Unit-III

Fault Analysis - Balanced fault

A fault in a circuit is any failure which interferes with the normal flow of current. The fault occur in power system due to insulation failure of equipments, flash over of lines, due to permanent damage to conductors or due to a accidental faulty operations.

falling of a tree over the line - unexpected faulty operation

Fault analysis is an important part of power system analysis. The fault can be broadly classified into shunt fault (short circuit) and series fault (open conductors). short circuit studies are performed to determine bus voltage and current flowing in the lines during various types of faults.

(i) symmetrical (or) Balanced faults

(ii) unsymmetrical fault (or) unBalanced faults

* Line to ground
* Line to Line
* Double Line to ground.

When the n/w is symmetrically faulted, the phase currents and phase voltages possess three phase symmetry. [equal in magnitude and phase shift (120°)]

The symmetrical fault is called as 3-phase fault and it is the most severe fault and most amenable to calculate. The symmetrical fault conditions are analyzed on per phase basis using thevenin's theorem (or) using bus impedance matrix. The unsymmetrical fault are analyzed using symmetrical components.

The relative frequency of occurrence of various types of faults in the power systems in the order of decreasing severity as follows:

3 φ fault = 5% severe
L - L - G fault = 10% severe
L - L fault = 15% less severe
L - G fault = 70% very less severe

Need or Importance for short circuit study:

* S/m protected against heavy flow of SC currents by disconnecting the faulty section from health section by means of CB.

* Short circuit study is essential to estimate the magnitude of fault current for the proper choice of CB and protective relay.

To design or develop the protective schemes for various part of the power system.

Assumptions in fault analysis.

* Representing each machine by a constant voltage source behind proper reactances which may be x'', x' and x.
* Prefault load currents are neglected
* Transformer taps are assumed to be nominal.
* A symmetric 3ϕ power s/m is considered
* Shunt capacitance and series capacitance of transmission lines are ignored and neglected respectively.
* Shunt elements in the transformer model that account for magnetising current and core loss are neglected
* The negative sequence impedance are assumed to be the same as that of positive sequence impedance for the alternators.

$$Z^+ = Z^-$$

Symmetrical short circuits

* Symmetrical fault is defined as the simultaneous short circuit across all three phases. It occurs infrequently.
* The reactance of syn. Generator under short circuit condition is a time varying quantity and for analysis, three reactances were defined.

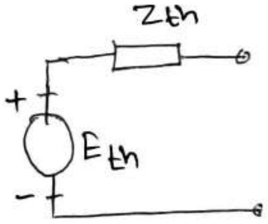
X_d'' = Subtransient reactance for first few cycles used to determining the interrupting capacity of the circuit Breaker.

X_d' = Transient reactance for the next 30 cycles used for relay setting and coordination, transient stability study.

X_d = Steady state synchronous Reactance.

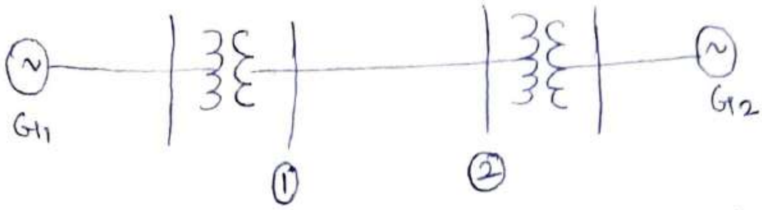
The duration of SC current depends on the time of operation of the CB. SC represent structural change in n/w ≡ to addition of Impedance at the fault location. The post fault voltages and current in the n/w are obtained by → superposing these changes on the prefault voltages and currents.

Thevenin's Theorem and Applications



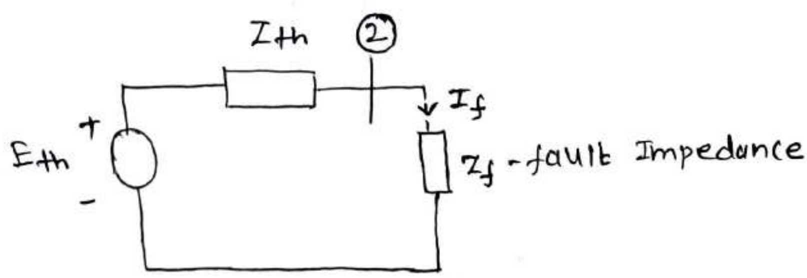
* The fault current can be evaluated
* The bus voltages and line currents during the fault can be determined
* Post fault voltages and current can be obtained by using prefault voltages and currents.

Short circuit analysis of two-bus system:



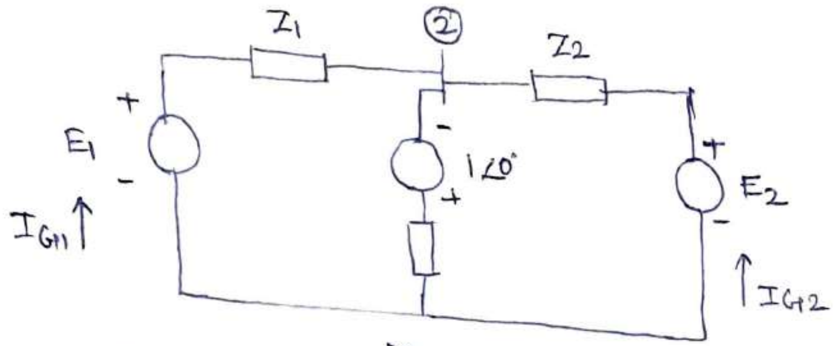
In power system, loads are specified and the load currents are unknown.

* Prefault bus voltages are obtained from results of the power flow solution.
* Loads are expressed by constant admittance using the prefault bus voltages
* Replace reactances of synchronous machine by their subtransient/transient value.
* Draw reactance diagram for the circuit (SC) short circuit
* Draw thevenin's equivalent viewed from the faulted bus.



Fault current = $\frac{E_{th}}{Z_{th} + Z_f}$

* Determine current contributed by the two generator



$$I_{G1} = I_f \times \frac{Z_2}{Z_1+Z_2} \quad ; \quad I_{G2} = I_f \times \frac{Z_1}{Z_1+Z_2}$$

* Determine post fault bus Voltages using

$$V_i^f = V_i^0 + \Delta V = V^0 + (-Z_{i2} \times I_{Gi})$$

V_i^0 ↓ Prefault voltage

$$V_2^f = V_2^0 + \Delta V = V_2^0 + Z_f I_f - 1\angle 0^\circ$$

* Determine post fault line flows.

$$I_{ij} = \frac{V_i - V_j}{Z_{ij(series)}}$$

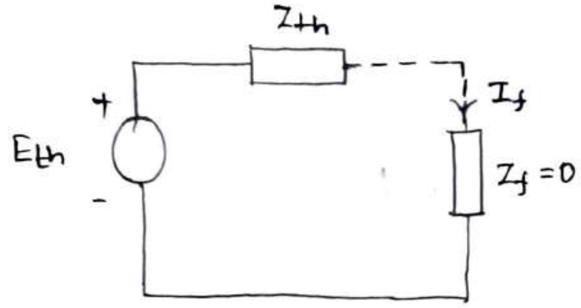
fault current flowing in the line connecting i^{th} and j^{th} bus

where, V_i, V_j] Bus Voltages

$Z_{ij(series)}$ = Series Impedance between bus i and j

Short circuit capacity (or) Fault level (or) Fault MVA

Fault level at a bus is defined as the product of the magnitude of the prefault bus voltage and the post fault current. It is used for determining the dimension of a bus bar, and the interrupting capacity of a circuit breaker.



$$\text{fault current} = \frac{E_{th}}{Z_{th}}$$

$$I_f = \frac{E_{th}}{X_{th}} \text{ p.u}$$

E_{th} = prefault voltages

$$\text{Base current} = \frac{MVA_b}{\sqrt{3} \times kV_b} \times 10^3$$

Fault in kA = I_f in pu $\times$ Base current

$$I_f = \frac{E_{th}}{X_{th}} \times \frac{MVA_b \times 10^3}{\sqrt{3} \times kV_b}$$

$$\text{Short circuit capacity (SCC)} = |E_{th}| \times |I_f|$$

$$= |E_{th}| \times \frac{|E_{th}|}{X_{th}}$$

$$= \frac{|E_{th}|^2}{X_{th}} \text{ p.u MVA}$$

Prefault voltage $\simeq 1\angle 0^\circ$

$$SCC = \frac{1}{X_{th}} \text{ p.u MVA}$$

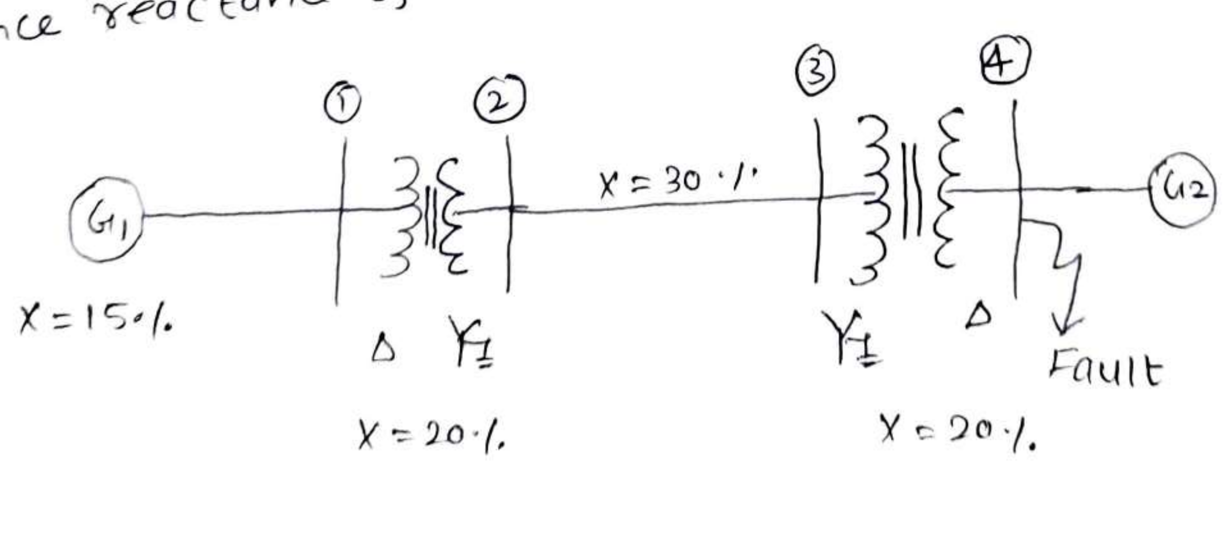
$$SCC = \frac{1}{X_{th}} \times MVA_b \text{ MVA}$$

where, MVA_b = Base MVA.

Solid fault or Bolted fault:

A fault represent a structural n/w change equivalent with that caused by the addition of an impedance at the place of fault. If the fault impedance is zero, then the fault is called as bolted (or) solid fault.

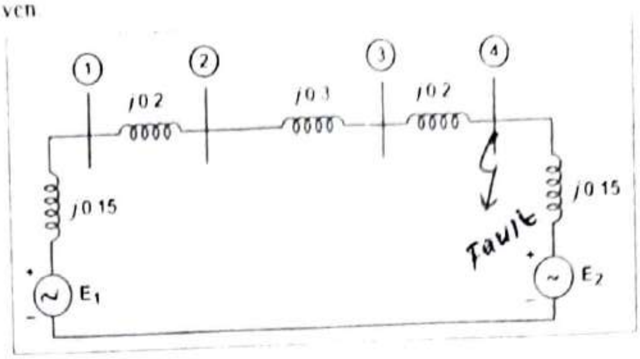
For the two bus system as shown in figure. Determine the fault current at the fault point and in other element and post fault voltage, for a bolted fault at bus 4. The sub transient reactance of the generators and positive sequence reactance of other elements are given.



