{44}} = 0.1607j - \frac{0.0715 \times 0.0715}{0.343j} = j0.145$$

$$Z_{12\,new} = Z_{12} - \frac{Z_{14}Z_{42}}{Z_{44}} = 0.0892j - \frac{0.0715j(-0.0715j)}{0.343j} = j0.104$$

$$Z_{13\,new} = Z_{13} - \frac{Z_{14}Z_{43}}{Z_{44}} = 0.0892j - \frac{0.0715j \times (-0.1715j)}{0.343j} = j0.125$$

$$Z_{21\,new} = Z_{12\,new} = j0.104$$

$$Z_{22\,new} = Z_{22} - \frac{Z_{24}Z_{42}}{Z_{44}} = 0.1607j - \frac{(-0.0715j \times -0.0715j)}{0.343j} = j0.1458$$

$$Z_{23\,new} = Z_{23} - \frac{Z_{24}Z_{43}}{Z_{44}} = 0.1607j - \frac{(-0.0715j) \times (-0.1715j)}{0.343j} = j0.125$$

$$Z_{31\,new} = Z_{13\,new} = j0.125$$

$$Z_{32\,new} = Z_{23\,new} = j0.125$$

$$Z_{33} = Z_{33} - \frac{Z_{34}Z_{43}}{Z_{44}} = 0.2607j - \frac{(-0.175j \times -0.175j)}{0.343j} = j0.171$$

$$Z_{bus} = \begin{bmatrix} j0.145 & j0.104 & j0.125 \\ j0.104 & j0.1458 & j0.125 \\ j0.125 & j0.125 & j0.171 \end{bmatrix}$$

18. The parameters of a 4 bus system are as under :-

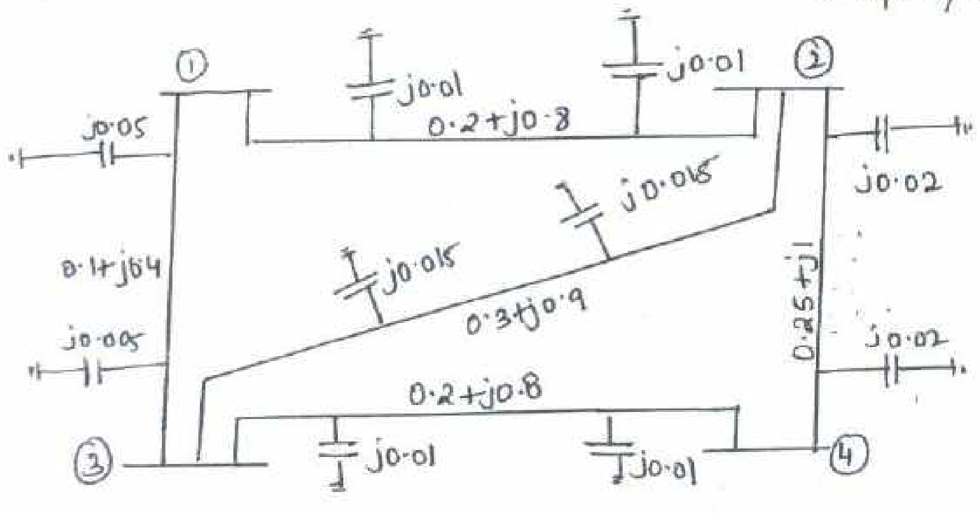
Bus code	Line impedance (PU)	Charging admittance (PU)
1-2	0.2+j0.8	j0.02
2-3	0.3+j0.9	j0.03
2-4	0.25+j1	j0.04
3-4	0.2+j0.8	j0.02
1-3	0.1+j0.4	j0.01

Draw the network and find the bus admittance matrix

(Nov/Dec 2011)

(April/May 2010)

