Unit 2

Power flow Analysis

Power flow analysis is one of the basic tools used in power system studies. It is concerned with the steady state analysis of the system when it is working under a normal balanced operation.

The power flow analysis is also called as load flow analysis. It is helps to identify the overloaded (or) underloaded lines and transformers as well as overvoltage (or) undervoltage buses in the system.

Load flow analysis is performed on a symmetrical steady state operating condition of a power system under normal mode of operation. The solution of load flow gives bus voltages and line/transformer power flow for a given load condition. This information is essential for long term and operational planning.

The load flow studies are very important for planning, economic scheduling, control and operations of existing system as well as planning its future expansion depends upon knowing the effect of interconnections, new loads, new generating stations or new transmission lines etc., before they are installed.

The information obtained from the load flow studies are the magnitude and phase angle of voltages at each bus, active and reactive power flow in each line, and power loss in the line. It also gives the initial conditions of the system when the transient behaviour of the system is to be studied.

Importance of power flow analysis in planning and operation of power system.

Long term planning: load flow analysis help in investigating the effectiveness of alternative plans and choosing the best plan for system expansion to meet the projected operating state.

operational planning: It helps in choosing the best unit commitment plan and generation schedules to run the system efficiently for the next day's load condition without violating the bus voltages and line flow operating limits.

Steps for load flow study:

1. Representation of system by single line diagram
2. Determining the impedance diagram using the information in single line diagram
3. Formulation of network equations
4. solution of network equations

Statement of power flow problem

Pratical load flow problem can be stated as follow:

"The network configuration, complex power demands for all buses, real power generation schedules and voltage magnitudes of all the P-V buses and Voltage magnitude of the slack bus".

To determine:

* Bus Admittance matrix
* Bus Voltage phase angles of all buses except the slack bus and bus voltage magnitude of all the P-Q buses.

State vector $X = [V_1, V_2, \ldots\ldots, V_N, \delta_1, \delta_2 \ldots \delta_N]$

Ideal load flow problem

The network configuration [line impedance and half line charging admittance] and all the bus power injections.

$$P_i = P_G - P_D$$

P_G = Generated Power @ bus

P_D = Bus demand

P_i = Bus power Injection

$$X = [V_1, V_2 \ldots V_N, \delta_1, \delta_2 \ldots \delta_N]^T$$

once the voltage at all the buses are known then we can compute slack bus power, power flow in the transmission line and power in the transmission line.

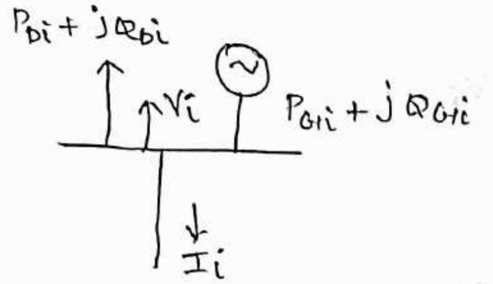
Classification of buses:

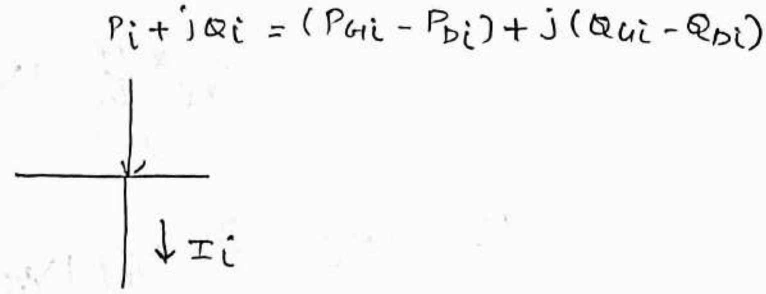
The power system is associated with four quantities they are Real power (P), Reactive Power (Q), Voltage Magnitude |V|, and phase angle of voltage (δ). In load flow problem, two quantities are specified for each bus and the remaining two quantities are obtained by solving the load flow eqn's. The types of Buses are

* Slack bus (or) Swing Bus (or) Reference Bus
* Load Bus (or) P-Q Bus
* Generator Bus (or) P-V Bus (or) Voltage Controlled Bus

Bus	Quantities specified	Quantities to be specified
Slack	$\lvert V\rvert, \delta$	P, Q
P-V	P, $\lvert V\rvert$	Q, δ
P-Q	P, Q	$\lvert V\rvert, \delta$

Development of power flow model in complex variable form and polar variable form.





Net power injection into the bus i,

$$S_i = S_{Gi} - S_{Di}$$
$$= P_i + jQ_i$$
$$= V_i I_i^*$$

$$[I] = [Y][V]$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

In general,

$$Y_{ij} = |Y_{ij}| \angle \theta_{ij} = |Y_{ij}| \cos \theta_{ij} + j|Y_{ij}| \sin \theta_{ij}$$

$$I_i = Y_{i1} V_1 + Y_{i2} V_2 + \dots + Y_{iN} V_N$$
$$= \sum_{j=1}^{N} Y_{ij} V_j$$

$$S_i = P_i + jQ_i = V_i I_i^*$$
$$S_i^* = P_i - jQ_i = V_i^* I_i$$

$$P_i - jQ_i = V_i^* \sum_{j=1}^{N} Y_{ij} V_j$$

$$= V_i^* \sum_{j=1}^{N} |Y_{ij}| \angle \theta_{ij} V_j$$

$$P_i - jQ_i = \sum_{j=1}^{N} |V_i| |Y_{ij}| |V_j| \angle(\theta_{ij} + \delta_j - \delta_i)$$

Equating real and reactive parts

$$P_i = \sum_{j=1}^{N} |V_i||Y_{ij}||V_j| \cos(\theta_{ij} + \delta_j - \delta_i)$$

$$Q_i = -\sum_{j=1}^{N} |V_i||Y_{ij}||V_j| \sin(\theta_{ij} + \delta_j - \delta_i)$$

∴ The P_i and Q_i can be written as,

$$P_i = |V_i|^2 |Y_{ii}| \cos\theta_{ii} + \sum_{\substack{j=1 \\ \neq i}}^{N} |V_i||Y_{ij}||V_j| \cos(\theta_{ij} + \delta_j - \delta_i)$$

$$Q_i = -|V_i|^2 |Y_{ii}| \sin\theta_{ii} - \sum_{\substack{j=1 \\ \neq i}}^{N} |V_i||Y_{ij}||V_j| \sin(\theta_{ij} + \delta_j - \delta_i)$$

The above P_i and Q_i are called as polar form of the power flow equations (or) static load flow equation.

Solution to load flow problem

The load flow methods are given by,

* Gauss Seidal load flow method
* Newton Raphson load flow method
* Fast decoupled load flow method

Reason for using Bus admittance Matrix in Gauss Seidal method:

Using Bus admittance matrix (Y_{BUS}) is amenable to digital computer analysis, because it could be formed and modified for network changes in subsequent cases and require less computation time and memory.

When the Generator bus (P-V bus) is assumed to be load bus (P-Q bus) during load flow analysis?

If the reactive power (Q) limit of the corresponding generator bus is violated, then it is assumed to be load bus and the flat voltage start to be 1.0 pu

Advantages

Gauss seidal	Newton Raphson method
* Calculations are simple. so the programming task is less	* Faster, more reliable and the result are accurate
* memory requirement is less	* require less number of iteration for convergence
* useful for small size system	* Suitable for large system
	* Number of iterations are independent of size of the system.

Dis-advantages

Gauss seidal	Newton Raphson method
* Require large number of iteration to reach convergence	* programming logic is more complex than Gauss Seidal method.
* Not suitable for large systems	* Memory requirement is more
* Convergence time increases with size of the system	* Number of calculations per iteration are higher than Gauss seidal method

Comparison of Gauss seidal and Newton Raphson method of load flow:

Gauss Seidal	Newton-Raphson
* Variables are expressed in Rectangular co-ordinate	* Variables are expressed in polar co-ordinate
* Number of mathematical operations per iteration will be lesser.	* Number of mathematical operation per iteration will be more than Gauss seidal method.
* G-S method has linear convergence characteristic	* N-R method has quadratic convergence characteristic.
* Number of iteration increases with number of buses	* Number of iteration remain constant, doesnot depend on size of the system.
* Convergence is affected by choice of slack bus and presence of series capacitor	* Convergence is not affected by any of the component or buses in the system.
* Number of iteration is more to get level of accuracy in the solution (30 or more)	* Number of iteration is less to get level of accuracy in the solution (3 to 5).
* Less memory requirement	* more memory requirement

Gauss seidal load flow method:

Also known as the method of successive displacement. For flat voltage start:

$\delta_i = 0$ for $i = 1, 2 \ldots N$ (for all buses except slack bus)

$|V_i| = 1.0$ (for all p-v buses)

$|V_i| = |V_i|_{spec}$ for all p-v and slack bus.

The formula to determine the voltage at buses,

$$V_i^{new} = \frac{1}{Y_{ii}}\left[\frac{P_i - jQ_i}{V_i^{*(old)}} - \sum_{j=1}^{i-1} Y_{ij} V_j^{new} - \sum_{j=i+1}^{N} Y_{ij} V_j^{old}\right]$$

Computation of reactive power generation,

$$Q_i^{calc} = -\mathrm{Im}\left[\sum_{j=1}^{N} V_i^{*} Y_{ij} V_j\right]$$

In Gauss seidal method, the number of iterations required for convergence can be reduced if the corrections in bus voltage computed at each iteration is multiplied by a factor greater than unity, called as accleration factor. to bring the voltage closer to the value to which it is converging.

$$V_{i,acc}^{new} = V_i^{old} + \underset{\substack{\downarrow \\ \text{Accleration factor}}}{\alpha}\left[V_i^{new} - V_i^{old}\right]$$

Computation of Slack Bus power:

$$S_i = P_i - jQ_i = V_i^* \sum_{j=1}^{N} Y_{ij} V_j$$

Computation of line flows and line loss:

$$S_{ij} = P_{ij} + jQ_{ij}$$

$$= V_i [V_i^* - V_j^*] Y_{ij\,series}^* + |V_i^2| Y_{pi}^*$$

$$S_{ji} = V_j [V_j^* - V_i^*] Y_{ij\,series}^* + |V_j|^2 Y_{pj}^*$$

$$= P_{ji} + jQ_{ji}$$

Real power loss $= P_{ij} + P_{ji} = P_{loss}$

Reactive power loss $= Q_{ij} + Q_{ji} = Q_{loss}$

Algorithm for iteration method:

Step 1: Form Y-bus matrix

Step 2: Assume $V_k = V_{k(spec)} \angle 0°$ (For Generator Bus)

Step 3: Assume $V_k = 1\angle 0° = 1+j0$ (For load buses)

Step 4: Set iteration count = 1 (iter = 1)

Step 5: Let bus number $i = 1$

Step 6: If 'i' refers to generator Bus go to step no 7, otherwise go to step 8.

Step 7(a): If 'i' refers to the slack bus go to step 9. Otherwise go to step 7(b).

Step 7(b): Compute Q_i using the formula Q_i^{calc}

$$Q_{Gi} = Q_i^{calc} + Q_{Li}$$

Check for Q limit violation,

If $Q_{i(min)} < Q_{Gi} < Q_{i(max)}$ then $Q_i^{calc} = Q_{i(spec)}$

If $Q_{Gi} < Q_{i(min)}$ then $Q_{i(spec)} = Q_{i(min)} - Q_{Li}$

If $Q_{Gi} > Q_{i(max)}$ then $Q_{i(spec)} = Q_{i(max)} - Q_{Li}$

If Q_{limit} is violated, then treat this P-~~V~~ as P-Q bus till convergence obtained.

Step 8: Compute the voltage using V_i^{new}

Step 9: If i is less than number of buses, increment i by 1 and goto step 6.

Step 10: Compare two successive iteration value for V_i. If V_i^{new} & V_i^{old} < tolerance go to step 12

Step 11: Update the new voltage value by acceleration factor $V_{i,acc}^{new}$.
then iter = iter + 1 ; goto step 5

Step 12: Compute relevant quantities

Slack Bus power (S_i)

Line flow (S_{ij})

Power loss (P_{loss}, Q_{LOSS})

Step 13: Stop the execution

Step 4 : Set iteration count = 1 (iter = 1).

Step 5 : Let bus number $i = 1$.

Step 6 : If 'i' refers to generator bus go to step no. 7, otherwise go to step 8.

Step 7(a) : If 'i' refers to the slack bus go to step 9. Otherwise go to step 7(b).

Step 7(b) : Compute Q_i using,

$$Q_i^{cal} = -\text{Im}\left[\sum_{j=1}^{N} V_i^* Y_{ij} V_j\right]$$

$$Q_{Gi} = Q_i^{cal} + Q_{Li}$$

Check for Q limit violation.

If $Q_{i\,(min)} < Q_{Gi} < Q_{i\,(max)}$, then $Q_{i\,(spec)} = Q_i^{cal}$.

If $Q_{i\,(min)} < Q_{Gi}$, then $Q_{i\,(spec)} = Q_{i\,(min)} - Q_{Li}$

If $Q_{i\,(max)} < Q_{Gi}$, then $Q_{i\,(spec)} = Q_{i\,(max)} - Q_{Li}$

If Q_{limit} is violated, then treat this bus as P–Q bus till convergence is obtained.