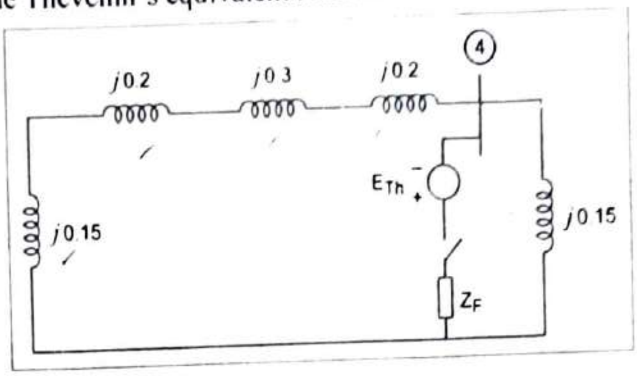
☺ Solution :

Step 1 : Draw reactance diagram.

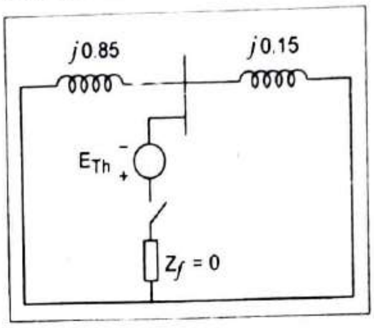
Note Positive sequence reactances are given. So directly draw the reactance diagram and substitute p.u values as given.



Step 2 : Determine Thevenin's equivalent reactance.



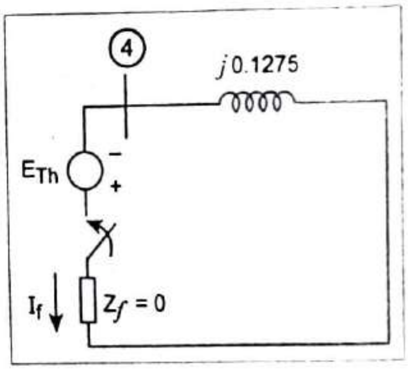
Note Voltage sources should be short circuited.



$j0.85$ and $j0.15$ are in parallel.

$$\therefore\ Z_{Th} = \frac{j0.85 \times j0.15}{j0.85 + j0.15} = j0.1275$$

Step 3 : Draw Thevenin's equivalent circuit.

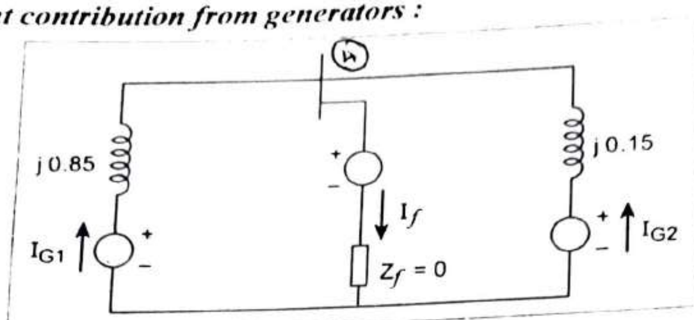


Prefault voltage $E_{Th} = 1 \angle 0°$

$$\text{Fault current } I_f = \frac{E_{Th}}{Z_{Th} + Z_f} = \frac{1 \angle 0°}{j0.1275 + 0}$$

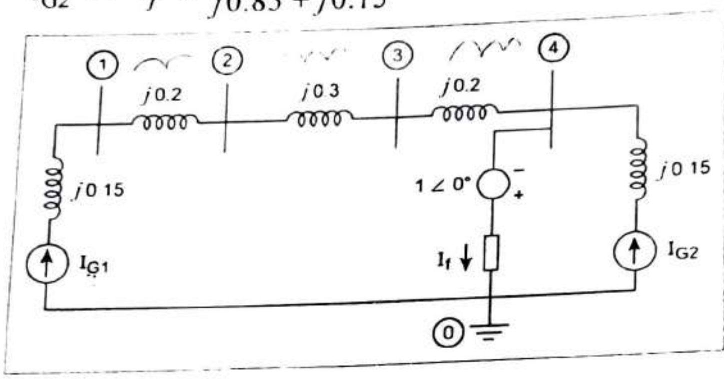
$$= -j7.843 = 7.843 \angle -90° \text{ p.u.}$$

Step 4 : Current contribution from generators :



$$I_{G1} = I_f \times \frac{j0.15}{j0.85 + j0.15} = 1.176 \angle -90^\circ \text{ p.u.}$$

$$I_{G2} = I_f \times \frac{j0.85}{j0.85 + j0.15} = 6.667 \angle -90^\circ \text{ p.u.}$$



Bus voltage changes (ΔV) :

$$\Delta V_1 = 0 - j0.15 \times I_{G1}$$
$$= 0 - j0.15 \times 1.176 \angle -90^\circ = -0.1764$$
$$\Delta V_2 = -j(0.2 + 0.15)\, I_{G1}$$
$$= 0 - j0.35 \times I_{G1} = -0.4116$$
$$\Delta V_3 = 0 - (j0.15 + j0.2 + j0.3)\, I_{G1}$$
$$= 0 - j0.65 \times I_{G1} = -0.7644$$
$$\Delta V_4 = -1 \angle 0^\circ$$

Post fault voltages :

$$V_1^f = V_1^0 + \Delta V_1 = 1 \angle 0^\circ + (-0.1764) = 0.8236 \text{ p.u.}$$
$$V_2^f = V_2^0 + \Delta V_2 = 1 \angle 0^\circ + (-0.4116) = 0.5884 \text{ p.u.}$$
$$V_3^f = V_3^0 + \Delta V_3 = 1 \angle 0^\circ + (-0.7644) = 0.2356 \text{ p.u.}$$
$$V_4^f = V_4^0 + \Delta V_4 = 1 \angle 0^\circ - 1 \angle 0^\circ = 0 \text{ p.u.}$$

Line flows (Post fault) :

$$I_{12} = \frac{V_1 - V_2}{Z_{12}} = \frac{0.8236 - 0.5884}{j0.2} = -j1.176 \text{ p.u.}$$

$$I_{23} = \frac{V_2 - V_3}{Z_{23}} = \frac{0.5884 - 0.2356}{j0.3} = -j1.176 \text{ p.u.}$$

$$I_{34} = \frac{V_3 - V_4}{Z_{34}} = \frac{0.2356 - 0}{j0.2} = -j1.178 \text{ p.u.}$$

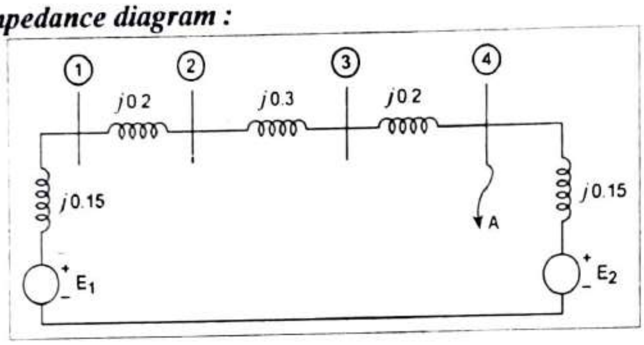
Short circuit capacity : $I_f = \dfrac{E_{Th}^2}{X_{Th}}$

$$SCC = |E_{Th}|\,|I_f| = \frac{|E_{Th}|^2}{X_{Th}} \text{ p.u. MVA}$$

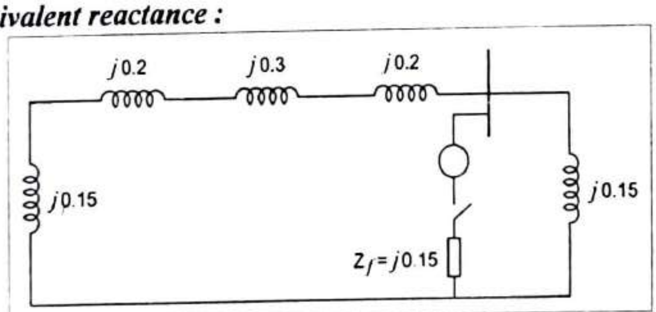
Short circuit capacity $= \dfrac{1}{0.1275} = 7.843$ p.u. MVA

Example 8.2 ***For the previous example, determine the fault current, bus voltages, line currents during the fault when a 3ϕ fault with a fault impedance $Z_f = j0.15$ p.u occurs on Bus 4.***

☺ ***Solution : Impedance diagram :***

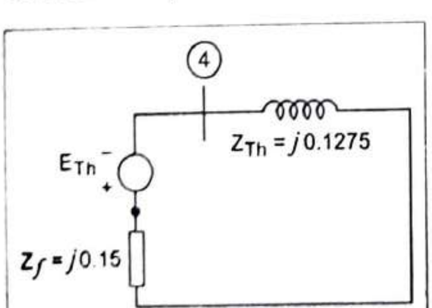


Thevenin's equivalent reactance :



$$Z_{Th} = \frac{j0.15 \times j0.85}{j0.15 + j0.85} = j0.1275$$

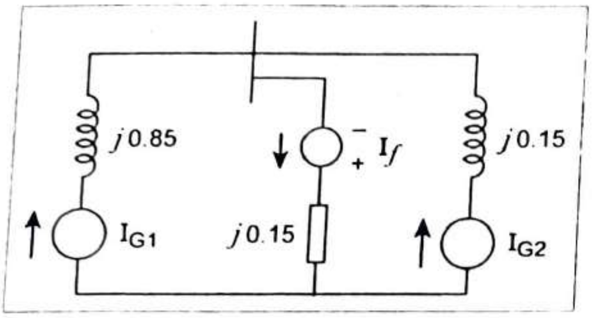
Thevenin's equivalent circuit :



Prefault voltage (E_{Th}) $= 1\angle 0°$

$$\text{Fault current } I_f = \frac{E_{Th}}{Z_{Th} + Z_f} = \frac{1\angle 0°}{j0.1275 + j0.15} = -j3.604 = 3.604\angle -90° \text{ p.u.}$$