Soln:-



Half line charging admittance $y_{i0} = \frac{\text{line charging admittance}}{2}$

$$Y_{11} = y_{12} + y_{13} + y_{10}$$

$$= \frac{1}{0.2+j0.8} + \frac{1}{0.1+j0.4} + j0.01 + j0.05$$

$$\boxed{Y_{11} = 0.8822 - j3.514}$$

$$Y_{12} = Y_{21} = -y_{12} = -\frac{1}{0.2+j0.8}$$

$$\boxed{Y_{12} = Y_{21} = -0.294 + j1.176}$$

$$Y_{13} = Y_{31} = -y_{13} = \frac{-1}{0.1+j0.4}$$

$$\boxed{Y_{13} = Y_{31} = -0.5882 + j2.353}$$

$$\boxed{Y_{14} = Y_{41} = 0}$$

$$Y_{22} = Y_{21} + Y_{24} + Y_{23} + Y_{20}$$

$$= \frac{1}{0.2+j0.8} + \frac{1}{0.25+j1} + \frac{1}{0.3+j0.9} + j0.01 + j0.02 + j0.015$$

$$\boxed{Y_{22} = 0.862 - j3.072}$$

$$Y_{23} = Y_{32} = -Y_{23} = \frac{-1}{0.3+j0.4}$$

$$\boxed{Y_{23} = Y_{32} = -0.333 + j1}$$

$$Y_{24} = Y_{42} = -Y_{24} = \frac{-1}{0.25+j1}$$

$$\boxed{Y_{24} = Y_{42} = -0.235 + j0.941}$$

$$Y_{33} = Y_{31} + Y_{34} + Y_{32} + Y_{30}$$

$$= \frac{1}{0.1+j0.4} + \frac{1}{0.2+j0.8} + \frac{1}{03+j0.9} + j0.05 + j0.015 + j0.01$$

$$\boxed{Y_{33} = 1.215 - j4.49}$$

$$Y_{44} = Y_{42} + Y_{43} + Y_{40}$$

$$= \frac{1}{0.25+j0.1} + \frac{1}{0.2+j0.8} + j0.01 + j0.02$$

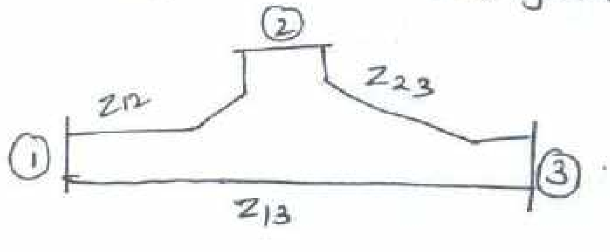
$$\boxed{Y_{44} = 0.529 - j2.087}$$

$$Y_{bus} = \begin{bmatrix} 0.8822-j3.514 & -0.2914+j1.176 & -0.5883+j2.353 & 0 \\ -0.294+j1.176 & 0.862-j3.072 & -0.333+j1 & -0.235+j0.941 \\ -0.588+j2.353 & -0.333+j1 & 1.215-j4.499 & -0.294+j1.176 \\ 0 & -0.235+j0.941 & -0.294+j1.176 & 0.529-j2.037 \end{bmatrix}$$

19. Determine Ybus for the 3-bus system shown in figure. The line series impedance as follows

Line (bus to bus)	Impedance (pu)
1-2	0.06+j0.18
1-3	0.03+j0.09
2-3	0.08+j0.24

(NOV/Dec 2013)



Soln:-

$$Y_{11} = Y_{12} + Y_{13}$$

$$= \frac{1}{0.06+j0.18} + \frac{1}{0.03+j0.09} = 1.666 - 5j + 3.33 - 10j$$

$$\boxed{Y_{11} = 4.996 - 15j}$$

$$Y_{12} = Y_{21} = -\frac{1}{0.06+j0.18}$$

$$\boxed{Y_{12} = Y_{21} = -1.666 + 5j}$$

$$Y_{13} = Y_{31} = -y_{13} = \frac{-1}{0.03+j0.09}$$

$$\boxed{Y_{13} - Y_{31} = -3.33+10j}$$

$$Y_{22} = y_{12} + y_{23}$$

$$= \frac{1}{0.06+j0.18} + \frac{1}{0.08+j0.24}$$

$$\boxed{Y_{22} = 2.916 - 8.7j}$$

$$Y_{23} = Y_{32} = -\frac{1}{0.08+j0.24}$$

$$\boxed{Y_{23} = Y_{32} = -1.25+3.7j}$$

$$Y_{33} = y_{13} + y_{23}$$

$$= \frac{1}{0.03+j0.09} + \frac{1}{0.08+j0.24}$$

$$\boxed{Y_{33} = 4.58 - 13.7j}$$

$$Y_{bus} = \begin{bmatrix} 4.996-15j & -1.666+5j & -3.33+10j \\ -1.66+5j & 2.916-8.7j & -1.25+3.7j \\ -3.33+10j & -1.25+3.7j & 4.58-13.7j \end{bmatrix}$$