Step 8 : Compute V_i using the equation,

$$V_i^{new} = \frac{1}{Y_{ii}}\left[\frac{P_{i\,(spec)} - Q_{i\,(spec)}}{V_i^{old\,*}} - \sum_{i=1}^{j-1} Y_{ij} V_j^{new} - \sum_{i=j+1}^{n} Y_{ij} V_i^{old}\right]$$

Step 9 : If i is less than number of buses, increment i by 1 and go to step 6.

Step 10 : Compare two successive iteration values for V_i.

If $V_i^{new} - V_i^{old} <$ tolerance, go to step 12.

Step 11 : Update the new voltage as

$$V^{new} = V^{old} + \alpha\,(V^{new} - V^{old})$$

$$V^{old} = V^{new}$$

iter = iter + 1 ; go to step 5

Step 12 : Compute relevant quantities.

Slack bus power, $S_1 = P_i - j\,Q_i = V_1^* I = V_i^* \sum_{j=1}^{N} Y_{ij} V_j$

Line flows $S_{ij} = P_{ij} + j\,Q_{ij}$

$$= V_i[\,V_i^* - V_j^*\,]\,Y_{ij\ \text{series}}^* + |V_i|^2\,Y_{ii}^*$$

$$P_{Loss} = P_{ij} + P_{ji}$$

$$Q_{Loss} = Q_{ij} + Q_{ji}$$

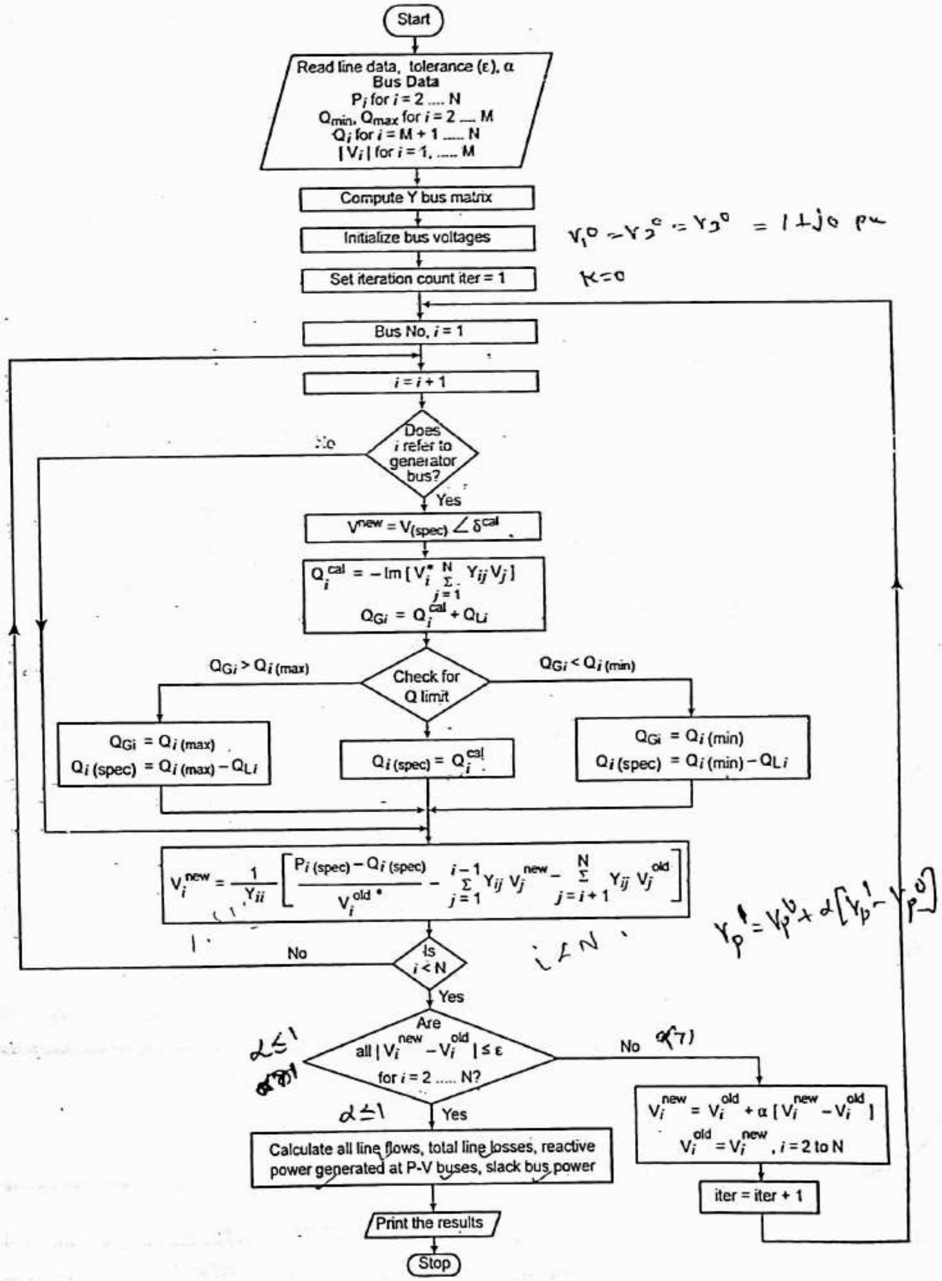
Step 13 : Stop the execution.

$Y_{11} V_1$

$Y_{12} V_2$

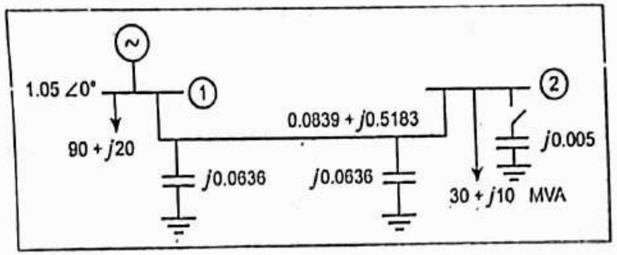
$Y_{13} V_3$

6.7.6. FLOW CHART FOR GAUSS-SEIDEL METHOD INCLUDING PV BUS ADJUSTMENT



SOLVED EXAMPLES

Example 6.1 *Perform power flow of one iteration for the system as shown in Fig., using Gauss-Seidal method. Determine slack bus power, line flows and line losses. Take base MVA as 100. ($\alpha = 1.1$).*



Solution :

Step 1 : Formulate Y_{bus}.

When the switch is open, there is no connection of capacitor at bus 2.

Take the bus 2 as load bus. (PV) bus

$$Y_{bus} = \begin{bmatrix} 0.3044 - j1.816 & -0.3044 + j1.88 \\ -0.3044 + j1.88 & 0.3044 - j1.816 \end{bmatrix}$$

Step 2 : Initialize bus voltages.

$$V_1^{old} = 1.05 \angle 0° \text{ p.u}$$

$$V_2^{old} = 1.0 \angle 0° \text{ p.u}$$

Step 3 : Calculate V_2^{new}.

$$P_2 = -30 \text{ MW} = \frac{-30}{100} \text{ p.u} = -0.3 \text{ p.u}$$

$$Q_2 = -10 \text{ MVAR} = \frac{-10}{100} \text{ p.u} = -0.1 \text{ p.u}$$

$$V_2^{new} = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{V_2^{old*}} - Y_{21} V_1^{new}\right]$$

$$= \frac{1}{0.3044 - j1.816}\left[\frac{-0.3 + j0.1}{1.0 \angle -0°} - (-0.3044 + j1.88)\, 1.05\right]$$

$$= 1.0054 - j0.1577 = 1.018 \angle -8.915°$$

Step 4 : Calculate V_2^{new} Using acceleration factor.

$$V_{2\,acc}^{new} = V_2^{old} + \alpha[V_2^{new} - V_2^{old}]$$

$$= 1.0 + 1.1[1.0054 - j0.1577 - 1]$$

$$= 1.0059 - j0.173 = 1.0207 \angle -9.78°$$

Step 5 : Slack-bus power.

$$S_1 = P_1 - jQ_1 = V_1^*[Y_{11} V_1 + Y_{12} V_2]$$

$$= 1.05 \angle -0° [(0.3044 - j1.816)\, 1.05 + (-0.3044 + j1.88)(1.0207 \angle -9.78°)]$$

$$= 0.3556 + j0.0388 \text{ p.u} = 35.56 + j3.88 \text{ MVA}$$

$$P_1 = 35.56 \text{ MW}, \quad Q_1 = -3.88 \text{ MVAR}$$

Real power generation $P_{G1} = P_1 + P_{L1}$

$$= 35.56 + 90 = 125.56 \text{ MW}$$

Reactive power generation $Q_{G1} = Q_1 + Q_{L1}$

$$= -3.88 + 20 = 16.12 \text{ MVAR}$$

Step 6 : Line flows.

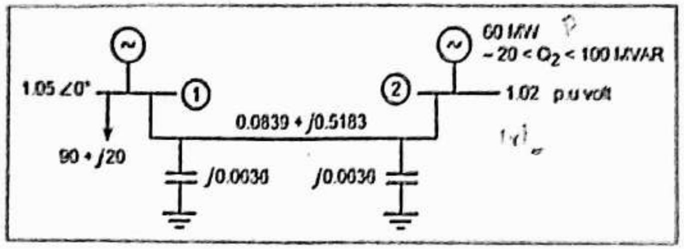
Bus From	Bus To	$S_{ij} = P_{ij} + jQ_{ij} = V_i[V_i^* - V_j^*]Y_{ij\,series}^* + \lvert V_i \rvert^2 Y_{Pi}^*$
1	2	$S_{12} = V_1[V_1^* - V_2^*]Y_{12\,series}^* + \lvert V_1 \rvert^* Y_{10}^*$ $= 1.05[1.05 \angle -0° - (1.0059 + j0.173)] \times$ $\boxed{(0.3044 + j1.88)} + 1.05^2 \times (-j0.0636)$ $= 0.3556 - j0.0383$ p.u $P_{12} = 0.3556$ p.u $= 35.56$ MW $Q_{12} = -0.0383$ p.u $= -3.83$ MVAR
2	1	$S_{21} = V_2[V_2^* - V_1^*]Y_{12\,series}^* + \lvert V_2 \rvert^2 Y_{20}^*$ $= (1.0059 - j0.173)[1.0059 + j0.173 - 1.05] \times$ $(0.3044 + j1.88) + 1.0207^2 \times (-j0.0636)$ $= -0.3459 - j0.038$ p.u $P_{21} = -0.3459$ p.u $= -34.59$ MW $Q_{21} = -0.038$ p.u $= -3.8$ MVAR

Step 7 : Transmission line loss ($S_{ij\,Loss} = S_{ij} + S_{ji}$)

$$P_{12\,Loss} = P_{12} + P_{21} = 35.56 - 34.59 = 0.97 \text{ MW}$$

$$Q_{12\,Loss} = Q_{12} + Q_{21} = -3.83 + (-3.8) = -7.63 \text{ MVAR}$$

Example 6.2 *Using Gauss-Seidal method, determine bus voltages, slack bus power, line flows and line losses for the Fig. shown.*



Solution :

Step 1 : Formulate Y_{bus}.

$$Y_{bus} = \begin{bmatrix} 0.3044 - j1.816 & -0.3044 + j1.88 \\ -0.3044 + j1.88 & 0.3044 - j1.816 \end{bmatrix}$$

Step 2 : Initialize bus voltages.

$$V_1^{old} = 1.05 \angle 0° \text{ p.u}$$
$$V_2^{old} = 1.02 \angle 0° \text{ p.u}$$

Step 3 : Calculate Q_2.

$$Q_2^{cal} = -\text{Im}\left\{ V_2^* \left[V_1 Y_{21} + V_2 Y_{22} \right] \right\}$$
$$= -\text{Im}\left\{ 1.02 \angle -0° \left[1.05 \angle 0° \times (-0.3044 + j1.88) + 1.02 \angle 0° (0.3044 - j1.816) \right] \right.$$
$$= -0.124 \text{ p.u} = -12.4 \text{ MVAR}$$

Check for Q_{limit} Violation :

$$Q_{2\,(min)} < Q_2^{cal} < Q_{2\,(max)}$$
$$-20 < -12.4 < 100$$

∴ This bus acts as generator bus.

$$Q_2 = -12.4 \text{ MVAR} = -0.124 \text{ p.u}$$

Step 4 : Calculate V_2^{new}.

$$V_2^{new} = \frac{1}{Y_{22}} \left[\frac{P_2 - j Q_2}{V_2^{old\,*}} - Y_{21} V_1^{new} \right]$$
$$= \frac{1}{0.3044 - j1.816} \left[\frac{0.6 + j0.124}{1.02 \angle -0°} - (-0.3044 + j1.88) \cdot 1.05 \right]$$
$$= \frac{0.9078 - j1.8524}{0.3044 - j1.816} = 1.073 + j0.32 = 1.12 \angle 16.61°$$
$$V_2^{new} = V_{2\,(spec)} \angle \delta_2^{cal} = 1.02 \angle 16.61°$$

Step 5 : Using acceleration factor,

$$V_{2\,acc}^{new} = V_2^{old} + \alpha \left[V_2^{new} - V_2^{old} \right]$$
$$= 1.02 + 1.1 \left[1.02 \angle 16.61° - 1.02 \right]$$
$$= 1.02 + 1.1 \left[0.977 + j0.292 - 1.02 \right]$$
$$= 0.973 + j0.32 = 1.02 \angle 18.24°$$

Bus voltages are

$$V_1 = 1.05 \angle 0°$$
$$V_2 = 1.02 \angle 18.24°$$

Step 6 : Slack bus power

$$S_1 = P_1 - j Q_1 = V_1^* \left[Y_{11} V_1 + Y_{12} V_2 \right]$$
$$= 1.05 \angle -0° \left[(0.3044 - j1.816) \times 1.05 + (-0.3044 + j1.88)(1.02 \angle 18.24°) \right]$$
$$= -0.607 - j0.18 \text{ p.u}$$
$$= -60.7 - j18 \text{ p.u}$$
$$\therefore P_1 = -60.7 \text{ MW}, \quad Q_1 = 18 \text{ MVAR}$$

Real power generation $P_{G1} = P_1 + P_{L1}$

$$= -60.7 + 90 = 29.3 \text{ MW}$$

Reactive power generation $Q_{G1} = Q_1 + Q_{L1}$

$$= 18 + 20 = 38 \text{ MVAR}$$

Step 7 : Line flows.