Current contribution from generators :



$$I_{G1} = \frac{I_f \times j0.15}{j0.15 + j0.85}$$

$$= -j0.5406 = 0.5406 \angle -90° \text{ p.u.}$$

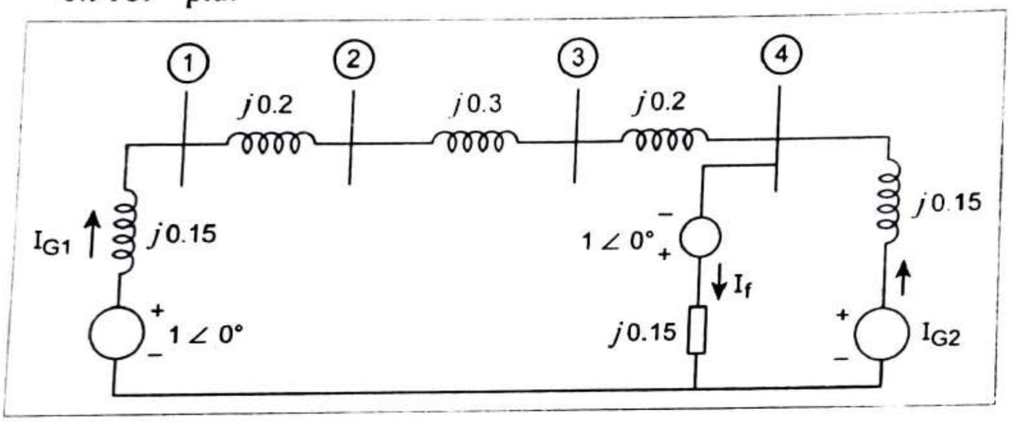
$$I_{G2} = \frac{I_f \times j0.85}{j0.85 + j0.15}$$

$$= -j3.0634 = 3.0634 \angle -90° \text{ p.u.}$$

Post fault bus voltage :

$$V_1^f = V_1^0 + \Delta V_1 = 1 \angle 0° + (-j0.15 \times I_{G1}) = 1 - j0.15 \times -j0.5406$$

$$= 0.9189 \text{ p.u.}$$



$$V_2^f = V_2^0 + \Delta V_2 = 1 \angle 0° + (-j0.35)\, I_{G1} = 1 \angle 0° - j0.35 \times -j0.5406$$

$$= 0.8108 \text{ p.u.}$$

$$V_3^f = V_3^0 + \Delta V_3 = 1 \angle 0° + (-j0.65)\, I_{G1} = 1 - j0.65 \times (-j0.5406)$$

$$= 0.6486 \text{ p.u.}$$

$$V_4^f = V_4^0 + \Delta V_4 = V_4^0 + Z_f I_f - 1 \angle 0°$$

$$= 1 \angle 0° + j0.15 \times -j3.604 - 1$$

$$= 0.5406 \text{ p.u.}$$

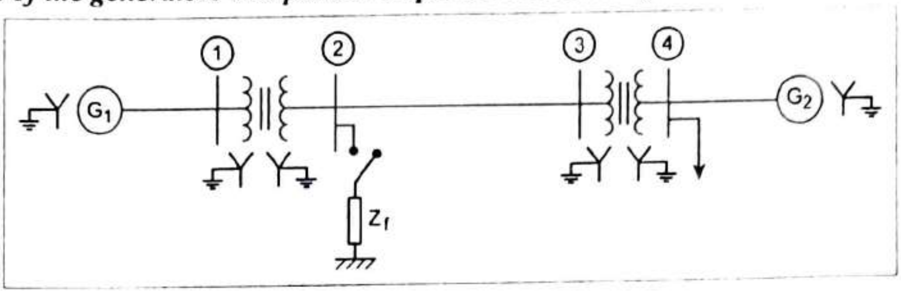
Line flows :

$$I_{12} = \frac{V_1 - V_2}{Z_{12}} = \frac{0.9189 - 0.8108}{j0.2} = -j0.5406 \text{ p.u.}$$

$$I_{23} = \frac{V_2 - V_3}{Z_{23}} = \frac{0.8108 - 0.6486}{j0.3} = -j0.5406 \text{ p.u.}$$

$$I_{34} = \frac{V_3 - V_4}{Z_{34}} = \frac{0.6486 - 0.5406}{j0.2} = -j0.54 \text{ p.u.}$$

Example 8.3 *For the two bus system shown in Fig., determine the fault current at the fault point and in other elements for a fault at bus 2 with $Z_f = 0$. The subtransient reactance of the generators and positive sequence reactance of other elements are given.*



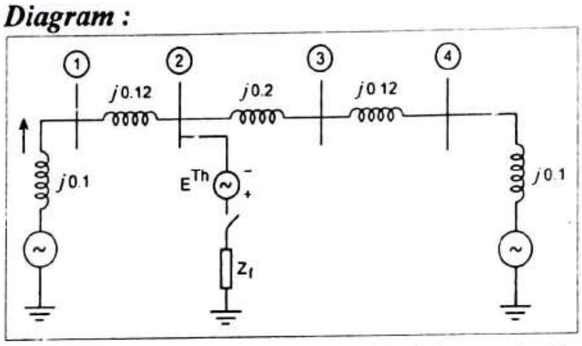
☺ ***Solution :***

Generator X = 10%

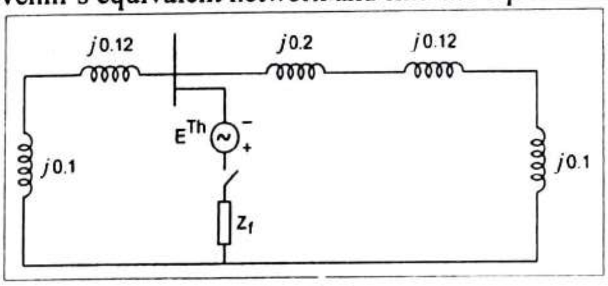
Transmission line X = 20%

Transformer X = 12%

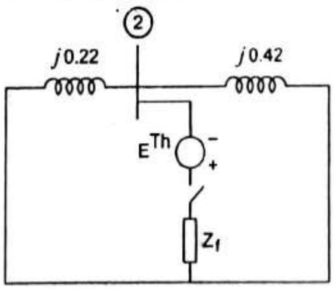
Step 1 : Reactance Diagram :



Step 2 : Draw Thevenin's equivalent network and find the equivalent reactance.



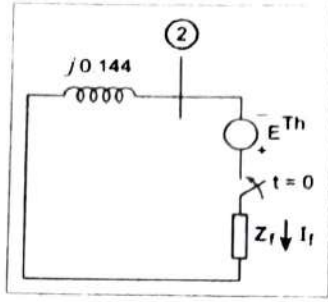
Note Voltage source must be short circuited.



$j0.22$ and $j0.42$ are in parallel.

$$\frac{j0.22 \times j0.42}{j0.22 + j0.42} = j0.144$$

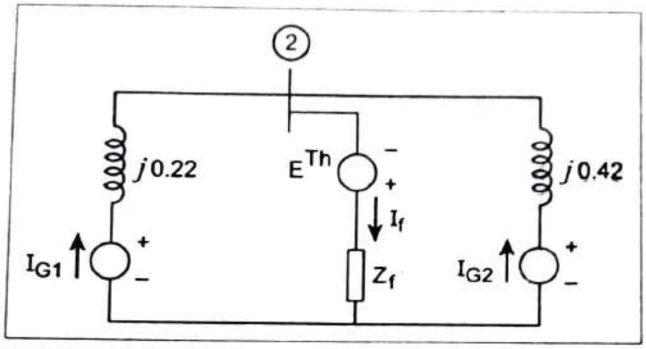
Thevenin's equivalent circuit is,



Assuming bolted fault or solid fault, $Z_f = 0$. Prefault voltage is $1 \angle 0°$.

$$I_f = \frac{E_{Th}}{Z_f + Z_{22}} = \frac{E_{Th}}{Z_f + Z_{Th}} = \frac{1 \angle 0°}{0 + j0.144} = 6.94 \angle -90° \text{ p.u.}$$

Step 3 : Current contribution from generators :



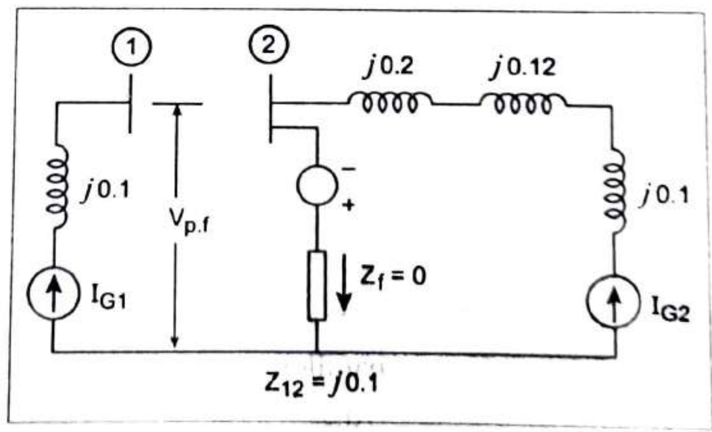
In general, $\boxed{I_G = I_{Total} \times \frac{Z_{parallel}}{Z_{Total}}}$

$$I_{G1} = I_f \times \frac{j0.42}{j0.22 + j0.42} = -j4.55 \text{ p.u.} = 4.55 \angle -90° \text{ p.u.}$$

$$I_{G2} = I_f \times \frac{j0.22}{j0.42 + j0.22} = -j2.39 \text{ p.u.} = 2.39 \angle -90° \text{ p.u.}$$

Step 4 : Post fault voltages

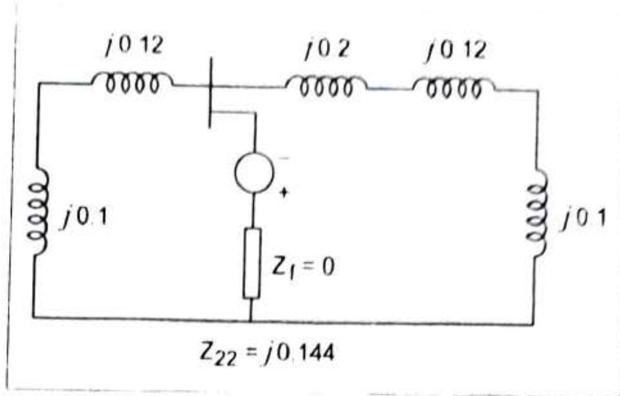
$$V_1^f = 1.0 - Z_{12} I_{G1}$$
$$= 1.0 - j0.1 \times 4.55 \angle -90°$$
$$= 0.545 \text{ p.u.}$$



$$V_2^f = 1.0 - Z_{22}\, I_f$$

$$= 1.0 - j0.144 \times 6.94 \angle -90°$$

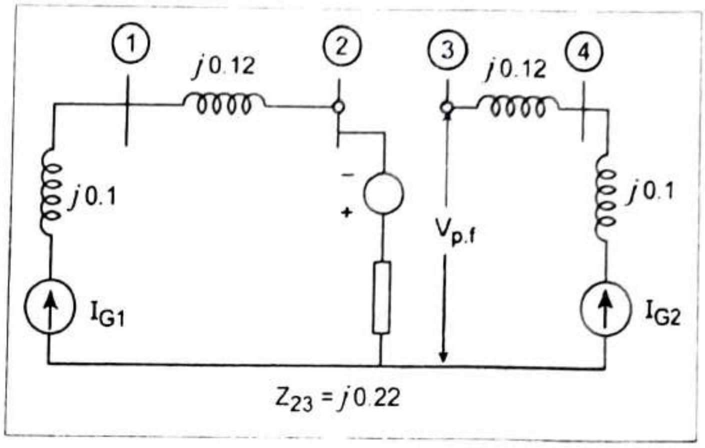
$$= 0$$



$$V_3^f = 1.0 - Z_{32}\, I_{G2}$$

$$= 1.0 - j0.22 \times 2.39 \angle -90°$$

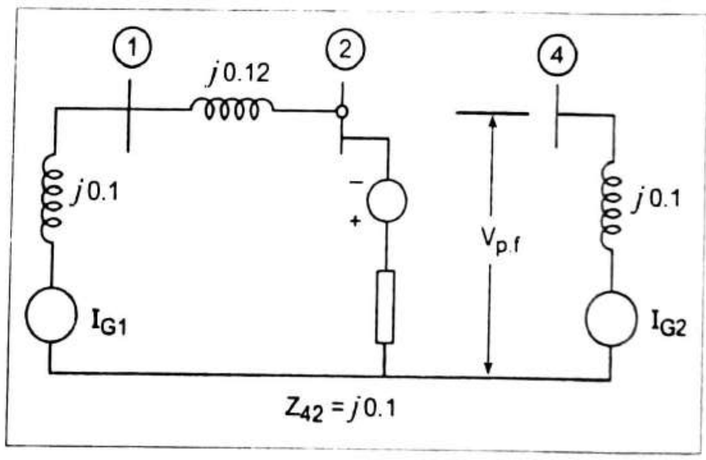
$$= 0.4742 \text{ p.u.}$$



$$V_4^f = 1.0 - Z_{42}\, I_{G2}$$

$$= 1.0 - j0.1 \times 2.39 \angle -90°$$

$$= 0.761 \text{ p.u.}$$



Step 5 : Line flow (post fault line currents)

$$I_{12} = \frac{V_1 - V_2}{X_{12}} = \frac{0.545 - 0}{0.12} = 4.542 \text{ p.u.}$$

$$I_{32} = \frac{V_3 - V_2}{X_{32}} = \frac{0.4742 - 0}{0.2} = 2.371 \text{ p.u.}$$

$$I_{43} = \frac{V_4 - V_3}{X_{43}} = \frac{0.761 - 0.4742}{0.12} = 2.4 \text{ p.u.}$$

$$I_{42} = \frac{V_4 - V_2}{X_{42}} = \frac{0.761 - 0}{0.32} = 2.378 \text{ p.u.}$$

Step 6 : Short circuit capacity :

$$|I_f| = \frac{|E_{Th}|^2}{X_{Th}}$$

SCC or fault level of a bus is defined as the product of the prefault voltage and post fault current.

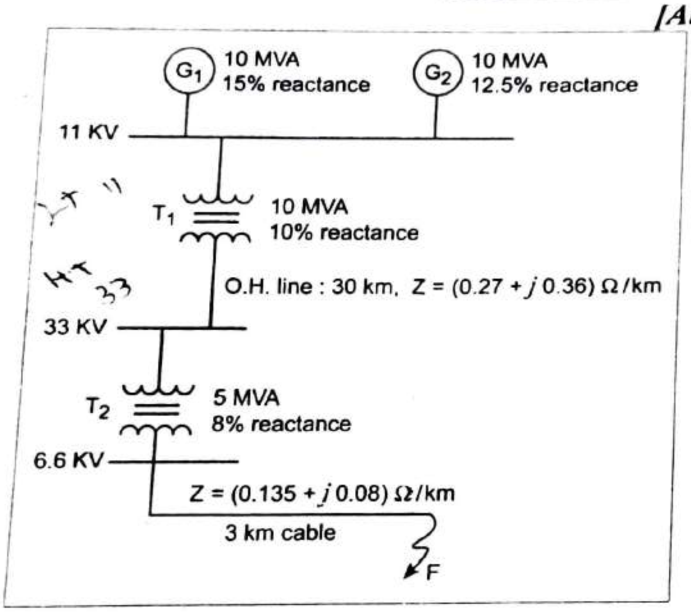
$$SCC = |V_{Th}| \cdot |I_f| = \frac{|E_{Th}|^2}{X_{Th}} \text{ p.u. MVA}$$

$$E_{Th} = 1 \angle 0^\circ \text{ (prefault voltage)}$$

$$\therefore \ SCC = \frac{1}{X_{Th}} \text{ p.u. MVA} = \frac{1}{0.144} = 6.94 \text{ p.u. MVA}$$

Example 8.4 ***For the radial network shown in Fig., three phase fault occurs at F. Determine the fault current and the line voltage at 11 KV bus under fault conditions.***

[Anna Univ., Dec. 2004]



☺ Solution :

Step 1 : Draw reactance diagram.

$$\text{Base MVA} = 10 \text{ MVA}$$

$$\text{Base KV} = 11 \text{ KV}$$

Generator 1 : $KV_{b\ new} = 11$ KV

$$Z_{new} = Z_{given} \times \left[\frac{KV_{b\ given}}{KV_{b\ new}}\right]^2 \times \left[\frac{MVA_{b\ new}}{MVA_{b\ given}}\right]$$

$$= j0.15 \times \left(\frac{11}{11}\right)^2 \times \left(\frac{10}{10}\right) = j0.15 \text{ p.u.}$$

Generator 2 : $KV_{b\ new} = 11$ KV

$$Z_{new} = j0.125 \times \left(\frac{11}{11}\right)^2 \times \left(\frac{10}{10}\right) = j0.125 \text{ p.u.}$$

Transformer 1 : Primary side $KV_{b\,new} = 11\ KV$

$$Z_{new} = j0.1 \times \left(\frac{11}{11}\right)^2 \times \left(\frac{10}{10}\right) = j0.1$$

Transmission line :

$$KV_{b\,new} = 11 \times \frac{33}{11} = 33\ KV$$

$$Z = \frac{Z_{actual}}{Z_{base}} = \frac{Z_{actual}}{KV_b^2} \times MVA_b$$

$$Z_{actual} = 0.27 + j0.36\ \ \Omega/km = (0.27 + j0.36) \times 30\ \Omega$$

$$Z_{new} = \frac{0.27 + j0.36}{33^2} \times 10 \times 30\ \text{p.u.}$$

$$= 0.0744 + j0.0992\ \text{p.u.}$$

Transformer 2 :

$$KV_{b\,new} = 33\ KV\ \text{(primary)}$$

$$Z_{new} = j0.08 \times \left(\frac{33}{33}\right)^2 \times \left(\frac{10}{5}\right) = j0.16\ \text{p.u.}$$

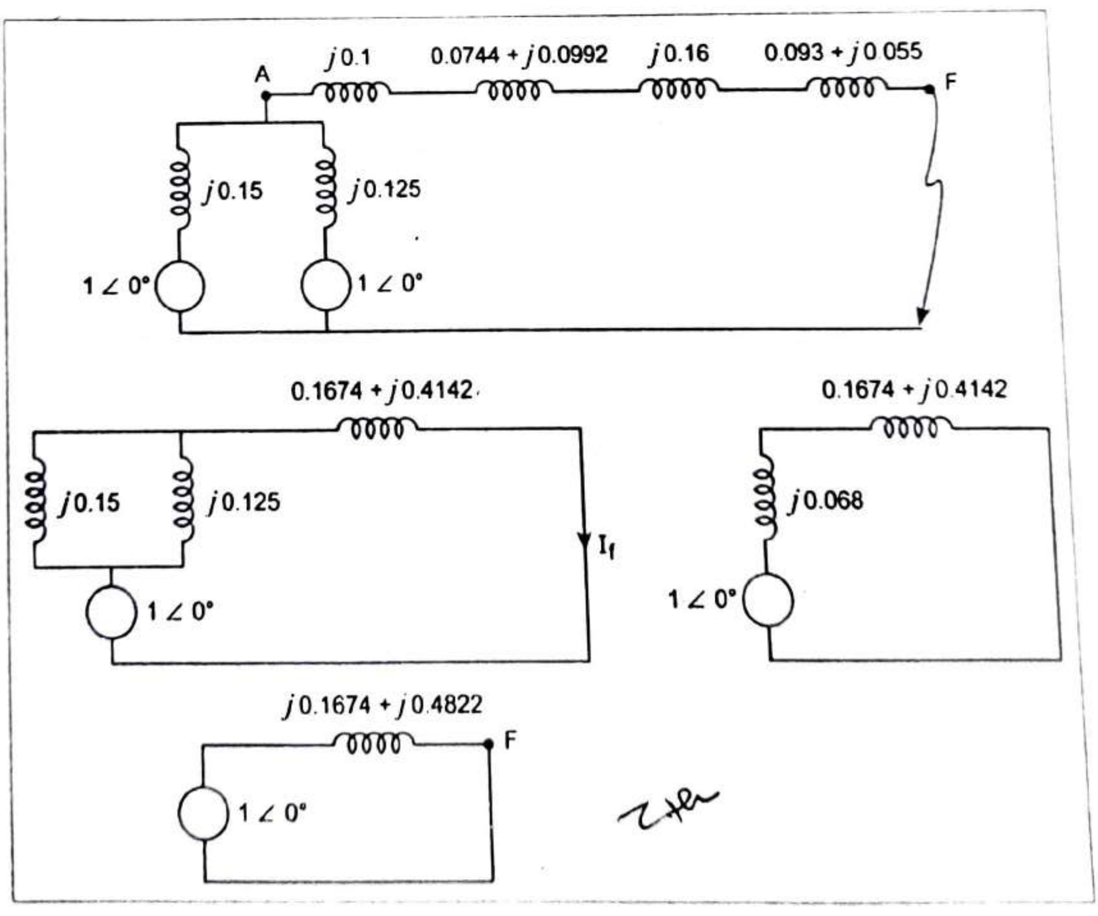
Cable :

$$Z_{actual} = (0.135 + j0.08)\ \Omega/km = (0.135 + j0.08) \times 3\ \Omega$$

$$KV_{b\,new} = 33 \times \frac{6.6}{33} = 6.6\ KV\ \text{(sec of Transformer 2)}$$

$$Z_{new} = \frac{Z_{actual}}{KV_b^2} \times MVA_b = \frac{(0.135 + j0.08)}{6.6^2} \times 3 \times 10$$

$$= 0.093 + j0.055\ \text{p.u.}$$



$$I_f = \frac{E_{Th}}{Z_{Th}} = \frac{1 \angle 0^\circ}{0.1674 + j0.4822} = 0.6425 - j1.85$$

$$= 1.959 \angle -70.85^\circ \text{ p.u.}$$

$$\text{Base current } I_B = \frac{MVA_b}{\sqrt{3} \times KV_b} = \frac{10 \times 10^3}{\sqrt{3} \times 6.6} = 874.77 \text{ Amp}$$

[KV_b for cable because fault point at F]

$$I_f = 1.959 \angle -70.85^\circ \text{ p.u.} \times I_B$$

$$= 1.959 \angle -70.85^\circ \times 874.77 = 1714 \text{ Amp}$$

Voltage at 11 KV bus :

$$Z_{AF} = j0.1 + 0.0744 + j0.0992 + j0.16 + 0.093 + j0.55$$

$$= 0.1674 + j0.4142$$

$$= 0.447 \angle 67.99^\circ \text{ p.u.}$$

$$\text{Voltage at 11 KV bus} = Z_{AF\,p.u.} \times I_{F\,p.u.}$$

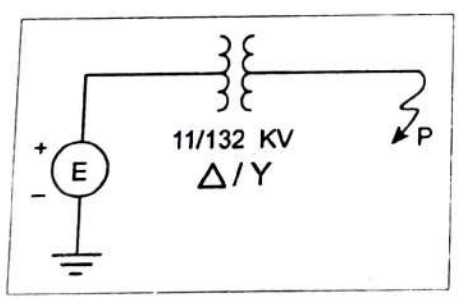
$$= 0.447 \angle 67.99^\circ \times 1.959 \angle -70.85^\circ$$

$$= 0.874 - j0.0436 = 0.875 \text{ p.u.}$$

$$= 0.875 \times \text{Base voltage}$$

$$= 0.875 \times 11 = 9.615 \text{ KV}$$

Example 8.5 ***A 60 MVA, Y connected 11 KV synchronous generator is connected to a 60 MVA, 11/132 KV Δ/Y transformer. The subtransient reactance X_d'' of the generator is 0.12 p.u. on a 60 MVA base, while the transformer reactance is 0.1 p.u. on the same base. The generator is unloaded when a symmetrical fault is suddenly place at point P as shown in Fig. Find the subtransient symmetrical fault current in p.u amperes and actual amperes on both sides of the transformer. Phase to neutral voltage of the generator at no load is 1.0 p.u.***



Solution : Prefault voltage $|E| = 1.0$ p.u. ; $X_d'' = 0.12$ p.u.