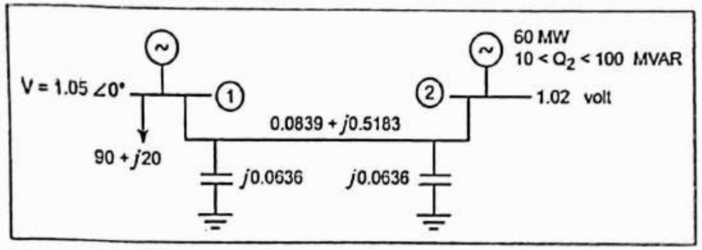
Bus From	Bus To	$S_{ij} = P_{ij} + j\,Q_{ij} = V_i \left[V_i^* - V_j^* \right] Y_{ij\,series}^* + \lvert V_i \rvert^2 Y_{Pi}^*$ where $Y_{Pi} = Y_{io}$
1	2	$S_{12} = V_1 \left[V_1^* - V_2^* \right] Y_{12\,series}^* + \lvert V_1 \rvert^2 Y_{10}^*$ $= 1.05 \left[1.05 \angle -0° - (0.973 - j0.32) \right] \times (0.3044 + j1.88) + 1.05^2 \times (-j0.0636)$ $= -0.607 + j0.18$ p.u $P_{12} = -0.607$ p.u $= -60.7$ MW, $Q_{12} = 0.18$ p.u $= 18$ MVAR
2	1	$S_{21} = V_2 \left[V_2^* - V_1^* \right] Y_{21\,series}^* + \lvert V_2 \rvert^2 Y_{20}^*$ $= (0.973 + j0.32) \left[0.973 - j0.32 - 1.05 \right] \times (0.3044 + j1.88) + 1.02^2 \times (-j0.0636)$ $= 0.64 - j0.1167$ p.u $P_{21} = 0.64$ p.u $= 64$ MW $Q_{21} = -0.1167$ p.u $= -11.67$ MVAR

Step 8 : Transmission loss $(S_{ij\,Loss} = S_{ij} + S_{ji})$

$$P_{12\,Loss} = P_{12} + P_{21} = -60.7 + 64 = 3.3 \text{ MW}$$

$$Q_{12\,Loss} = Q_{12} + Q_{21} = 18 - 11.67 = 6.33 \text{ MVAR}$$

Example 6.3 ***Using Gauss-Seidal method, determine bus voltages, for the Fig. shown. Take Base MVA–100, $\alpha = 1.1$.***



Solution :

Step 1 : Formulate Y_{bus}.

$$Y_{bus} = \begin{bmatrix} 0.3044 - j1.816 & -0.3044 + j1.88 \\ -0.3044 + j1.88 & 0.3044 - j1.816 \end{bmatrix}$$

Step 2 : Initialize bus voltages.

$$V_1^{old} = 1.05 \angle 0° \text{ p.u}$$

$$V_2^{old} = 1.02 \angle 0° \text{ p.u}$$

Step 3 : Calculate Q value for generator bus.

$$Q_2^{cal} = -\text{Im}\{V_2^* [V_1 Y_{21} + V_2 Y_{22}]\}$$

$$= -\text{Im}\{1.02 [1.05 \angle 0° \times (-0.3044 + j1.88) + 1.02 \angle 0° (0.3044 - j1.816)]\}$$

$$= -0.124 \text{ p.u}$$

Check for Q_{limit} violation :

$$Q_{2\,(min)} = 10 \text{ MVAR}, \quad Q_{2\,(min)} = \frac{10}{100} = 0.1 \text{ p.u}$$

$$Q_2^{cal} < Q_{2\,(min)}$$

i.e., $-0.124 < 0.1$

$\therefore$ Bus 2 will act as load bus.

$$Q_2 = Q_{2\,(min)} = 0.1 \text{ p.u}$$

$$V_2^{old} = 1.0 \angle 0° \text{ p.u}$$

$$P_2 = 60 \text{ MW} = 0.6 \text{ p.u}$$

Step 4 : Calculate V_2^{new}

$$V_2^{new} = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{V_2^{old*}} - Y_{21} V_1^{new}\right]$$

$$= \frac{1}{0.3044 - j1.816}\left[\frac{0.6 - j0.1}{1.0 \angle 0°} - (-0.3044 + j1.88)\, 1.05\right]$$

$$= \frac{1}{1.842 \angle -80.49°}[0.9196 - j2.074]$$

$$= \frac{1}{1.842 \angle -80.49°}[2.2687 \angle -66.087°]$$

$$= 1.2316 \angle 14.403° = 1.193 + j0.306$$

Step 5 : Using acceleration factor.

$$V_{2\,acc}^{new} = V_2^{old} + \alpha [V_2^{new} - V_2^{old}]$$

$$= 1 + 1.1 [1.193 + j0.306 - 1]$$

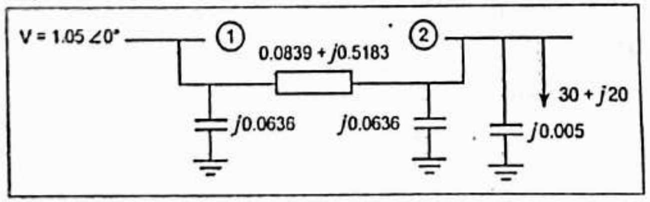
$$= 1.2098 + j0.33$$

$$= 1.254 \angle 15.26°$$

Bus voltages are $V_1 = 1.05 \angle 0°$

$$V_2 = 1.254 \angle 15.26°$$

Example 6.4 ***Using Gauss-Seidal method, determine bus voltages and reactive power generation for the Fig. shown. Take Base MVA = 100.***



Solution :

Step 1 : Formulate Y_{bus}.

$$Y_{bus} = \begin{bmatrix} Y_{10} + Y_{12} & -Y_{12} \\ -Y_{12} & Y_{20} + Y_{2C} + Y_{12} \end{bmatrix}$$

where Y_{2C} = Admittance of shunt element

$$= \begin{bmatrix} 0.3044 - j1.816 & -0.3044 + j1.88 \\ -0.3044 + j1.88 & 0.3044 - j1.811 \end{bmatrix}$$

Step 2 : Initialize bus voltages.

$V_1^{old} = 1.05\angle 0°$ p.u (slack bus)

$V_2^{old} = 1.0\angle 0°$ p.u (P-V bus because capacitor generates reactive power)

Step 3 : Calculate Q_2.

$$Q_2^{cal} = -\text{Im}\{V_2^*[V_1 Y_{21} + V_2 Y_{22}]\}$$

$$= -\text{Im}\{1.0\angle 0°[1.05\times(-0.3044 + j1.88) + 1.0(0.3044 - j1.811)]$$

$$= -0.163 \text{ p.u}$$

Step 4 : Calculate V_2^{new}.

$$P_2 = P_{G2} - P_{L2} = \frac{-30}{100} = -0.3 \text{ p.u}; \quad Q_2 = -0.163 \text{ p.u}$$

$$V_2^{new} = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{V_2^{old*}} - Y_{21}V_1^{new}\right]$$

$$= \frac{1}{0.3044 - j1.811}\left[\frac{-0.3 + j0.163}{1.0\angle -0°} - (-0.3044 + j1.88)\times 1.05\right]$$

$$= 0.973 - j0.1611 = 0.986\angle -9.4°$$

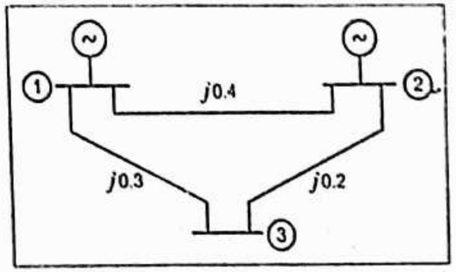
$$= 1.0\angle -9.4°$$

Reactive power generation $Q_{G2} = Q_2 + Q_{L2}$

$= -0.163 + 0.2 = 0.037$ p.u $= 3.7$ MVAR

Reactive bus power $Q_2 = -0.163$ p.u MVAR $= -16.3$ MVAR

Example 6.5 *For the system shown in Fig., determine the voltages at the end of the first iteration by Gauss-Seidel method and also find the slack bus power, line flows, transmission loss. Assume base MVA as 100.*



Solution :

Bus No.	Voltage	Generator P	Generator Q	Load P	Load Q	Q_{min} MVAR	Q_{max} MVAR
1.	$1.05\angle 0°$ p.u	–	–	–	–	–	–
2.	1.02 p.u	0.3 p.u	–	–	–	–10	100
3.	–	–	–	0.4 p.u	0.2 p.u	–	–

Step 1 : From Y bus.

$$Y_{bus} = \begin{bmatrix} \frac{1}{Y_{11}} & \frac{-1}{Y_{12}} & \frac{-1}{Y_{13}} \\ \frac{-1}{Y_{21}} & \frac{1}{Y_{22}} & \frac{-1}{Y_{23}} \\ \frac{-1}{Y_{31}} & \frac{-1}{Y_{32}} & \frac{1}{Y_{33}} \end{bmatrix} = \begin{bmatrix} \frac{1}{j0.4}+\frac{1}{j0.3} & \frac{-1}{j0.4} & \frac{-1}{j0.3} \\ \frac{-1}{j0.4} & \frac{1}{j0.4}+\frac{1}{j0.2} & \frac{-1}{j0.2} \\ \frac{-1}{j0.3} & \frac{-1}{j0.2} & \frac{1}{j0.3}+\frac{1}{j0.2} \end{bmatrix}$$

$$= \begin{bmatrix} -j5.8333 & j2.5 & j3.3333 \\ j2.5 & -j7.5 & j5 \\ j3.3333 & j5 & -j8.3333 \end{bmatrix}$$

Step 2 : Initialize bus voltages.

$V_1^{old*} = 1.05\angle 0°$ p.u [Bus 1 is a slack bus *i.e.*, V and δ is specified]

$V_2^{old*} = 1.02\angle 0°$ p.u [Bus 2 is a PV bus *i.e.*, P and V is specified]

$V_3^{old*} = 1.0\angle 0°$ p.u [Bus 3 is a load bus *i.e.*, P and Q is specified]

Note For slack bus, the specified voltage will not change in any iteration.

For generation bus, calculate V_i^{new} using the formula and write

$$V_i^{new} = V_{specified}\angle\delta_{calculated\ value}$$

Step 3 : Calculate Q value for all generator buses.

$$Q_i^{cal} = -\text{Im}\left\{V_i^{old*}\left[\sum_{j=1}^{i-1} Y_{ij}V_j^{new} + \sum_{j=i}^{N} Y_{ij}V_j^{old}\right]\right\}$$

$$Q_2^{cal} = -\text{Im}\left\{V_2^{old*}\left[Y_{21}V_1^{new} + Y_{22}V_2^{old} + Y_{23}V_3^{old}\right]\right\}$$

$$= -\text{Im}\left\{1.02\angle 0°[j2.5\times 1.05\angle 0° + (-j7.5\times 1.02\angle 0°) + j5\times 1\angle 0°]\right\}$$

$$= -\text{Im}\left\{1.02\angle 0°[j2.625 - j7.65 + j5]\right\}$$

-Im{Re+jIm}

-Im

$$\boxed{Q_2^{cal} = 0.025 \text{ p.u}}$$

Now $Q_{2(min)} \le Q_2^{cal} \le Q_{2(max)}$

i.e., Q_2^{cal} is within the specified limit.