Base MVA = 60 MVA

Base KV = 11 KV

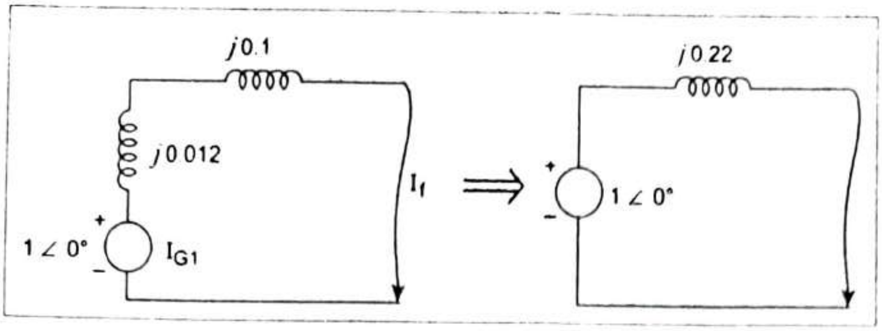
Generator :

$$Z_{p.u\,new} = j0.12 \times \left[\frac{11}{11}\right]^2 \times \left[\frac{60}{60}\right] = j0.12$$

Transformer :

$$Z_{p.u\,new} = j0.1 \times \left[\frac{11}{11}\right]^2 \times \left[\frac{60}{60}\right] = j0.1$$

Reactance diagram :



$$I_{F\,p.u.} = \frac{E_{Th}}{Z_{Th}} = \frac{1 \angle 0^\circ}{j0.22} = -j4.54 \text{ p.u.}$$

$$|I_{F\,p.u.}| = 4.54 \text{ p.u.}$$

Primary side of transformer :

$$\left.\begin{array}{r}\text{Base current on low voltage}\\ \text{side of the transformer}\end{array}\right\} = \frac{\text{Base MVA}}{\sqrt{3} \times KV_b} = \frac{60}{\sqrt{3} \times 11} = 3.149 \text{ KA}$$

$$[KV_b = 11 \text{ KV}]$$

$$\text{Actual current on L.V. side of transformer} = I_{F\,p.u.} \times \text{Base current}$$

$$= 4.54 \times 3.149 = 14.297 \text{ KA}$$

Secondary side of transformer :

$$\text{Base } KV_{new} = KV_{old} \times \frac{\text{H.V. side rating}}{\text{L.V. side rating}}$$

$$= 11 \times \frac{132}{11} = 132 \text{ KV}$$

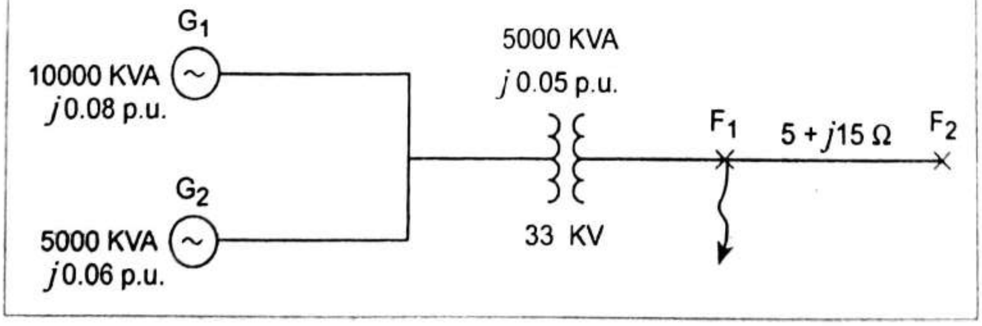
$$\text{Base current} = \frac{\text{Base MVA}}{\sqrt{3} \times KV_b} = \frac{60}{\sqrt{3} \times 132} = 0.262 \text{ KA}$$

$$\text{Actual current} = I_{F\,p.u.} \times \text{Base current}$$

$$= 4.54 \times 0.262 = 1.189 \text{ KA}$$

Example 8.6 ***A 3ϕ transmission line operating at 33 KV and having resistance and reactance of 5 ohms and 15 ohms respectively is connected to the generating station bus-bar through a 5000 KVA step up transformer which has a reactance of 0.05 p.u. Connected to the bus-bars are two alternators, are 10,000 KVA having 0.08 p.u. reactance and another 5000 KVA having 0.06 p.u. reactance. Calculate the KVA at a short circuit fault between phases occurring at the high voltage terminals of the transformers.***

Solution :



$$\text{Base MVA} = 10 \text{ MVA}$$
$$\text{Base KV} = 33 \text{ KV}$$

Transmission line :

$$Z_{p.u.}^{new} = \frac{Z_{p.u}^{old}}{KV_b^2} \times MVA$$
$$= \frac{5 + j15}{33^2} \times 10 = 0.0459 + j0.1377 \quad \text{p.u.}$$

Transformer :

$$Z_{p.u.\ new} = Z_{p.u.\ given} \times \left[\frac{KV_{b\ given}}{KV_{b\ new}}\right]^2 \times \left[\frac{MVA_{b\ new}}{MVA_{b\ given}}\right]$$
$$Z_{p.u.\ new} = j0.05 \times \left[\frac{33}{33}\right]^2 \times \left[\frac{10}{5}\right] = j0.1 \quad \text{p.u.}$$

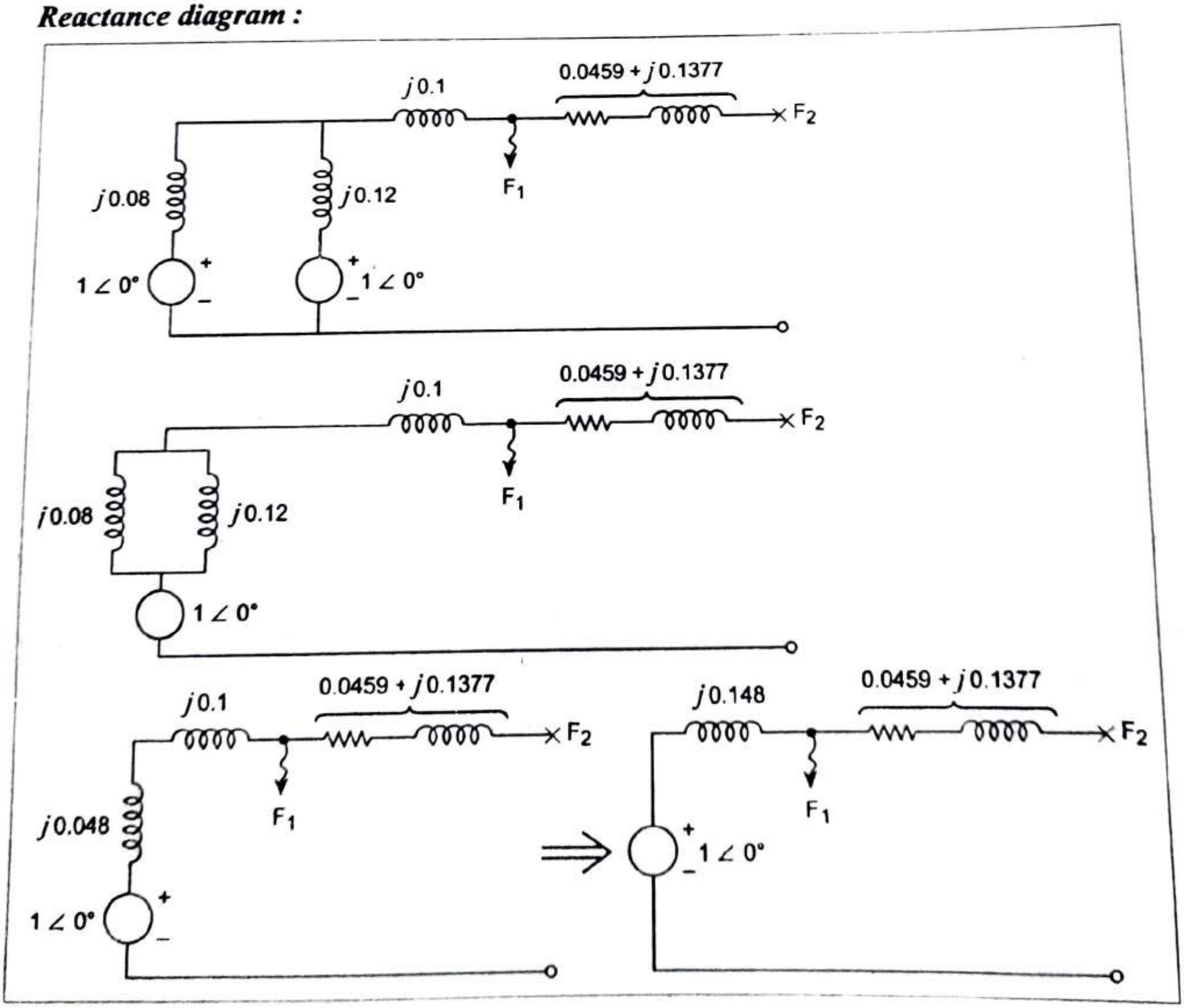
Generator 1 :

$$Z_{p.u.\ new} = j0.08 \times \left[\frac{33}{33}\right]^2 \times \left[\frac{10}{10}\right] = j0.08 \quad \text{p.u.}$$

Generator 2 :

$$Z_{p.u.\ new} = j0.06 \times \left[\frac{33}{33}\right]^2 \times \left[\frac{10}{5}\right] = j0.12 \quad \text{p.u.}$$

Reactance diagram :



(a) Total impedance upto the fault F_1, $Z_{1\,p.u}$ $= j0.148$ p.u.

Short circuit KVA fed into the fault at F_1 $= |KVA_{1\,SC}| = \dfrac{|KVA_b|}{|Z_{1\,p.u}|}$

$$= \frac{10,000}{0.148} = 67567.56 \text{ KVA}$$

$$= 67.568 \text{ MVA}$$

(b) Total impedance upto the fault F_2, $Z_{2\,p.u}$ $= 0.0459 + j0.2857$

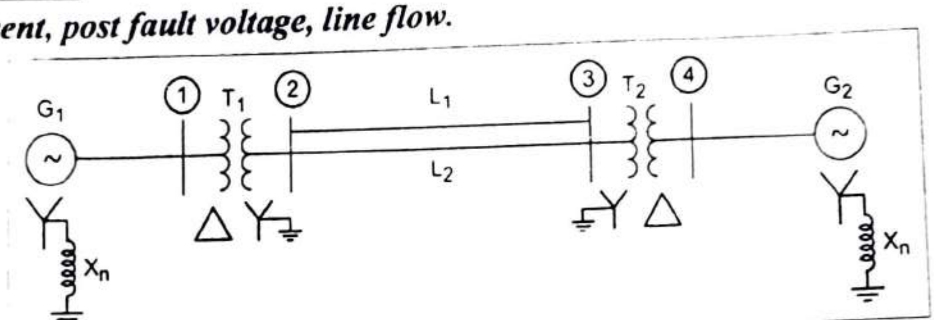
$|Z_{2\,p.u}| = 0.289$ p.u.