Step 4 : Calculate V_i^{new}.

$$V_1^{new} = 1.05\angle 0° \text{ p.u}$$

$$V_i^{new} = \frac{1}{Y_{ii}}\left[\frac{P_i - jQ_i}{V_i^{old*}} - \sum_{j=1}^{i-1} Y_{ij} V_j^{new} - \sum_{j=i+1}^{N} Y_{ij} V_j^{old}\right]$$

$$V_2^{new} = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{V_2^{old*}} - Y_{21} V_1^{new} - Y_{23} V_3^{old}\right]$$

$$P_2 = 0.3 \text{ p.u (Given)}; \quad Q_2 = 0.025 \text{ p.u}$$

$$V_2^{new} = \frac{1}{-j7.5}\left[\frac{0.3 - j0.025}{1.02\angle 0^\circ} - j2.5 \times 1.05\angle 0^\circ - j5 \times 1\angle 0^\circ\right]$$

$$= 1.0199 + j0.0392$$

$$= 1.0207\angle 2.2^\circ$$

$$V_2^{new} = V_{2\,(spec)}\angle \delta_2^{cal} = 1.02\angle 2.2^\circ = 1.0192 + j0.0392$$

$$P_3 = P_{G3} - P_{L3} = 0 - 0.4 = -0.4 \text{ p.u}$$

$$Q_3 = Q_{G3} - Q_{L3} = 0 - 0.2 = -0.2 \text{ p.u}$$

$$V_3^{new} = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{V_3^{old*}} - Y_{31} V_1^{new} - Y_{32} V_2^{new}\right]$$

$$= \frac{1}{-j8.3333}\left[\frac{-0.4 + j0.2}{1.0\angle 0^\circ} - j3.3333 \times 1.05\angle 0^\circ - j5 \times 1.02\angle 2.2^\circ\right]$$

$$= \frac{1}{-j8.3333}[-0.4 + j0.2 - j3.4999 - j5.096 + 0.196]$$

$$= 1.0075 - j0.0244 = 1.0078\angle -1.39^\circ$$

$$V_3^{new} = 1.0075 - j0.0244$$

$$= 1.0078\angle -1.39^\circ$$

Step 5 : Slack Bus Power

$$S_1 = P_1 - jQ_1 = V_1^* \sum_{j=1}^{N} Y_{ij} V_j$$

$$S_1 = V_1^*[Y_{11}V_1 + Y_{12}V_2 + Y_{13}V_3]$$

$$= 1.05[-j5.8333 \times 1.05\angle 0^\circ + j2.5 \times (1.0192 + j0.0392) + j3.3333(1.0075 - j0.0244)]$$

$$= -0.0175 - j0.2295 \text{ p.u}$$

$$P_1 = -0.0175 \text{ p.u} = -1.75 \text{ MW}$$

$$Q_1 = 0.2295 \text{ p.u} = 22.95 \text{ MVAR}$$

Step 6 : Line Flow

$$S_{ij} = P_{ij} + jQ_{ij} = V_i[V_i^* - V_j^*]Y_{ij\,series}^* + |V_i|^2 Y_{P_i}^*$$

Line flow from bus 1 to 2.

$$S_{12} = P_{12} + jQ_{12} = V_1[V_1^* - V_2^*]Y_{12\,series}^*$$

$$= 1.05[(1.05\angle -0^\circ) - 1.0192 + j0.0392]j2.5$$

$$= -0.1029 + j0.0808 \text{ p.u}$$

$$S_{21} = P_{21} + jQ_{21} = V_2[V_2^* - V_1^*]Y_{21\,series}^*$$

$$= 1.0192 + j0.0392[1.0192 - j0.0392 - 1.05]j2.5$$

$$= 0.1029 - j0.0746 \text{ p.u}$$

$$S_{23} = P_{23} + jQ_{23}$$

$$S_{23} = V_2[V_2^* - V_3^*]Y_{23\,series}^*$$

$$= 1.0192 + j0.0392[1.0192 - j0.0392 - 1.0075 - j0.0244]j5$$

$$= 0.3218 + j0.072 \text{ p.u}$$

$$S_{32} = P_{32} + jQ_{32}$$

$$S_{32} = V_3[V_3^* - V_2^*]Y_{32\,series}^*$$

$$= 1.0075 - j0.0244[1.0075 + j0.0244 - 1.0192 + j0.0392]j5$$

$$= -0.3218 - j0.0512 \text{ p.u}$$

$$S_{13} = P_{13} + jQ_{13}$$

$$S_{13} = V_1[V_1^* - V_3^*]Y_{13\,series}^*$$

$$= 1.05[1.05\angle -0^\circ - 1.0075 - j0.0244]j3.3333$$

$$= 0.085 + j0.148 \text{ p.u}$$

$$S_{31} = P_{31} + jQ_{31}$$

$$S_{31} = V_3[V_3^* - V_1^*]Y_{31\,series}^*$$

$$= 1.0075 - j0.0244 \times [1.0075 + j0.0244 - 1.05] \times j3.3333$$

$$= -0.085 - j0.1407 \text{ p.u}$$

Transmission Loss

$$S_{ij\,Loss} = S_{ij} + S_{ji}$$

For line 1–2,

$$S_{12\,Loss} = P_{12\,Loss} + jQ_{12\,Loss} = S_{12} + S_{21}$$

$$S_{12\,Loss} = -0.1029 + j0.0808 + 0.1029 - j0.0746$$

$$= 0 + j0.0061$$

$$P_{12\,Loss} = 0, \quad Q_{12\,Loss} = 0.0061 \text{ p.u} = 0.61 \text{ MVAR}$$

For line 2–3,

$$S_{23\,Loss} = P_{23\,Loss} + jQ_{23\,Loss} = S_{23} + S_{32}$$

$$= 0.3218 + j0.072 + (-0.3218 - j0.0512)$$

$$= 0 + j0.021$$

$$P_{23\,Loss} = 0, \quad Q_{23\,Loss} = 0.021 \text{ p.u} = 2.1 \text{ MVAR}$$

For line 1–3,

$$S_{13\,Loss} = P_{13\,Loss} + j\,Q_{13\,Loss} = S_{13} + S_{31}$$
$$= 0.085 + j0.148 + [-0.085 - j0.1407]$$
$$= 0 + j0.00726$$
$$P_{13\,Loss} = 0, \quad Q_{13\,Loss} = 0.00726 \text{ p.u} = 0.726 \text{ MVAR}$$

Example 6.6 ***Resolve the previous example 6.5, the reactive power constraint on generator bus-2 be changed to $10 \le Q_2 \le 100$. Determine slack bus power. [Q_2 in MVAR].***

☺ ***Solution :***

Step 1 :

$$Y_{bus} = \begin{bmatrix} -j5.8333 & j2.5 & j3.3333 \\ j2.5 & -j7.5 & j5 \\ j3.3333 & j5 & -j8.3333 \end{bmatrix}$$

Step 2 : Initialize bus voltages.

$$V_1^{old} = 1.05 \angle 0° \text{ p.u}$$
$$V_2^{old} = 1.02 \angle 0° \text{ p.u}$$
$$V_3^{old} = 1.0 \angle 0° \text{ p.u}$$

Step 3 : Calculate Q value for generator bus.

$$Q_2^{cal} = -\text{Im}\left[1.02 \angle -0° \,[\,j2.5 \times 1.05 \angle 0° + (-j7.5 \times 1.02 \angle 0°) + j5 \times 1 \angle 0°]\right]$$
$$= 0.025 \text{ p.u}$$

$Q_2^{cal} < Q_{2\,(min)}$ [Q_2 exceeds the limit, ∴ Bus 2 will act as load bus, *i.e.*, $V_2^{old} = 1.0 \angle 0°$]

$$\text{Substituting } Q_2 = Q_{2\,(min)} = 10 \text{ MVAR}$$
$$= \frac{10}{100} = 0.1 \text{ p.u}$$
$$V_2^{old} = 1.0 \angle 0°$$

Step 4 : Calculate V_i^{new}.

$$V_2^{new} = \frac{1}{Y_{22}}\left[\frac{P_2 - j\,Q_2}{V_2^{old\,*}} - Y_{21}\,V_1^{new} - Y_{23}\,V_3^{old}\right]$$
$$= \frac{1}{-j7.5}\left[\frac{0.3 - j0.1}{1 \angle 0°} - j2.5 \times 1.05 \angle 0° - j5 \times 1 \angle 0°\right]$$
$$= 1.03 + j0.04 = 1.0308 \angle 2.22°$$

$$V_3^{new} = \frac{1}{Y_{33}}\left[\frac{P_3 - j\,Q_3}{V_3^{old\,*}} - Y_{31}\,V_1^{new} - Y_{32}\,V_2^{new}\right]$$
$$= \frac{1}{-j8.3333}\left[\frac{-0.4 + j0.2}{1.0 \angle -0°} - j3.3333 \times 1.05 \angle 0° - j5 \times 1.0308 \angle 2.22°\right]$$
$$= 1.014 - j0.024$$
$$= 1.014 \angle -1.36°$$

$$\boxed{\begin{aligned} V_1^{new} &= 1.05 \angle 0° \\ V_2^{new} &= 1.0308 \angle 2.22° \\ V_3^{new} &= 1.014 \angle -1.36° \end{aligned}}$$

Slack Bus Power :

$$S_1 = P_1 - j\,Q_1 = V_1^*\,[\,Y_{11}\,V_1 + Y_{12}\,V_2 + Y_{13}\,V_3\,]$$
$$= 1.05 \angle -0° \,[-j5.8333 \times 1.05 \angle 0° + j2.5 \times (1.0308 \angle 2.22°) + j3.3333\,(1.014 \angle -1.36°)]$$
$$= -0.0206 - j0.1794 \text{ p.u}$$
$$P_1 = -0.0206 \text{ p.u} = -2.06 \text{ MW}$$
$$Q_1 = 0.1794 \text{ p.u} = 17.94 \text{ MVAR}$$

Example 6.7 ***Resolve example 6.6, the reactive power constraint on generator bus 2 be changed to $-0.04 < Q_2 < 0.01$. [Given Q_2 in p.u]***

☺ ***Solution :***

Step 1 : Formulate Y-bus.

$$Y_{bus} = \begin{bmatrix} -j5.8333 & j2.5 & j3.3333 \\ j2.5 & -j7.5 & j5 \\ j3.3333 & j5 & -j8.3333 \end{bmatrix}$$

Step 2 : Initialize bus voltages.

$$V_1^{old} = 1.05 \angle 0° \text{ p.u}$$
$$V_2^{old} = 1.02 \angle 0° \text{ p.u}$$
$$V_3^{old} = 1.0 \angle 0° \text{ p.u}$$

Step 3 : Calculate Q value for generator bus.

$$Q_2^{cal} = -\text{Im}\,[1.02 \angle -0° \,(j2.5 \times 1.05 \angle 0° + (-j7.5 \times 1.02) + j5 \times 1 \angle 0°)\,]$$
$$= 0.025 \text{ p.u}$$

$Q_2^{cal} > Q_{2\,(max)}$ [Q_2 exceeds the limit; ∴ Bus 2 will act as load bus, *i.e.*, $V_2^{old} = 1.0 \angle 0°$]

$$\text{Substituting } Q_2 = Q_{2\,(max)} = 0.01 \text{ p.u}$$

Step 4 : Calculate V_i^{new}.

$$V_2^{new} = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{V_2^{old*}} - Y_{21}V_1^{new} - Y_{23}V_3^{old}\right]$$

$$= \frac{1}{-j7.5}\left[\frac{0.3 - j0.01}{1.0\angle 0°} - j2.5 \times 1.05\angle 0° - j5 \times 1\angle 0°\right]$$

$$= 1.018 + j0.04 = 1.0187\angle 2.25°$$

$$V_3^{new} = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{V_3^{old*}} - Y_{31}V_1^{new} - Y_{32}V_2^{new}\right]$$

$$= \frac{1}{-j8.3333}\left[\frac{-0.4 + j0.2}{1.0\angle 0°} - j3.3333 \times 1.05\angle 0° - j5\,(1.018 + J0.04)\right]$$

$$= 1.0068 - j0.024 = 1.0071\angle -1.37°$$

$$\boxed{\begin{aligned} V_1^{new} &= 1.05 + j0 \\ V_2^{new} &= 1.0187\angle 2.25° \\ V_3^{new} &= 1.0071\angle -1.37° \end{aligned}}$$

Slack Bus Power :

$$S_1 = P_1 - jQ_1 = V_1^*[Y_{11}V_1 + Y_{12}V_2 + Y_{13}V_3]$$

$$= 1.05\angle -0°\,[-j5.8333 \times 1.05\angle 0° + j2.5 \times (1.018 + j0.04) + j3.3333\,(1.0068 - j0.024)]$$

$$= -0.021 - j0.2352 \text{ p.u}$$

$$P_1 = -0.021 \text{ p.u} = -2.1 \text{ MW}$$

$$Q_1 = 0.2352 \text{ p.u} = 23.52 \text{ MVAR}$$

Line Flows :

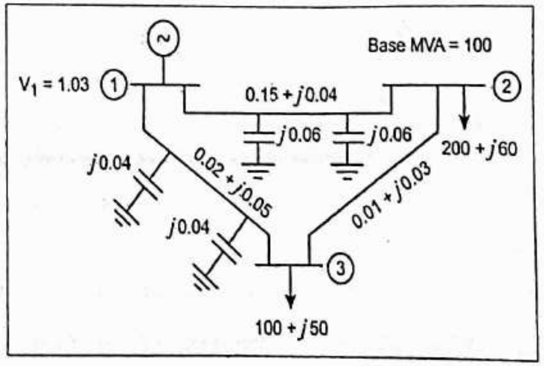
Bus No.		$S_{ij} = P_{ij} + jQ_{ij} = V_i[V_i^* - V_j^*]Y_{ij\ series}^* + \|V_i\|^2 Y_{Pi}^*$
From	To	
1	2	$S_{12} = 1.05\angle 0°\,[1.05\angle -0° - 1.018 + j0.04]\,j2.5$ $= -0.105 + j0.084$
2	1	$S_{21} = (1.018 + j0.04)\,[1.018 - j0.04 - 1.05]\,j2.5$ $= 0.105 - j0.0774$
2	3	$S_{23} = (1.018 + j0.04)\,[1.018 - j0.04 - 1.0068 - j0.024] \times j5$ $= 0.323 + j0.069$