Short circuit KVA fed into the fault at F_2 $= |KVA_{2\,SC}| = \dfrac{|KVA_b|}{|Z_{2\,p.u}|}$

$$= \frac{10000}{0.289} = 34602 \text{ KVA}$$

$$= 34.602 \text{ MVA}$$

Example 8.7 ***A symmetrical fault occurs on bus 4 of system shown in Fig. Compute the fault current, post fault voltage, line flow.***

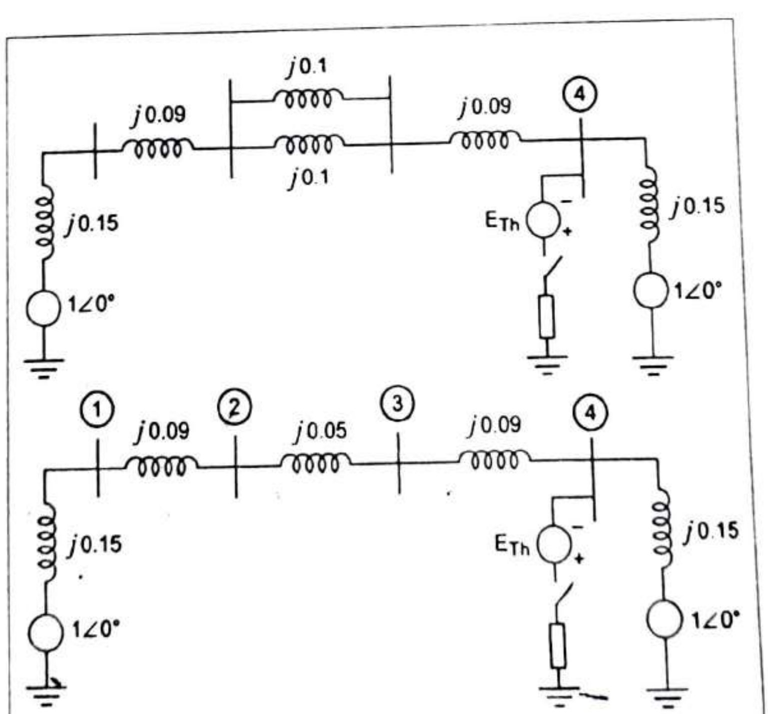


Generator G_1, G_2: 100 MVA, 20 KV, $X^+ = 15\%$

Transformer T_1, T_2: $X_{leak} = 9\%$

Transmission line L_1, L_2: $X^+ = 10\%$

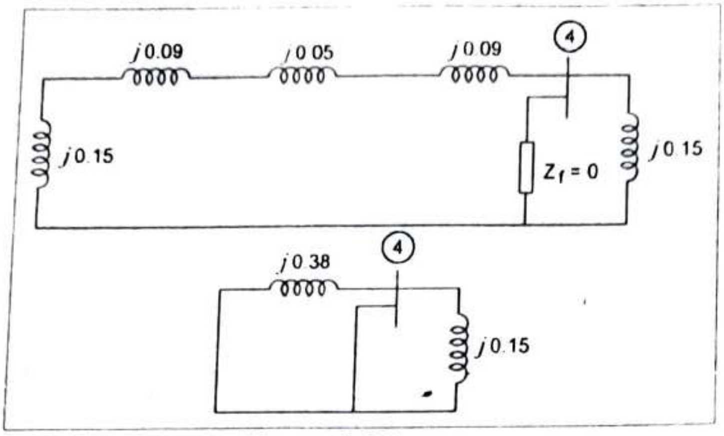
Solution :



$j0.1$ and $j0.1$ are connected in parallel.

$$\frac{j0.1 \times j0.1}{j0.1 + j0.1} = j0.05$$

Thevenin's reactance :



$$Z_{44} = j0.15 \ // \ j0.38 = \frac{j0.15 \times j0.38}{j0.15 + j0.38} = j0.1075$$

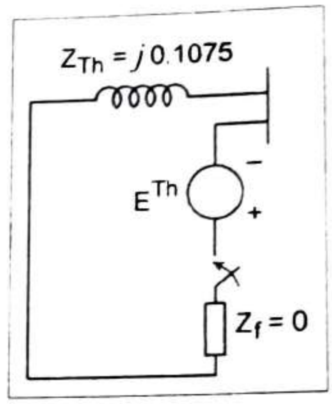
$$I_f = \frac{E_{Th}}{Z_{Th} + Z_f} = \frac{E_{Th}}{Z_{44} + Z_f} = \frac{1 \angle 0°}{j0.1075} = 9.3 \angle -90° \text{ p.u.}$$

Actual current in KA = p.u. value × Base current

$$= 9.3 \times \frac{MVA}{\sqrt{3} \times KA}$$

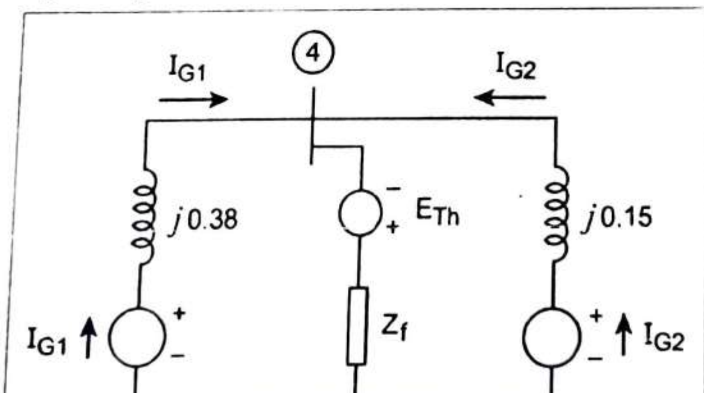
$$= 9.3 \times \frac{100}{\sqrt{3} \times 20} = 26.85 \text{ KA}$$

Actual current] = If * IB



Thevenin's equivalent circuit

Current contribution from generators :



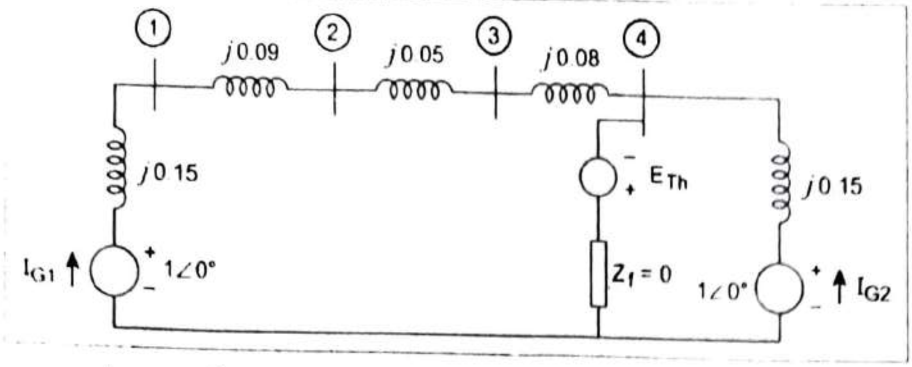
$$I_{G1} = I_f \times \frac{j0.15}{j0.38 + j0.15} = 2.632 \angle -90°$$

$$I_{G2} = I_f \times \frac{j0.38}{j0.38 + j0.15} = 6.67 \angle -90°$$

Post fault bus voltages :

$$V_1^f = V_1^0 + \Delta V_1 = 1 \angle 0° + (-j0.15) \times I_{G1}$$

$$= 1 + (-j0.15) \times 2.632 \angle -90° = 0.6052 \text{ p.u.}$$



$$V_2^f = V_2^0 + \Delta V_2 = 1\angle 0° + (-j0.15 - j0.09) \times I_{G1} = 0.3683 \quad \text{p.u.}$$

$$V_3^f = V_3^0 + \Delta V_3 = 1\angle 0° + (-j0.15 - j0.09 - j0.05) \times I_{G1}$$

$$= 0.2367 \quad \text{p.u.}$$

$$V_4^f = V_4^0 + \Delta V_4 = 1\angle 0° - 1\angle 0° = 0$$

Line flows :

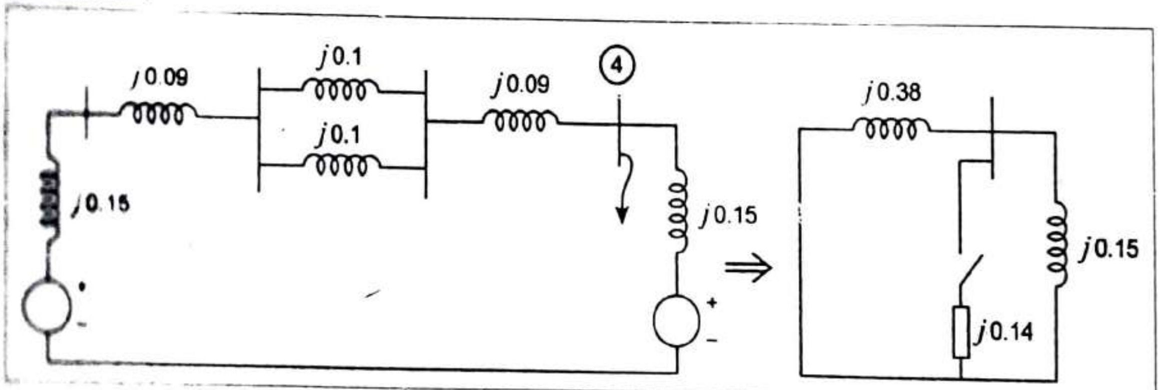
$$I_{12} = \frac{V_1 - V_2}{Z_{12}} = \frac{0.6052 - 0.3683}{j0.09} = -j2.632 \quad \text{p.u.}$$

$$I_{23} = \frac{V_2 - V_3}{Z_{23}} = \frac{0.3683 - 0.2367}{j0.05} = -j2.632 \quad \text{p.u.}$$

$$I_{34} = \frac{V_3 - V_4}{Z_{34}} = \frac{0.2367 - 0}{j0.09} = -j2.6 \quad \text{p.u.}$$

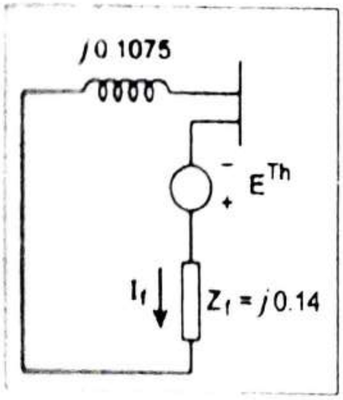
Example 8.8 ***For the example 8.7, when $Z_f = j0.14$ p.u., determine fault current and current supplied by the generators.***

Solution :



$$Z_{Th} = \frac{j0.38 \times j0.15}{j0.38 + j0.15} = j0.1075 \quad \text{p.u.}$$

Thevenin's equivalent circuit :



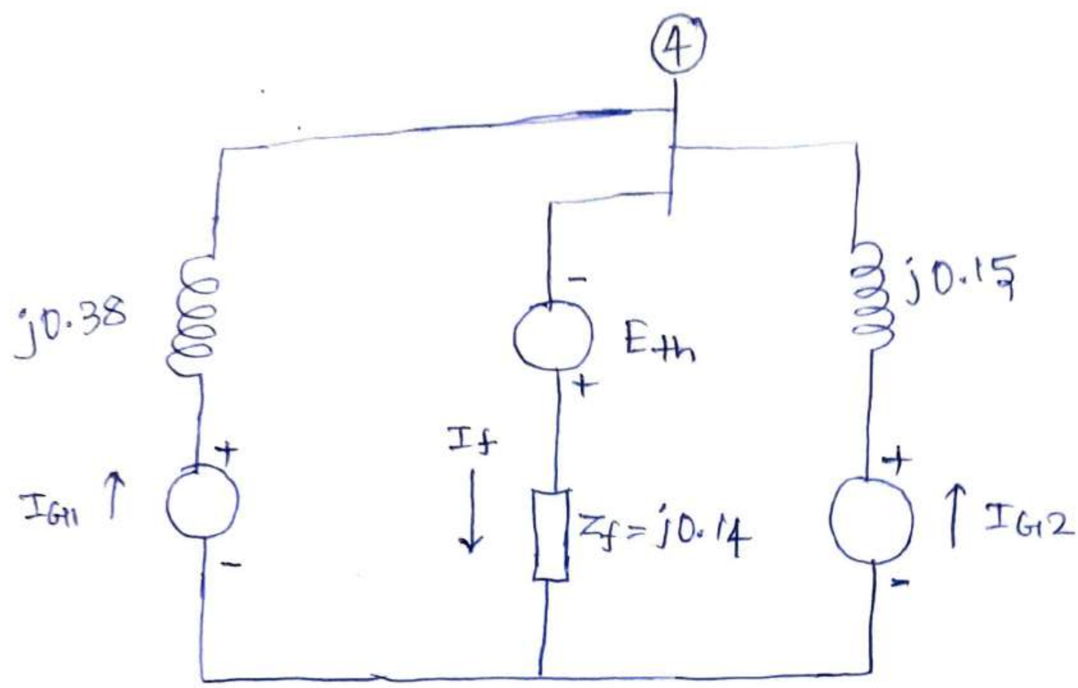
$$\text{Fault current } I_f = \frac{E_{Th}}{Z_{Th} + Z_f} = \frac{1\angle 0°}{j0.1075 + j0.14}$$

$$= -j4.04 \quad \text{p.u.} = 4.04\angle -90° \quad \text{p.u.}$$

$$\text{Actual fault current} = \text{p.u. value} \times \text{Base current}$$

$$= 4.04 \times \frac{\text{MVA}}{\sqrt{3}\ \text{KV}} = \frac{4.04 \times 100}{\sqrt{3} \times 20} = 11.66 \quad \text{KA}$$

Current contribution from the generator:



$$I_{G1} = \frac{I_{f\,p.u} \times j0.15}{j0.15 + j0.38} = 1.1434 \angle -90^\circ$$

$$I_{G2} = I_{f\,p.u} \times \frac{j0.38}{j0.15 + j0.38} = 2.8966 \angle -90^\circ$$

Systematic short circuit computation (Z Bus in phase):

Fault Analysis using Z-Bus Matrix:

Step 1: Each machine is represented by a constant voltage source behind proper reactance (X_d'', X_d', X_d). Transmission line reactance are expressed in per unit on a common base MVA.