Bus No.		$S_{ij} = P_{ij} + jQ_{ij} = V_i[V_i^* - V_j^*]Y_{ij\ series}^* + \|V_i\|^2 Y_{Pi}^*$
From	To	
3	2	$S_{32} = (1.0068 - j0.024)\,[1.0068 + j0.024 - 1.018 + j0.04] \times j5$ $= -0.323 - j0.048$
1	3	$S_{13} = 1.05\angle 0°\,[1.05\angle -0° - 1.0068 - j0.024] \times j3.3333$ $= 0.0839 + j0.1512$
3	1	$S_{31} = (1.0068 - j0.024)\,[1.0068 + j0.024 - 1.05] \times j3.3333$ $= -0.0839 - j0.143$

Transmission Loss :

$$S_{12\,Loss} = S_{12} + S_{21}$$
$$= -0.105 + j0.084 + 0.105 - j0.0774$$
$$= 0 + j0.0066$$
$$P_{12\,Loss} = 0,\ Q_{12\,Loss} = 0.0066 \text{ p.u} = 0.66 \text{ MVAR}$$
$$S_{23\,Loss} = S_{23} + S_{32}$$
$$= 0.323 + j0.069 - 0.323 - j0.048$$
$$= 0 + j0.021$$
$$P_{23\,Loss} = 0,\ Q_{23\,Loss} = 0.021 \text{ p.u} = 2.1 \text{ MVAR}$$
$$S_{13\,Loss} = S_{13} + S_{31}$$
$$= 0.0839 + j0.1512 - 0.0839 - j0.143$$
$$= 0 + j0.0082$$
$$P_{13\,Loss} = 0,\ Q_{13\,Loss} = 0.0082 \text{ p.u} = 0.82 \text{ MVAR}$$

Example 6.8 ***Perform Gauss-Seidel load flow for the system shown in Fig. and the bus data is given in Table. Determine bus voltages, slack bus power, line flows, and transmission line losses.***



☺ ***Solution : Step 1 :*** Formulate Y_{bus} matrix.

$$Y_{bus} = \begin{bmatrix} Y_{12}+Y_{13}+Y_{10} & -Y_{12} & -Y_{13} \\ -Y_{12} & Y_{12}+Y_{23}+Y_{20} & -Y_{23} \\ -Y_{13} & -Y_{23} & Y_{31}+Y_{32}+Y_{30} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{0.15+j0.04}+\frac{1}{0.02+j0.05}+j0.06+j0.04 & \frac{-1}{0.15+j0.04} & \frac{-1}{0.02+j0.05} \\ \frac{-1}{0.15+j0.04} & \frac{1}{0.15+j0.04}+\frac{1}{0.01+j0.03}+j0.06 & \frac{-1}{0.01+j0.03} \\ \frac{-1}{0.02+j0.05} & \frac{-1}{0.01+j0.03} & \frac{1}{0.01+j0.03}+\frac{1}{0.02+j0.05}+j0.04 \end{bmatrix}$$

$$= \begin{bmatrix} 13.1206-j18.8011 & -6.2241+j1.6598 & -6.8966+j17.2414 \\ -6.2241+j1.6598 & 16.2241-j31.5998 & -10+j30 \\ -6.8966+j17.2414 & -10+j30 & 16.8966-j47.2014 \end{bmatrix}$$

Step 2 : Initialize bus voltages.

$V_1^{old} = 1.03 \angle 0^\circ$ p.u (slack bus)

$V_2^{old} = 1.0 \angle 0^\circ$ p.u (PQ bus)

$V_3^{old} = 1.0 \angle 0^\circ$ p.u (PQ bus)

Step 3 : Calculate V_2^{new}.

$$P_2 = -P_{L2} = \frac{-200}{100} = -2; \quad Q_2 = -Q_{L2} = \frac{-60}{100} = -0.6 \text{ p.u}$$

$$V_2^{new} = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{V_2^{old*}} - Y_{21}V_1^{new} - Y_{23}V_3^{old}\right]$$

$$= \frac{1}{16.2241-j31.5998}\left[\frac{-2+j0.6}{1.0\angle -0^\circ} - (-6.2241+j1.6598)\,1.03\angle 0^\circ - (-10+j30)\times 1.0\angle 0^\circ\right]$$

$$= \frac{14.4108-j31.1096}{16.2241-j31.5998} = 0.9644 - j0.0391$$

$$= 0.965 \angle -2.32^\circ$$

Given $\alpha = 1.2$,

$$V_2^{new} = 1.0 + 1.2\,[0.9644 - j0.0391 - 1.0]$$

$$= 0.9573 - j0.047 = 0.9584 \angle -2.81^\circ$$

Calculate V_3^{new} :

$$P_3 = P_{G3} - P_{L3} = \frac{160-60}{100} = -1 \text{ p.u}$$

$$Q_3 = 0 - Q_{L3} = \frac{-50}{100} = -0.5 \text{ p.u}$$

$$V_3^{new} = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{V_3^{old*}} - Y_{31}V_1^{new} - Y_{32}V_2^{new}\right]$$

$$= \frac{1}{16.8966-j47.2014}\left[\frac{-1+j0.5}{1\angle -0^\circ} - (-6.8966+j17.2414)\times 1.03 - (-10+j30)\,[0.9573-j0.047]\right]$$

$$= 0.9682 - j0.0443 = 0.969 \angle -2.61^\circ \text{ p.u}$$

Given $\alpha = 1.2$

$$V_3^{new} = V_3^{old} + \alpha\,[V_3^{new} - V_3^{old}]$$

$$= 1 + j0 + 1.2\,[0.9682 - j0.0443 - (1+j0)]$$

$$= 0.96184 - j0.05316 = 0.9633 \angle -3.16^\circ$$

Step 4 : Slack Bus Power :

$$S_1 = P_1 - jQ_1 = V_1^*\,[Y_{11}V_1 + Y_{12}V_2 + Y_{13}V_3]$$

$$= 1.03\angle -0^\circ\,[(13.1206-j18.8011)\,1.03\angle 0^\circ + (-6.2241+j1.6598)\times 0.9584\angle -2.81^\circ + (-6.8966+j17.2414)\,(0.9633\angle -3.16^\circ$$

$$= 1.03\angle -0^\circ\,[(13.1206-j18.8011)\,1.03 + (-6.2241+j1.6598)\times (0.9572-j0.047) + (-6.8966+j17.2414)\,(0.9618-j0.053$$

$$= 1.9731 - j0.5516 \text{ p.u}$$

$$= 197.31 - j55.16 \text{ MVA}$$

$$P_1 = 197.31 \text{ MW}$$

$$Q_1 = 55.16 \text{ MVAR}$$

Step 5 : Line Flows :

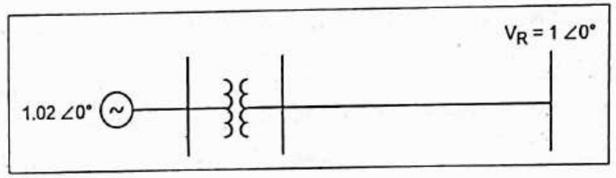
Bus No. From	Bus No. To	$S_{ij} = P_{ij} + jQ_{ij} = V_i\,[V_i^* - V_j^*]\,Y_{ij\,series}^* + \lvert V_i\rvert^2 Y_{Pi}^*$
1	2	$S_{12} = 1.03\,[1.03 - (0.9573 + j0.047)]\,(6.22 + j1.66) + 1.03^2\,(-j0.06)$ $= 0.5461 - j0.2405$ p.u $P_{12} = 0.5461$ p.u $= 54.61$ MW, $Q_{12} = -0.2405$ p.u $= -24.05$ MVAR
2	1	$S_{21} = (0.9573 - j0.047)\,[0.9573 + j0.047 - 1.30]\,(6.22 + j1.66) + 0.9584^2\,(-j0.06$ $= -0.4995 + j0.1341$ p.u $P_{21} = -0.4995$ p.u $= -49.95$ MW $Q_{21} = 0.1341$ p.u $= 13.41$ MVAR

Bus No.		$S_{ij} = P_{ij} + j\,Q_{ij} = V_i\,[V_i^* - V_j^*]\,Y_{ij\,series}^* + \vert V_i\vert^2\,Y_{Pi}^*$
From	To	
2	3	$S_{23} = (0.9573 - j0.047)\,[0.9573 + j0.047 - (0.9618 + j0.053)] \times (10 + j30) + 0.9584^2 \times 0$ $= 0.1201 - j0.1930$ p.u $P_{23} = 0.1201$ p.u $= 12.01$ MW, $Q_{23} = -0.193$ p.u $= -19.3$ MVAR
3	2	$S_{32} = (0.9618 - j0.053)\,[0.9618 + j0.053 - (0.9573 + j0.047)] \times (10 + j30) + 0$ $= -0.1195 + j0.1947$ p.u $P_{32} = -0.1195$ p.u $= -11.95$ MW, $Q_{32} = 0.1947$ p.u $= 19.47$ MVAR
1	3	$S_{13} = 1.03\,[1.03 - (0.9618 + j0.053)]\,(6.9 + j17.24) + 1.03^2\,(-j0.04)$ $= 1.426 + j0.7919$ p.u $P_{13} = 1.426$ p.u $= 142.6$ MW $Q_{13} = 0.7919$ p.u $= 79.19$ MVAR
3	1	$S_{31} = (0.9618 - j0.053)\,(0.9618 + j0.053 - 1.03)\,(6.9 + j17.24) + 0.9633^2 \times (-j0.04)$ $= -1.374 - j0.7429$ p.u $P_{13} = -1.374$ p.u $= -137.4$ MW $Q_{13} = -0.7429$ p.u $= -74.29$ MVAR

Step 6 : Transmission Line Loss

Bus No.		$S_{ij\,Loss} = S_{ij} + S_{ji}$
From	To	
1	2	$P_{12\,Loss} = P_{12} + P_{21} = 54.61 - 49.95 = 4.66$ MW $Q_{12\,Loss} = Q_{12} + Q_{21} = -24.05 + 13.41 = -10.64$ MVAR
2	3	$P_{23\,Loss} = P_{23} + P_{32} = 12.01 - 11.95 = 0.06$ MW $Q_{23\,Loss} = Q_{23} + Q_{32} = -19.3 + 19.47 = 0.17$ MVAR
1	3	$P_{13\,Loss} = P_{13} + P_{31} = 142.6 - 137.4 = 5.2$ MW $Q_{13\,Loss} = Q_{13} + Q_{31} = 79.19 - 74.29 = 4.9$ MVAR

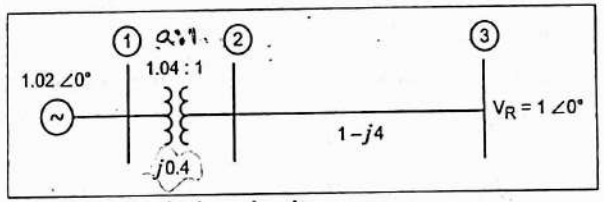
Example 6.9 ***For the system shown in Fig., find the voltage at receiving bus at the end of first iteration using Gauss-Seidal method, voltage at sending-end is 1.02 ∠0° p.u. Line admittance is 1.0 – j4 p.u, transformer reactance is j0.4 p.u and off nominal turns ratio is 1.04. Assume $V_R = 1\angle 0°$. Determine slack bus power, line flows, transformer flows.***



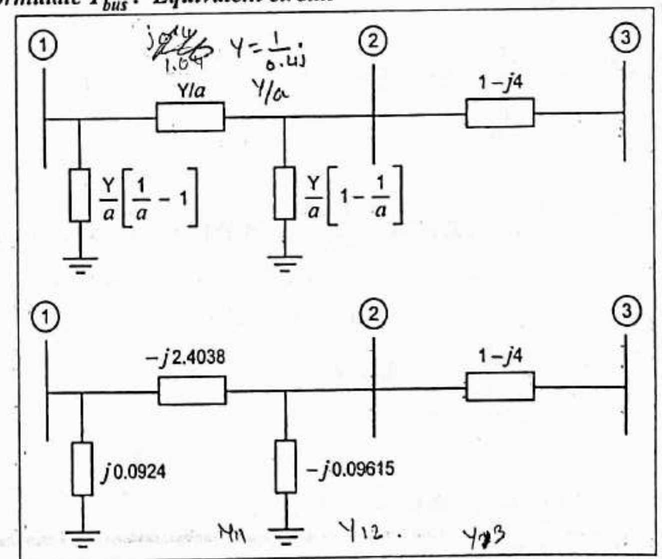
$Y = \frac{1}{Z}$.