Prefault Bus voltages - obtained from power flow solution.

Initial bus voltage $V_{bus}^{0} = \begin{bmatrix} V_1^0 \\ \vdots \\ V_q^0 \\ \vdots \\ V_N^0 \end{bmatrix}$

Step 2: Obtain Z-bus matrix using building algorithm. Assume one node as reference and short circuiting all the voltage sources. Determine the Z_{bus} using step by step bus building algorithm.

Step 3: Obtain the fault current.

$$\text{Fault current } I_f = \frac{V_q^0}{Z_{qq} + Z_f}$$

Z_{qq} → diagonal element of the Z_{bus} matrix.

Step 4: Obtain thevenin's network by inserting thevenin's voltage source V_q^0 in series with Z_f and compute change in bus voltages using network equation.

The current entering every bus is zero except faulted bus.

$$I_1 = I_2 \dots\dots = I_N = 0 \quad \text{except } I_q$$

$$I_q = -I_f \quad \text{[fault current is leaving the bus q]}$$

$$I_{bus} = \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_q \\ \vdots \\ I_N \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ -I_f \\ \vdots \\ I_N \end{bmatrix}$$

$$I_{Bus(F)} = Y_{bus} \cdot \Delta V_{bus}$$

solving ΔV_{bus},

$$\Delta V_{Bus} = Z_{bus} \cdot I_{bus(F)}$$

$$\begin{bmatrix} \Delta V_1 \\ \Delta V_2 \\ \vdots \\ \Delta V_q \\ \vdots \\ \Delta V_N \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \dots & Z_{1q} \dots & Z_{1N} \\ Z_{21} & Z_{22} \dots & Z_{2q} \dots & Z_{2N} \\ \vdots & & & \\ Z_{q1} & Z_{q2} \dots & Z_{qq} \dots & Z_{qN} \\ \vdots & & & \\ Z_{N1} & Z_{N2} \dots & Z_{Nq} \dots & Z_{NN} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \vdots \\ -I_f \\ \vdots \\ 0 \end{bmatrix}$$

Step 5: Post fault Bus voltages. - obtained by superposition of the prefault bus voltages and the changes in the bus voltages.

$$V_{bus(f)} = V_{bus}^{0} + \Delta V_{bus}$$

$$V_1^f = V_1^0 - Z_{1q} I_f$$

$$V_2^f = V_2^0 - Z_{2q} I_f$$

$$\vdots$$

$$V_q^f = V_q^0 - Z_{qq} I_f$$

$$\vdots$$

$$V_N^f = V_N^0 - Z_{Nq} I_f$$

In general, $V_i^f = V_i^0 - Z_{iq} I_f$

Bus voltage during the fault

$$V_i^f = V_i^0 - \frac{Z_{iq} V_q^0}{Z_{qq} + Z_f} \qquad i \neq q$$

$$V_q^f = \frac{Z_f}{Z_{qq} + Z_f} V_q^0 \qquad i = q$$

If the fault is solid or bolted then

$Z_f = 0$

Step 6:

Fault current $I_f = \dfrac{V_q^0}{Z_{qq}}$

$$V_q^f = 0$$

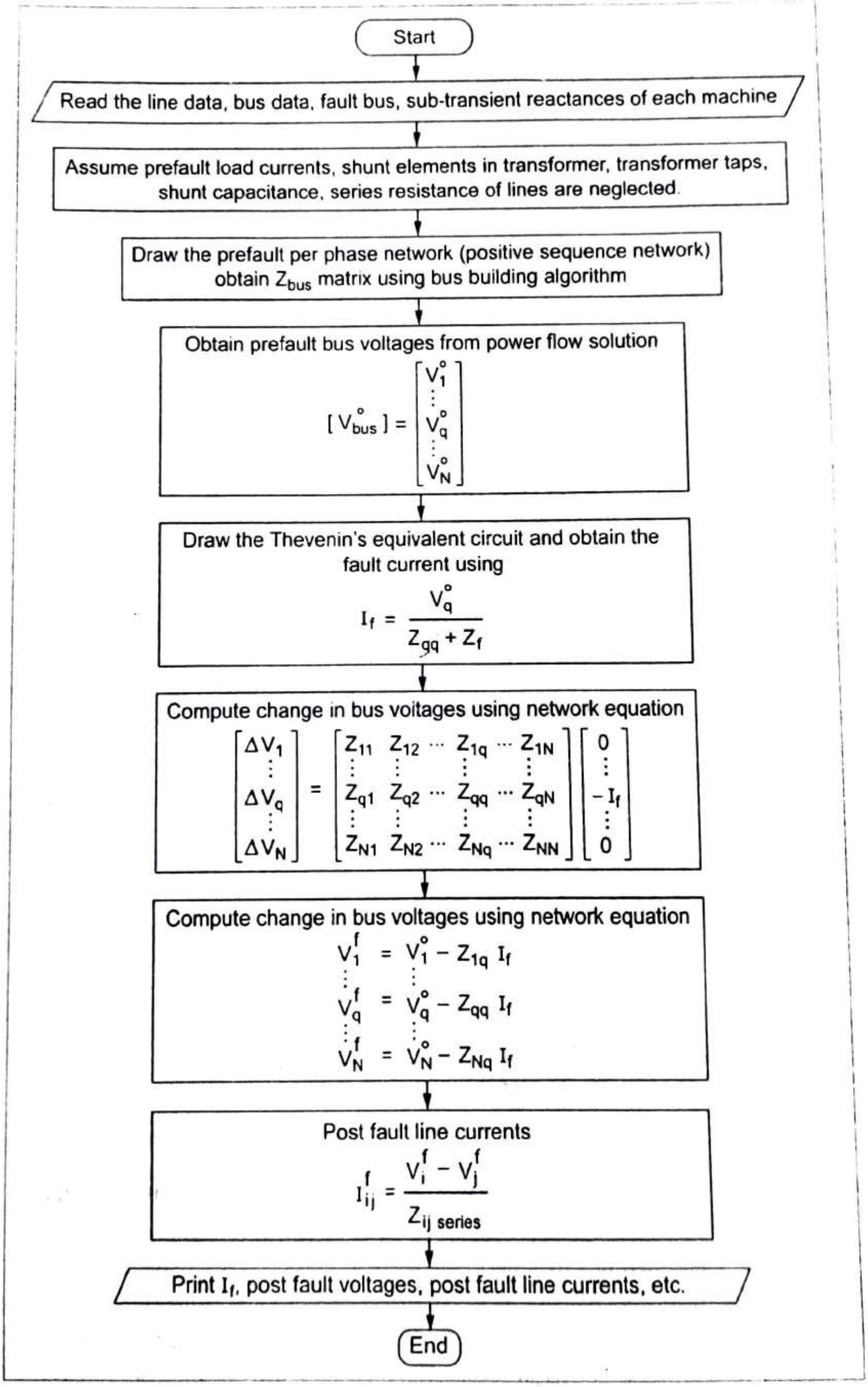
$$V_i^f = V_i^0 - \frac{Z_{iq}}{Z_{qq}} V_q^0 \; ; \; i \neq q$$

Post fault line currents

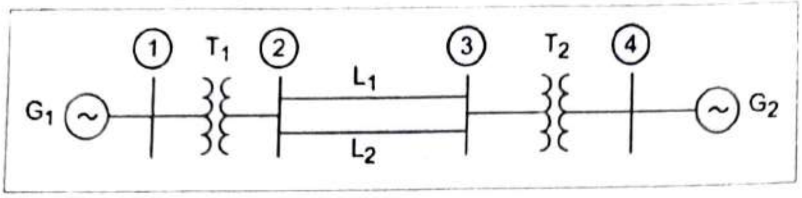
$$I_{ij}^f = \frac{V_i^f - V_j^f}{Z_{ij}(\text{series})}$$

very simple and practical. Thus all fault calculations are formulated in the bus frame of reference using bus impedance matrix Z_{bus}.

Symmetrical Fault Analysis using Z_{bus} (Flow chart)



Example 8.9 ***A symmetrical fault occurs on bus 4 of system shown in Fig. Determine the fault current, post fault voltages and line currents.***

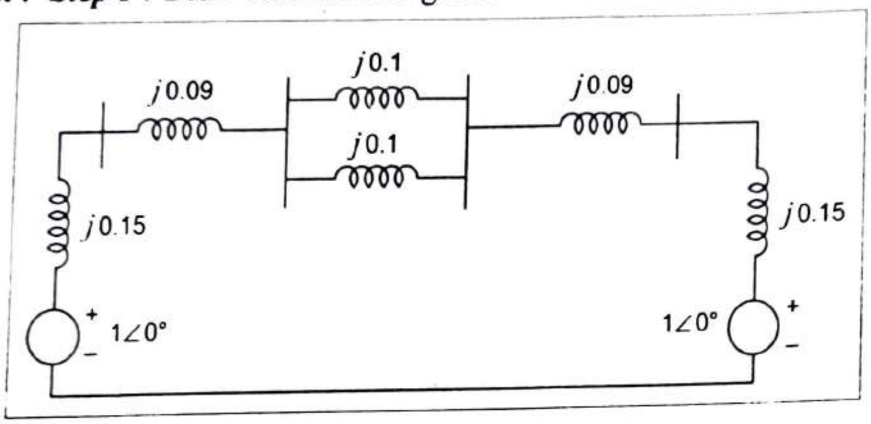


G_1, G_2 : *100 MVA, 20 KV,* $X^+ = 15\%$

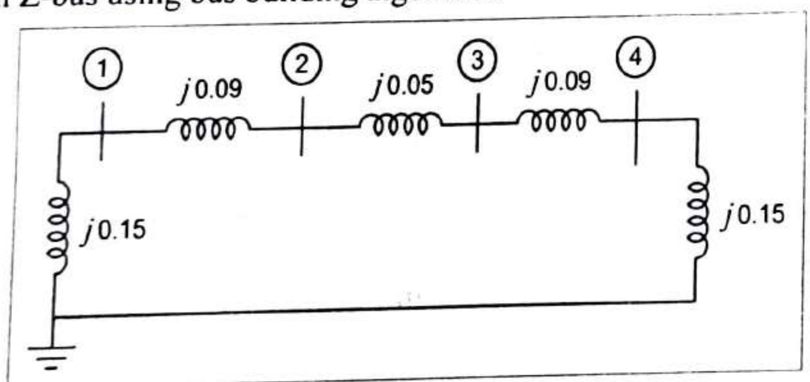
Transformer : $X_{leak} = 9\%$

L_1, L_2 : $X^+ = 10\%$

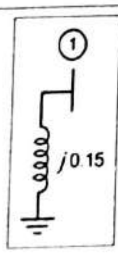
☺ ***Solution :*** ***Step 1 :*** Draw reactance diagram.