Impedance

☺ ***Solution :***



Step 1 : Formulate Y_{bus} : Equivalent circuit



$$Y_{bus} = \begin{bmatrix} -j2.3114 & j2.4038 & 0 \\ j2.4038 & 1-j6.5 & -(1-j4) \\ 0 & -(1-j4) & 1-j4 \end{bmatrix}$$

Step 2 : Initialize bus voltages :

$V_1 = 1.02\angle 0°$
$V_2 = 1.0\angle 0°$
$V_3 = 1.0\angle 0°$

Step 3 : Calculate V_2^{new} and V_3^{new}

$$P_2 = 0, \quad Q_2 = 0 \quad \text{[because no load at bus 2]}$$

$$V_2^{new} = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{V_2^{old*}} - Y_{21}V_1^{new} - Y_{23}V_3^{old}\right]$$

$$= \frac{1}{1-j6.5}\left[\frac{0-j0}{1\angle 0^\circ} - j2.4038 \times 1.02\angle 0^\circ + (1-j4)\times 1\angle 0^\circ\right]$$

$$= 0.9928 + j0.0011$$

$$= 0.9928\angle 0.06^\circ$$

$$P_3 = 0, \quad Q_3 = 0 \quad \text{[because no load at bus 3]}$$

$$V_3^{new} = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{V_3^{old*}} - Y_{31}V_1^{new} - Y_{32}V_2^{new}\right]$$

$$= \frac{1}{1-j4}\left[\frac{0-j0}{1\angle 0^\circ} - 0 + (1-j4)\times 0.9928\angle 0.06^\circ\right]$$

$$= 0.9928\angle 0.06^\circ$$

Step 4 : Slack bus power :

$$S_1 = P_1 - jQ_1 = V_1^*[Y_{11}V_1 + Y_{12}V_2 + Y_{13}V_3]$$

$$= 1.02[-j2.3114 \times 1.02 + j2.4038 \times 0.9928\angle 0.06^\circ + 0]$$

$$= -0.0026 + j0.0294$$

$$P_1 = -0.0026 \times 100 = -0.26 \text{ MW}$$

$$Q_1 = -0.0294 \times 100 = -2.94 \text{ MVAR}$$

Step 5 : Line flows :

Bus No.		$S_{ij} = P_{ij} + jQ_{ij} = V_i[V_i^* - V_j^*]Y_{ij\,series}^* + \lvert V_i\rvert^2 Y_{Pi}^*$
From	To	
2	3	$S_{23} = 0.9928\angle 0.06^\circ[0.9928\angle -0.06^\circ - 0.9928\angle -0.06^\circ]\times(1+j4)$ $= 0$
3	2	$S_{32} = 0$

Step 5 : Transformer flows :

Bus No.		$S_{ij} = P_{ij} + jQ_{ij} = \frac{V_i}{a}\left[\frac{V_i^*}{a} - V_j^*\right]Y_{ij\,series}^*$
From	To	
1	2	$S_{12} = \frac{V_1}{a}\left[\frac{V_1^*}{a} - V_2^*\right]Y_{12\,series}^*$ $P_{12} + jQ_{12} = \frac{1.02}{1.04}\left[\frac{1.02}{1.04} - 0.9928\angle -0.06^\circ\right]\times j2.5$ $= -0.00255 - j0.0295$ p.u
2	1	$S_{21} = V_2\left[V_2^* - \left(\frac{V_1^*}{a}\right)\right]Y_{ji\,series}^*$ $P_{21} + jQ_{21} = 0.9928\angle 0.06^\circ\left[0.9928\angle -0.06^\circ - \left(\frac{1.02}{1.04}\right)\right]j2.5$ $= 0.00255 + j0.0298$ p.u

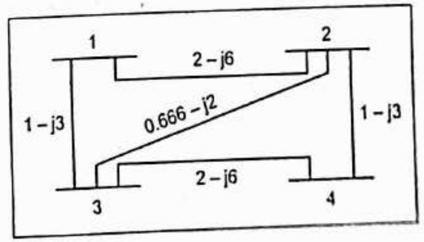
$$\text{Power Loss } P_{Loss} = P_{ij} + P_{ji} = -0.00255 + 0.00255 = 0$$

$$Q_{Loss} = Q_{ij} + Q_{ji} = -0.0295 + 0.0298 = 0.0003 \text{ p.u}$$

$$= 0.03 \text{ MVAR}$$

Example 6.9(a) *For the sample system shown in the Fig. the generators are connected at all four buses, while the loads are at buses 2 and 3. Values of real and reactive powers are listed in table. Bus 2 be a PV bus with $V_2 = 1.04$ p.u and bus 3 and bus 4 are PQ bus. Assuming a flat voltage start, find bus voltages and bus angles the end of first Gauss Seidal iterations. And consider the reactive power limit as $0.2 \le Q_2 \le 1$.*

Bus	P_p p.u	Q_p p.u	V_p p.u	Remarks
1	–	–	$1.04\angle 0^\circ$	Slack bus
2	0.5	–	1.04 p.u	PV bus
3	–1.0	0.5	–	PQ bus
4	0.3	–0.1	–	PQ bus



Solution:

Step 1: Formulate Y-bus matrix. The given values are admittances

$$Y_{bus} = \begin{bmatrix} 3-j9 & -2+j6 & -1+j3 & 0 \\ -2+j6 & 3.666-j11 & -0.666+j2 & -1+j3 \\ -1+j3 & -0.666+j2 & 3.666-j11 & -2+j6 \\ 0 & -1+j3 & -2+j6 & 3-j9 \end{bmatrix}$$

Step 2: Initialize bus voltages

$$V_1^{old} = 1.04\angle 0^\circ \text{ p.u (Slack bus)}$$

$$V_2^{old} = 1.04 \text{ p.u (PV bus)}$$

$$V_3^{old} = 1.0\angle 0^\circ \text{ p.u (PQ)}$$

$$V_4^{old} = 1.0\angle 0^\circ \text{ p.u (PQ)}$$

Step 3: Calculate Q_2 for the PV bus

$$Q_2^{cal} = -I_m\{V_2^{old} \times [Y_{21}V_1^{new} + Y_{22}V_2^{old} + Y_{23}V_3^{old} + Y_{24}V_4^{old}]$$

$$= -I_m\{1.04 \times [(-2+j6)\,1.04 + (3.666-j11) \times 1.04 + (-0.666+j2) \times 1.0 + (-1+j3) \times 1.0]\}$$

$$= -I_m\{0.069 - j0.208\} = 0.208 \text{ p.u}$$

$0.2 < Q_2^{cal} < 1$, within the limit

$\therefore$ Bus 2 acts as PV bus.

$$P_2 = 0.5 \text{ p.u}, \quad Q_2 = 0.208 \text{ p.u.}$$

Calculate V_2.

$$V_2^{new} = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{V_2^{old*}} - Y_{21}V_1^{new} - Y_{23}V_3^{old} - Y_{24}V_3^{old}\right]$$

$$= \frac{1}{3.666-j11}\left[\frac{0.5-j0.208}{1.04} - (-2+j6)\times 1.04 - (-0.666+j2)\times 1.0 - (-1+j3)\times 1.0\right]$$

$$= 1.051 + j0.0339 = 1.0518\angle 1.846^\circ = 1.0518\angle 0.032 \text{ rad}$$

$$\delta_2 = 0.032 \text{ rad}$$

$$V_2^{new} = 1.04\angle 0.032 \text{ rad} = 1.0395 + j0.0333$$

$$V_3^{new} = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{V_3^{old*}} - Y_{31}V_1^{new} - Y_{33}V_2^{new} - Y_{34}V_4^{old}\right]$$

$$= \frac{1}{3.666-j11}\left[\frac{-1.0-j0.5}{1.0} - (-1+j3)\times 1.04 - (-0.666+j2) \times [1.0395 - j0.0333] - (-2+j6)\times 1.0\right]$$

$$= 1.0317 - j0.0894 \text{ p.u}$$

$$V_4^{new} = \frac{1}{Y_{44}}\left[\frac{P_4 - jQ_4}{V_4^{old*}} - Y_{41}V_1^{new} - Y_{42}V_2^{new} - Y_{43}V_3^{new}\right]$$

$$= \frac{1}{3-j9}\left[\frac{0.3+j0.1}{1.0} - 0\times 1.04 - (-1+j3)\times 1.0395 + j0.0333 - (-2+j6)\times(1.0317 - j0.0894)\right]$$

$$= 1.0343 - j0.015 \text{ p.u}$$

6.8. NEWTON-RAPHSON METHOD

The Gauss-Seidel algorithm is very simple but the convergence becomes increasingly slow as the system size grows. The Newton-Raphson technique, converges equally fast for large as well as small systems, usually in less than 4 to 5 iterations but more functional evaluations are required. It has become very popular for large system studies.

The most widely used method for solving simultaneous non-linear algebraic equations is the N-R method. This method is a successive approximation procedure based on an initial estimate of the unknown and the use of Taylor's series expansion.

6.8.1. Load Flow Model in Real Variable Form

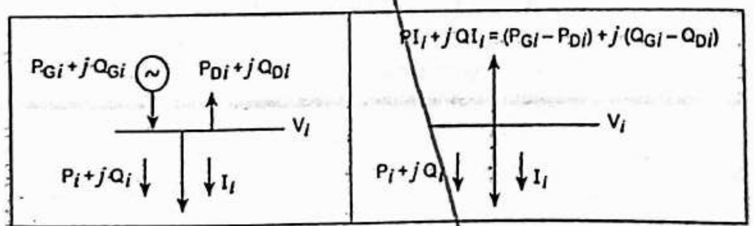


Fig. 6.7. Complex power balancing at a bus

Newton Raphson load flow method:

Gauss seidal algorithm - Very simple
- convergence becomes slow.

Newton Raphson method - convergence become fast
- Suitable for larger system

Most widely used method for solving simultaneous non-linear algebric equation. NR method is successive approximation procedure based on an initial estimate of the unknown.

$$\text{Real power } (P_i) = \sum_{j=1}^{N} |V_i||Y_{ij}||V_j| \cos(\theta_{ij} + \delta_j - \delta_i)$$

$$\text{Reactive power } (Q_i) = -\sum_{j=1}^{N} |V_i||Y_{ij}||V_j| \sin(\theta_{ij} + \delta_j - \delta_i)$$

$$\begin{bmatrix} \Delta P_2^{0} \\ \vdots \\ \Delta P_N^{0} \\ \hline \Delta Q_2^{0} \\ \vdots \\ \Delta Q_N^{0} \end{bmatrix} = \left[\begin{array}{ccc|ccc} \left(\frac{\partial P_2}{\partial \delta_2}\right)^0 & \cdots & \left(\frac{\partial P_2}{\partial \delta_N}\right)^0 & \left(\frac{\partial P_2}{\partial |V_2|}\right)^0 & \cdots & \left(\frac{\partial P_2}{\partial |V_N|}\right)^0 \\ \vdots & J_1 & \vdots & \vdots & J_2 & \vdots \\ \left(\frac{\partial P_N}{\partial \delta_2}\right)^0 & \cdots & \left(\frac{\partial P_N}{\partial \delta_N}\right)^0 & \left(\frac{\partial P_N}{\partial |V_2|}\right)^0 & \cdots & \left(\frac{\partial P_N}{\partial |V_N|}\right)^0 \\ \hline \left(\frac{\partial Q_2}{\partial \delta_2}\right)^0 & \cdots & \left(\frac{\partial Q_2}{\partial \delta_N}\right)^0 & \left(\frac{\partial Q_2}{\partial |V_2|}\right)^0 & \cdots & \left(\frac{\partial Q_2}{\partial |V_N|}\right)^0 \\ \vdots & J_3 & \vdots & \vdots & J_4 & \vdots \\ \left(\frac{\partial Q_N}{\partial \delta_N}\right)^0 & \cdots & \left(\frac{\partial Q_N}{\partial \delta_N}\right)^0 & \left(\frac{\partial Q_2}{\partial |V_n|}\right)^0 & \cdots & \left(\frac{\partial Q_N}{\partial |V_N|}\right)^0 \end{array}\right] \begin{bmatrix} \Delta \delta_2^{0} \\ \vdots \\ \Delta \delta_N^{0} \\ \hline |\Delta V_2|^{0} \\ \vdots \\ (\Delta V_n)^{0} \end{bmatrix}$$

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta |V| \end{bmatrix}$$

The diagonal and off diagonal elements of J_1 are:

$$\frac{\partial P_i}{\partial \delta_i} = \sum_{\substack{j=1 \\ \neq i}}^{N} |V_i||V_j||Y_{ij}| \sin(\theta_{ij} + \delta_j - \delta_i)$$

$$\frac{\partial P_i}{\partial \delta_j} = -|V_i||V_j||Y_{ij}| \sin(\theta_{ij} + \delta_j - \delta_i)$$

The diagonal and off-diagonal elements of J_2 are:

$$\frac{\partial P_i}{\partial |V_i|} = 2|V_i||Y_{ii}| \cos\theta_{ii} + \sum_{\substack{j=1 \\ \neq i}}^{N} |V_j||Y_{ij}| \cos(\theta_{ij} + \delta_j - \delta_i)$$

$$\frac{\partial P_i}{\partial |V_j|} = |V_i||Y_{ij}| \cos(\theta_{ij} + \delta_j - \delta_i)$$

The diagonal and off-diagonal elements of J_3 are

$$\frac{\partial Q_i}{\partial \delta_i} = \sum_{\substack{j=1 \\ \neq i}}^{N} |V_i||V_j||Y_{ij}| \cos(\theta_{ij} + \delta_j - \delta_i)$$

$$\frac{\partial Q_i}{\partial \delta_j} = -|V_i||V_j||Y_{ij}| \cos(\theta_{ij} + \delta_j - \delta_i) \quad (j \neq i)$$

The diagonal and off-diagonal elements of J_4 are

$$\frac{\partial Q_i}{\partial |V_i|} = -2|V_i||Y_{ii}| \sin\theta_{ii} - \sum_{\substack{j=1 \\ \neq i}}^{N} |V_j||Y_{ij}| \sin(\theta_{ij} + \delta_j - \delta_i)$$

$$\frac{\partial Q_i}{\partial |V_j|} = -|V_i||Y_{ij}| \sin(\theta_{ij} + \delta_j - \delta_i) \quad (j \neq i)$$

$$\Delta P_i = P_{i(spec)} - P_i^{calc}$$

$$\Delta Q_i = Q_{i(spec)} - Q_i^{calc}$$

The new estimates for bus voltages are;

$$\delta_i^{new} = \delta_i^{old} + \Delta\delta_i^{old}$$

$$V_i^{new} = V_i^{old} + \Delta |V_i|^{old}$$

Algorithm - Newton Raphson method:

1. Formulate Y-bus matrix
2. Assume flat start for starting voltage soln.

$\delta_i^0 = 0$ for $i = 1, \dots N$ (except slack bus)

$|V_i^0| = 1.0$, for $i = M+1, M+2 \dots N$

$|V_i| = |V_i|_{spec}$ for all PV bus and slack bus

3. For load buses, calculate P_i^{cal}, Q_i^{calc}
4. For PV buses, check Q-limit violation

If $Q_{i(min)} < Q_i^{calc} < Q_{i(max)}$ then bus act as P-V bus

If $Q_i^{calc} > Q_{i(max)}$ then $Q_{i(spec)} = Q_{i(max)}$

If $Q_i^{calc} < Q_{i(min)}$ then $Q_{i(spec)} = Q_{i(min)}$

the P-V bus act as P-Q bus

05. Calculate mismatch vector

$$\Delta P_i = P_{i(spec)} - P_i^{calc}$$

$$\Delta Q_i = Q_{i(spec)} - Q_i^{calc}$$

06. Compute

$$\Delta P_{i(max)} = \max |\Delta P_i| \quad i = 1, 2 \ldots N$$

$$\Delta Q_{i(max)} = \max |\Delta Q_i| \quad i = M+ \ldots N$$

07. Compute Jacobian matrix using $J = \begin{bmatrix} \frac{\partial P_i}{\partial \delta} & \frac{\partial P_i}{\partial |V|} \\ \frac{\partial Q_i}{\partial \delta} & \frac{\partial Q_i}{\partial |V|} \end{bmatrix}$

08. Obtain state correction vector

$$\begin{bmatrix} \Delta \delta \\ \Delta V \end{bmatrix} = [J]^{-1} \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix}$$

09. Update state vector using,

$$V^{new} = V^{old} + \Delta V$$

$$\delta^{new} = \delta^{old} + \Delta \delta$$

10. This procedure is continued until

$|\Delta P_i| < \varepsilon$ and $|\Delta Q_i| < \varepsilon$, otherwise go to step 3.

Why Bus admittance matrix is preferred [illegible]
flow?

1. Easy to formulate Y_{Bus} matrix
2. No need of taking inverse
3. Computation time is less
4. Matrix is symmetric, so calculation of upper or lower triangular matrix is enough
5. Each bus is connected to only a few nearby buses. So many off-diagonal elements are zero.

FLOW CHART :

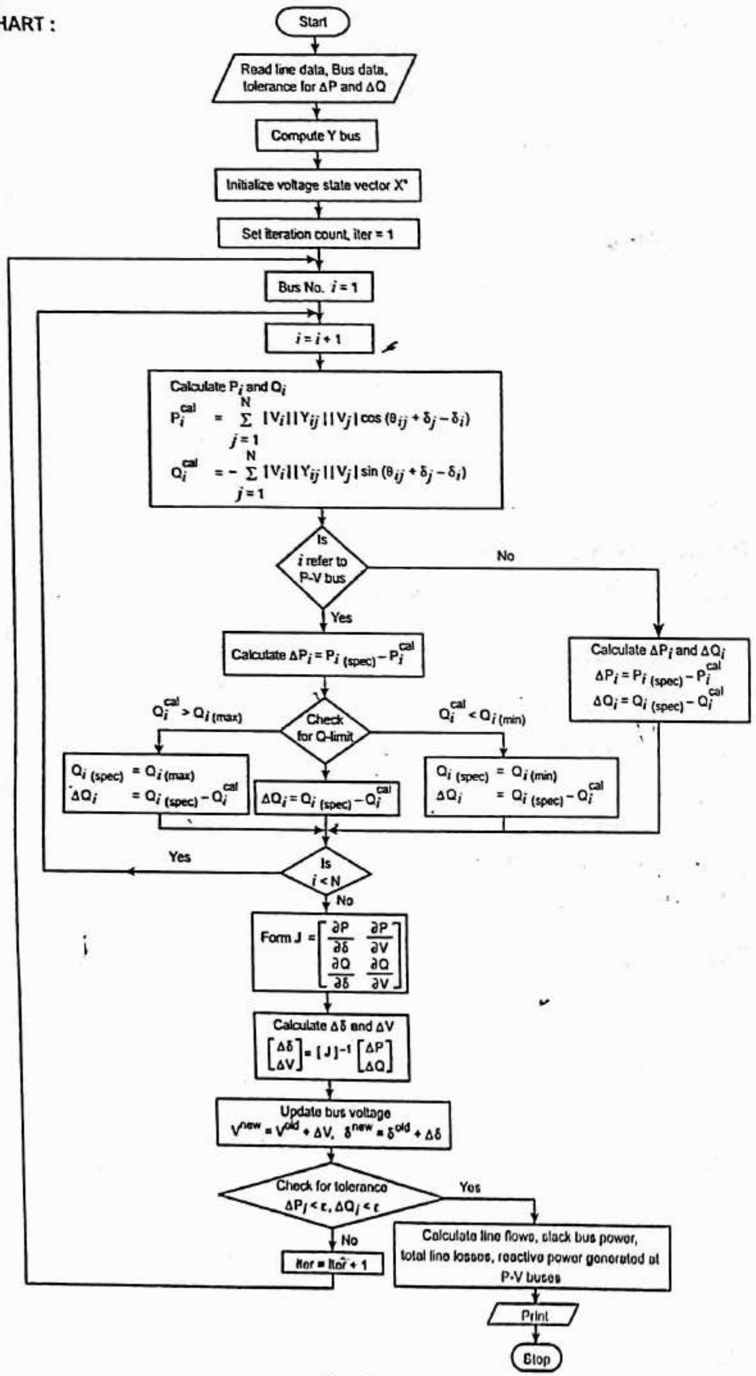


Fig. 6.8.

load flow Analysis

6. Compute $\Delta P_{i\,(max)} = \max|\Delta P_i|$; $\quad i = 1, 2, \ldots\ldots, N$ except slack

$\Delta Q_{i\,(max)} = \max|\Delta Q_i|$, $\quad i = M+1 \ldots\ldots N$

7. Compute Jacobian matrix using $J = \begin{bmatrix} \frac{\partial P_i}{\partial \delta} & \frac{\partial P_i}{\partial |V|} \\ \frac{\partial Q_i}{\partial \delta} & \frac{\partial Q_i}{\partial |V|} \end{bmatrix}$

8. Obtain state correction vector $\begin{bmatrix} \Delta\delta \\ \Delta V \end{bmatrix} = [J]^{-1} \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix}$

9. Update state vector using

$$V^{new} = V^{old} + \Delta V$$
$$\delta^{new} = \delta^{old} + \Delta\delta$$

10. This procedure is continued until

$|\Delta P_i| < \varepsilon$ and $|\Delta Q_i| < \varepsilon$, otherwise go to step 3.

Example 6.10 ***Perform two iteration of Newton Raphson load flow method and determine the power flow solution for the given system. Take base MVA as 100.***

Solution : ***Line Data :***

Line	Bus		R (p.u)	X (p.u)	Half line charging admittance $\left(\frac{Y_P}{2}\right)$ (p.u)
	From	To			
1	1	2	0.0839	0.5183	0.0636

Bus Data :

Bus	P_L	Q_L
1	90	20
2	30	10

slack bus — Generator PQ

① ② $\delta_2 V_2$

$$Y_{bus} = \begin{bmatrix} 0.3044 - j1.816 & -0.3044 + j1.88 \\ -0.3044 + j1.88 & 0.3044 - j1.816 \end{bmatrix}$$

$$Y_{bus} = \begin{bmatrix} 1.842 \angle -1.405 & 1.904 \angle 1.7314 \\ 1.904 \angle 1.7314 & 1.842 \angle -1.405 \end{bmatrix}$$

[*Note :* Use in rad mode]

Assume the initial value *i.e.*, $\delta = 0$, $V_1 = 1.0$.

$$[X] = \begin{bmatrix} \delta_2 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1.0 \end{bmatrix}$$

Step 3 : Calculate P_2^{cal}, Q_2^{cal}, ΔP_2 and ΔQ_2.

$$P_2^{cal} = |V_2| \{ |V_1||Y_2| \cos(\theta_{12} + \delta_2 - \delta_1) + |V_2||Y_{22}| \cos(\theta_{22} + \delta_2 - \delta_2) \}$$

$$= 1.0\,[1.05 \times 1.904 \cos(1.7314) + 1.842 \cos(-1.405)]$$
$$= 1.05 \times 1.904\,(-0.15991) + 1.842\,(0.16503)$$
$$= -0.015 \text{ p.u}$$
$$P_{2\,(spec)} = P_{G2} - P_L$$
$$= 0 - \frac{30}{100} = -0.3 \text{ p.u}$$
$$\Delta P_2 = P_{2\,(spec)} - P_2^{cal}$$
$$= -0.3 - (-0.015) = -0.285$$
$$Q_2^{cal} = -V_2 \{ |V_1||Y_{21}| \sin(\theta_{12} + \delta_1 - \delta_2) + |V_2||Y_{22}| \sin(\theta_{22} + \cancel{\delta_2} - \cancel{\delta_2}) \}$$
$$= -1.0\,[1.05 \times 1.904 \sin(1.7314) + 1.0 \times 1.842 \sin(-1.405)]$$
$$= -0.157 \text{ p.u}$$
$$\Delta Q_2 = Q_{2\,(spec)} - Q_2^{cal} = -0.1 - (-0.157)$$
$$= 0.057$$

$\left\{ \begin{array}{l} \cos\theta - \sin\theta \\ \sin\theta = \cos\theta \end{array} \right.$

Step 4 : Form Jacobian matrix.

$$\begin{bmatrix} \frac{\partial P_2}{\partial \delta_2} & \frac{\partial P_2}{\partial V_2} \\ \frac{\partial Q_2}{\partial \delta_2} & \frac{\partial Q_2}{\partial V_2} \end{bmatrix} \begin{bmatrix} \Delta\delta_2 \\ \Delta|V_2| \end{bmatrix} = \begin{bmatrix} \Delta P_2 \\ \Delta Q_2 \end{bmatrix}$$

$$\frac{\partial P_2}{\partial \delta_2} = |V_2||V_1||Y_{12}| \sin(\theta_{12} + \delta_1 - \delta_2) + |V_2|^2 |Y_{22}| \times 0$$
$$= 1.0 \times 1.05 \times 1.904 \sin(1.7314)$$
$$= 1.973$$
$$\frac{\partial P_2}{\partial V_2} = |V_1||Y_{21}| \cos(\theta_{12} + \delta_1 - \delta_2) + 2|V_2||Y_{22}| \cos(\theta_{22})$$
$$= 1.05 \times 1.904 \cos(1.7314) + 2 \times 1.842 \cos(-1.405)$$
$$= 0.289$$
$$\frac{\partial Q_2}{\partial \delta_2} = |V_2||V_1||Y_{21}| \cos(\theta_{12} + \delta_1 - \delta_2) - |V_2|^2 |Y_{22}| \times 0$$
$$= 1.05 \times 1.904 \times \cos(1.7314)$$
$$= -0.3197$$
$$\frac{\partial Q_2}{\partial V_2} = -|V_1||Y_{21}| \sin(\theta_{12} + \delta_1 - \delta_2) - 2|V_2||Y_{22}| \sin(\theta_{22})$$
$$= -1.05 \times 1.904 \sin(1.7314) - 2 \times 1.842 \sin(-1.405)$$
$$= 1.66$$

$\Delta P_2 = P_2^{spec} - P_2^{cal}$

Step 5 : Compute Δx,

$$\begin{bmatrix} \Delta\delta_2 \\ \Delta V_2 \end{bmatrix} = \begin{bmatrix} \frac{\partial P_2}{\partial \delta_2} & \frac{\partial P_2}{\partial V_2} \\ \frac{\partial Q_2}{\partial \delta_2} & \frac{\partial Q_2}{\partial V_2} \end{bmatrix}^{-1} \cdot \begin{bmatrix} \Delta P_2 \\ \Delta Q_2 \end{bmatrix}$$
$$= \begin{bmatrix} 1.973 & 0.289 \\ -0.3196 & 1.66 \end{bmatrix}^{-1} \times \begin{bmatrix} -0.285 \\ 0.057 \end{bmatrix}$$
$$= \begin{bmatrix} 0.493 & -0.086 \\ 0.0949 & 0.586 \end{bmatrix} \begin{bmatrix} -0.285 \\ 0.057 \end{bmatrix}$$
$$= \begin{bmatrix} -0.145 \\ 0.0064 \end{bmatrix}$$
$$X = X^\circ + \Delta X = \begin{bmatrix} 0 \\ 1.0 \end{bmatrix} + \begin{bmatrix} -0.145 \\ 0.0064 \end{bmatrix} = \begin{bmatrix} -0.145 \\ 1.0064 \end{bmatrix}$$

$\delta_2 =$

$V_2 =$

Iteration 2 : Compute mismatch vectors.

$$P_2^{cal} = 1.0064\,[1.05 \times 1.904 \cos(1.7314 + 0 + (-0.145)) + 1.0064 \times 1.842 \cos(-1.405)]$$
$$= -0.297$$
$$\Delta P_2 = P_{2\,(spec)} - P_2^{cal} = -0.3 - (-0.297) = -0.003$$
$$Q_2^{cal} = -\{1.0064\,[1.05 \times 1.904 \times \sin(1.7314 + 0 - (-0.145)) + 1.0064 \times 1.842 \times \sin(-1.405)]\}$$
$$= -0.078$$
$$\Delta Q_2 = Q_{2\,(spec)} - Q_2^{cal}$$
$$= -0.1 - (-0.078) = -0.021$$

Compute Jacobian matrix.

$$\frac{\partial P_2}{\partial \delta_2} = 1.0064 \times 1.05 \times 1.904 \sin(1.7314 + 0.145)$$
$$= 1.919$$
$$\frac{\partial P_2}{\partial V_2} = 1.05 \times 1.904 \cos(1.7314 + 0.145) + 2 \times 1.0064 \times 1.842 \times \cos(-1.405)$$
$$= 0.011$$
$$\frac{\partial Q_2}{\partial \delta_2} = 1.0064 \times 1.05 \times 1.904 \times \cos(1.7314 + 0.145)$$
$$= -0.605$$

$$\frac{\partial Q_2}{\partial V_2} = -1.05 \times 1.904 \sin(1.7314 + 0.145) - 2 \times 1.0064 \times 1.842 \times \sin(-1.405)$$

$$= 1.75$$

$$\begin{bmatrix} \Delta\delta_2 \\ \Delta V_2 \end{bmatrix} = \begin{bmatrix} 1.919 & 0.011 \\ -0.605 & 1.75 \end{bmatrix}^{-1} \begin{bmatrix} \Delta P_2 \\ \Delta Q_2 \end{bmatrix}$$

$$= \begin{bmatrix} 0.52 & -0.0033 \\ 0.179 & 0.57 \end{bmatrix} \begin{bmatrix} -0.003 \\ -0.021 \end{bmatrix} = \begin{bmatrix} -0.0015 \text{ rad} \\ -0.0125 \text{ p.u} \end{bmatrix}$$

$$\begin{bmatrix} \delta_2 \\ V_2 \end{bmatrix} = \begin{bmatrix} \delta_2^{old} \\ V_2^{old} \end{bmatrix} + \begin{bmatrix} \Delta\delta_2 \\ \Delta V_2 \end{bmatrix}$$

$$= \begin{bmatrix} -0.145 \\ 1.0064 \end{bmatrix} + \begin{bmatrix} -0.0015 \\ -0.0125 \end{bmatrix} = \begin{bmatrix} -0.1465 \text{ rad} \\ 0.994 \text{ p.u} \end{bmatrix} = \begin{bmatrix} -8.39^\circ \\ 0.994 \text{ p.u} \end{bmatrix}$$

$$V_2 = 0.994 \angle -8.39^\circ$$

Example 6.11 *In example 6.10, Bus 2 is a voltage controlled bus having the rating $P_G = 60$ MW, $V_2 = 1.02$ p.u.*

$-10 < Q_2 < 100$ MVAR. Carry out two iterations and determine bus voltage magnitudes.

Solution :

$$Y_{bus} = \begin{bmatrix} 1.842 \angle -80.49^\circ & 1.904 \angle 99.2^\circ \\ 1.904 \angle 99.2^\circ & 1.842 \angle -80.49^\circ \end{bmatrix}$$

$$X^\circ = [\delta_2] = 0 \text{ rad}$$

Compute θ_{12} in radians :

$$Y_{bus} = \begin{bmatrix} 1.842 \angle -1.405 & 1.904 \angle 1.7314 \\ 1.904 \angle 1.7314 & 1.842 \angle -1.405 \end{bmatrix}$$

Check for Q-limit :

$$Q_2^{cal} = -|V_2| \{ |V_1||Y_{21}| \sin(\theta_{12} + \delta_1 - \delta_2) + |V_2||Y_{22}| \sin\theta_{22} \}$$

$$= -1.02 \{1.05 \times 1.904 \sin(1.7314) + 1.02 \times 1.842 \sin(-1.405)\}$$

$$= -0.1239 \text{ p.u}$$

$$Q_{2(min)} < Q_2^{cal} < Q_{2(max)}$$

So the bus acts as generator bus.

Compute ΔP_2 :

$$P_2^{cal} = |V_2| \{ |V_1||Y_{21}| \cos(\theta_{12} + \delta_1 - \delta_2) + |V_2||Y_{22}| \cos(\theta_{22} + \delta_2 - \delta_2) \}$$

$$= 1.02 \{1.05 \times 1.904 \cos(1.7314) + 1.02 \times 1.842 \cos(-1.405)\}$$

$$= 1.02(-0.3197 + 0.3100)$$

$$= -0.009 \text{ p.u}$$

$$\Delta P_2 = P_{2(spec)} - P_2^{cal} = 0.6 - (-0.009) = 0.609 \text{ p.u}$$

Form Jacobian matrix :

$$J = \left[\frac{\partial P_2}{\partial \delta_2}\right] = |V_2| \{ |V_1||Y_{21}| + \sin(\theta_{12} + \delta_1 - \delta_2) \}$$

$$= 1.02 \times 1.05 \times 1.904 \sin(0 - 0 + 1.7314)$$

$$= 2.013$$

Compute $\Delta\delta$:

$$\Delta\delta_2 = [J]^{-1} [\Delta P_2]$$

$$= \frac{1}{2.013} \times 0.609 = 0.303$$

$$\delta_2^{new} = \delta_2^{old} + \Delta\delta_2 = 0 + 0.303 = 0.303 \text{ rad}$$

$$|V_2^{new}| = 1.02$$

Iteration 2 :

$$P_2^{cal} = 1.02 \{1.05 \times 1.904 \cos(1.7314 + 0 - 0.303) + 1.02 \times 1.842 \cos(-1.405)\}$$

$$= 0.606$$

$$\Delta P_2 = P_{2(spec)} - P_2^{cal} = 0.6 - 0.606 = -0.006$$

$$Q_2^{cal} = -1.02 \{1.05 \times 1.904 \sin(1.7314 + 0 - 0.303) + 1.02 \times 1.842 \times \sin(-1.405)\}$$

$$= -0.128$$

$$Q_{2(min)} < Q_2^{cal} < Q_{2(max)}$$

$\therefore$ This bus acts as generator bus.

Jacobian matrix :

$$J = \frac{\partial P_2}{\partial \delta_2} = |V_2| \{ |V_1||Y_{21}| \sin(\theta_{12} + \delta_1 - \delta_2) \}$$

$$= 1.02 [1.05 \times 1.904 \sin(1.7314 + 0 - 0.303)]$$

$$= 2.0185$$

$$\Delta\delta_2 = [J]^{-1} [\Delta P_2] = \frac{1}{2.0185} \times -0.006$$

$$= -0.003$$

$$\delta_2^{new} = \delta_2^{old} + \Delta\delta_2 = 0.303 + (-0.003)$$

$$\delta_2 = 0.3 \text{ rad} = 17.19^\circ$$

$$|V_2| = 1.02 \text{ p.u}$$

Example 6.12 ***In Example 6.10, bus 2 is a P-V bus having the rating $P_G = 60$ MW, $V_2 = 1.02$ p.u.***

$10 < Q_2 < 100$ MVAR, carry out one iteration. Perform load flow using Newton Raphson method to determine bus voltages.

☺ ***Solution :***

Step 1 : Form Y_{bus}.

$$Y_{bus} = \begin{bmatrix} 1.842 \angle -1.405 & 1.904 \angle 1.7314 \\ 1.904 \angle 1.7314 & 1.842 \angle -1.405 \end{bmatrix} \quad \theta_{12} \text{ in rad.}$$

Step 2 : Check for Q-limit violation.

$$Q_2^{cal} = -\left\{ |V_2| \, |V_1| \, |Y_{21}| \sin(\theta_{12} - \delta_2 + \delta_1) + |V_2|^2 |Y_{22}| \sin\theta_{22} \right\}$$

$$= -\left\{ 1.02 \times 1.05 \times 1.904 \times \sin(1.7314) + 1.02^2 \times 1.842 \times \sin(-1.405) \right\}$$

$$= -0.1239$$

$$Q_2^{cal} < Q_{2\,(min)}$$

$$Q_2 = Q_{2\,(min)} = \frac{10}{100} = 0.1 \text{ p.u MVAR}$$

Now bus 2 will act as load bus.

$$V_2 = 1.0 \angle 0 \text{ p.u}$$

Step 3 : Compute ΔP_2 and ΔQ_2.

$$P_2^{cal} = |V_2| |V_1| |Y_{21}| \cos(\theta_{12} - \delta_2 + \delta_1) + |V_2|^2 |Y_{22}| \cos\theta_{22}$$

$$= 1.0 \times 1.05 \times 1.904 \times \cos(1.7314) + 1.0^2 \times 1.842 \times \cos(-1.405)$$

$$= -0.0157$$

$$\Delta P_2 = P_{2\,(spec)} - P_2^{cal} = \frac{60}{100} - (-0.0157) = 0.6157$$

$$\Delta Q_2 = Q_{2\,(spec)} - Q_2^{cal} = 0.1 - (-0.1239) = 0.2239$$

Step 4 : Form Jacobian matrix.

$$\frac{\partial P_2}{\partial \delta_2} = |V_2| |V_1| |Y_{21}| \sin(\theta_{12} - \delta_2 + \delta_1)$$

$$= 1.0 \times 1.05 \times 1.904 \times \sin(1.7314 - 0 + 0)$$

$$= 1.973$$

$$\frac{\partial P_2}{\partial V_2} = |V_1| |Y_{12}| \cos(\theta_{12} - \delta_2 + \delta_1) + 2|V_2| |Y_{22}| \cos\theta_{22}$$

$$= 1.05 \times 1.904 \times \cos(1.7314) + 2 \times 1.0 \times 1.842 \times \cos(-1.405)$$

$$= 0.288$$

$$\frac{\partial Q_2}{\partial \delta_2} = |V_2| |V_1| |Y_{21}| \cos(\theta_{12} - \delta_2 + \delta_1)$$

$$= 1.0 \times 1.05 \times 1.904 \times \cos(1.7314)$$

$$= -0.3197$$

$$\frac{\partial Q_2}{\partial V_2} = -|V_1| |Y_{21}| \sin(\theta_{12} - \delta_2 + \delta_1) - 2|V_2| |Y_{22}| \sin\theta_{22}$$

$$= -1.05 \times 1.904 \sin(1.7314) - 2 \times 1.0 \times 1.842 \times \sin(-1.405)$$

$$= 1.66$$

Step 5 : Calculate $\Delta\delta$ and ΔV.

$$\begin{bmatrix} \Delta\delta_2 \\ \Delta V_2 \end{bmatrix} = \begin{bmatrix} 1.973 & -0.3197 \\ 0.288 & 1.66 \end{bmatrix}^{-1} \begin{bmatrix} \Delta P_2 \\ \Delta Q_2 \end{bmatrix}$$

$$= \frac{1}{3.367} \begin{bmatrix} 1.66 & 0.288 \\ -0.3197 & 1.973 \end{bmatrix} \begin{bmatrix} 0.6157 \\ 0.2239 \end{bmatrix}$$

$$= \begin{bmatrix} 0.493 & 0.0855 \\ -0.095 & 0.586 \end{bmatrix} \begin{bmatrix} 0.6157 \\ 0.2239 \end{bmatrix}$$

$$\begin{matrix} \Delta\delta_2 \\ \Delta V_2 \end{matrix} = \begin{bmatrix} 0.3227 \text{ rad} \\ 0.073 \text{ p.u} \end{bmatrix}$$

$$\delta_2^1 = \delta_2^0 + \Delta\delta_2 = 0 + 0.3227 = 0.3227 \text{ rad} = 18.49°$$

$$V_2^1 = V_2^0 + \Delta V_2 = 1.0 + 0.073 = 1.073$$

$$V_2^{new} = 1.073 \angle 18.49°$$

Advantages of G.S. Method

1. Calculations are simple and so the programming task is less.
2. The memory requirement is less.
3. Useful for small size system.

Disadvantages of G.S. Method

1. Requires large number of iterations to reach convergence.
2. Not suitable for large systems.
3. Convergence time increases with size of the system.