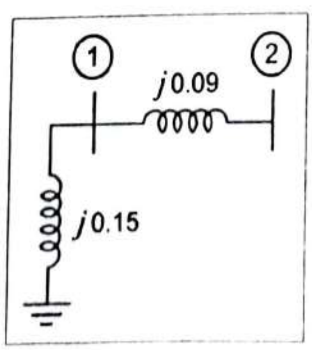
Step 2 : Form Z-bus using bus building algorithm.



$j0.1 \times j0.1 / (0.1+0.1) = 0.01/0.2$

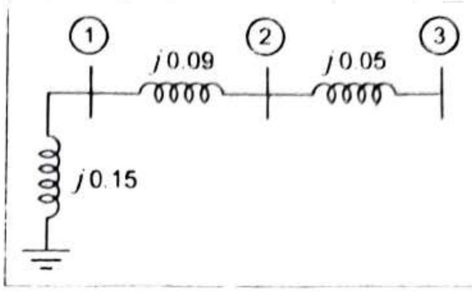


$$Z_{bus} = \begin{matrix} & 1 \\ 1 & [j0.15] \end{matrix}$$

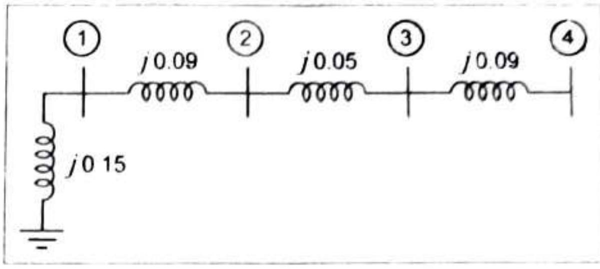


$$Z_{bus} = \begin{matrix} & 1 & 2 \end{matrix} \\ \begin{matrix} 1 \\ 2 \end{matrix} \begin{bmatrix} j0.15 & j0.15 \\ j0.15 & j0.24 \end{bmatrix}$$

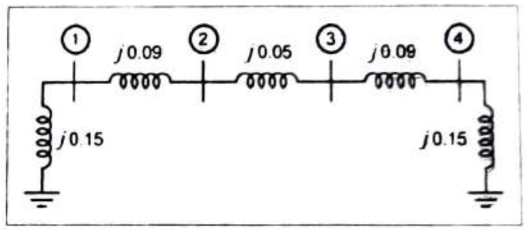
$$Z_{bus} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} j0.15 & j0.15 & j0.15 \\ j0.15 & j0.24 & j0.24 \\ j0.15 & j0.24 & j0.29 \end{bmatrix} \end{array}$$



$$Z_{bus} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} j0.15 & j0.15 & j0.15 & j0.15 \\ j0.15 & j0.24 & j0.24 & j0.24 \\ j0.15 & j0.24 & j0.29 & j0.29 \\ j0.15 & j0.24 & j0.29 & j0.38 \end{bmatrix} \end{array}$$



$$= \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} j0.15 & j0.15 & j0.15 & j0.15 & j0.15 \\ j0.15 & j0.24 & j0.24 & j0.24 & j0.24 \\ j0.15 & j0.24 & j0.29 & j0.29 & j0.29 \\ j0.15 & j0.24 & j0.29 & j0.38 & j0.38 \\ j0.15 & j0.24 & j0.29 & j0.38 & j0.53 \end{bmatrix} \end{array}$$



Eliminate node 5 using the relation,

$$Z_{ij\ new} = Z_{ij\ old} - \frac{Z_{i(n+1)}\ Z_{(n+1)j}}{Z_{(n+1)(n+1)}}$$

$$= \begin{bmatrix} j0.1075 & j0.172 & j0.068 & j0.0424 \\ j0.172 & j0.13 & j0.108 & j0.068 \\ j0.068 & j0.108 & j0.13 & j0.082 \\ j0.0424 & j0.13 & j0.082 & j1.075 \end{bmatrix}$$

Step 3 : Fault current $I_f = \frac{V^\circ}{Z_{qq} + Z_f} = \frac{1 \angle 0^\circ}{j0.1075} = 9.3 \angle -90^\circ$ p.u.

$$\text{Actual current in KA} = \text{p.u. value} \times \text{Base current}$$

$$= 9.3 \angle -90^\circ \times \frac{\text{MVA}}{\sqrt{3} \times \text{KV}}$$

$$= 9.3 \angle -90^\circ \times \frac{100}{\sqrt{3} \times 20} = 26.85 \text{ KA}$$

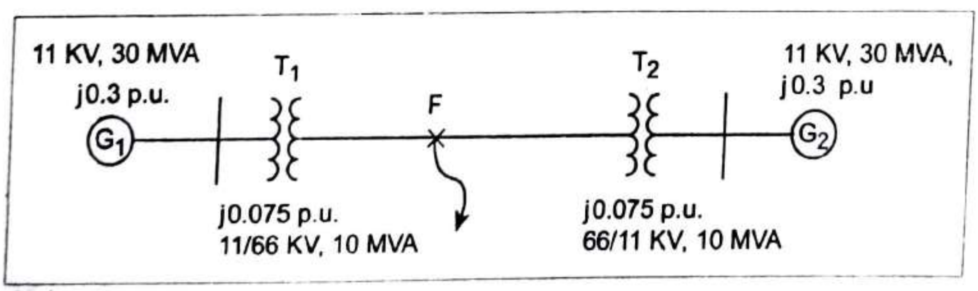
Step 4 : Post fault voltages

$$V_1^f = V^0 - Z_{14} I_f$$
$$= 1\angle 0° - j0.042 \times 9.3 \angle -90° = 0.6056 \text{ p.u.}$$
$$V_2^f = V^0 - Z_{24} I_f$$
$$= 1\angle 0° - j0.068 \times 9.3 \angle -90° = 0.3686 \text{ p.u.}$$
$$V_3^f = V^0 - Z_{34} I_f$$
$$= 1\angle 0° - j0.082 \times 9.3 \angle -90° = 0.2374 \text{ p.u.}$$
$$V_4^f = V^0 - Z_{44} \times I_f$$
$$= 1\angle 0° - j0.1075 \times 9.3 \angle -90° = 0 \text{ p.u.}$$

Step 5 : Post fault line currents.

$$I_{ij}^f = \frac{V_i^f - V_j^f}{Z_{ij\text{ series}}}; \quad I_{12}^f = \frac{V_1^f - V_2^f}{Z_{12}} = \frac{0.6056 - 0.3686}{j0.09}$$
$$|I_{12}^f| = 2.634 \text{ p.u.}$$
$$I_{23}^f = \frac{V_2^f - V_3^f}{Z_{23}}$$
$$|I_{23}^f| = \frac{0.3686 - 0.2374}{j0.05} = 2.63 \text{ p.u.}$$
$$I_{34}^f = \frac{V_3^f - V_4^f}{Z_{34}} = \frac{0.2374 - 0}{j0.09} = 2.637 \text{ p.u.}$$

Example 8.10 ***Generator G_1 and G_2 are identical and rated as 11 KV, 30 MVA and have a transient reactance of 0.3 p.u. at own MVA base. The transformers T_1 and T_2 are also identical and are rated 11/66 KV, 10 MVA and have a reactance of 0.075 per unit to their own MVA base. Then the line is 60 km long, each conductor has a reactance of 0.92 Ω/km. The 3 phase fault is assumed at point F, which is 25 km from generator G_1. Find the short circuit current.***



Solution : Choose common base MVA as 30 MVA, then the reactance of various elements are :

Generator 1, $Z_{p.u.\ new} = j0.3 \times \left[\frac{11}{11}\right]^2 \times \left[\frac{30}{30}\right] = j0.3$

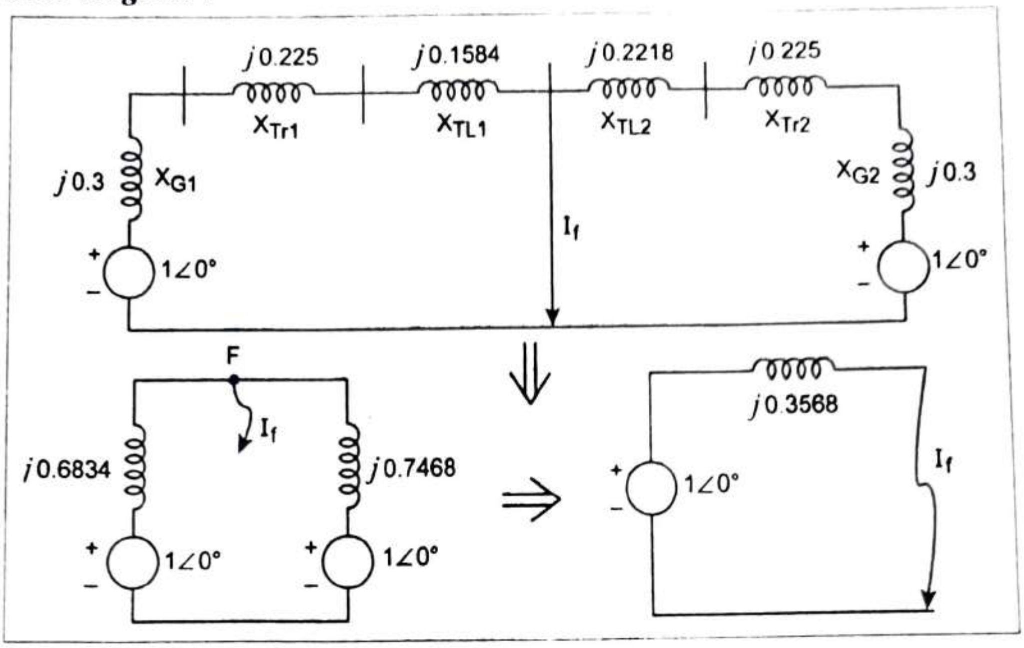
Transformer 1, $Z_{p.u.\ new} = j0.075 \times \left(\frac{11}{11}\right)^2 \times \left[\frac{30}{10}\right] = j0.225$ p.u.

$$\text{Transmission line, } Z_{p.u.\,new} = (j0.92 \times 25) \times \frac{30}{66^2} = j0.1584 \text{ p.u.}$$

$$\text{Transformer 2, } Z_{p.u.\,new} = j0.075 \times \left(\frac{66}{66}\right)^2 \times \left[\frac{30}{10}\right] = j0.225 \text{ p.u.}$$

$$\text{Generator 2, } Z_{p.u.\,new} = j0.3 \times \left[\frac{66}{66}\right]^2 \times \left[\frac{30}{30}\right] = j0.3 \text{ p.u.}$$

Reactance diagram :



$$\text{Fault current } I_f = \frac{E_{Th}}{X_{Th}} = \frac{1\angle 0°}{j0.3568} = -j2.8023 \text{ p.u.}$$

$$I_f = 2.8023 \angle -90°$$

Actual fault current in amperes is

$$= |I_f| \times I_{base}$$

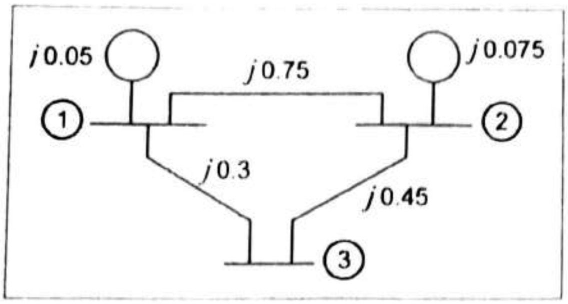
$$\text{Base current } I_{base} = \frac{30 \times 10^3}{\sqrt{3} \times 66} = 262.432 \text{ A}$$

$$\therefore \text{ Actual fault current} = 2.8023 \times 262.432$$

$$= 735.413 \text{ A}$$

Example 8.11 ***The one line diagram of a simple three-bus system is as shown in Fig. Each generator is represented by an emf behind the transient reactance. All impedances are expressed in p.u. on a common 100 MVA base, and***

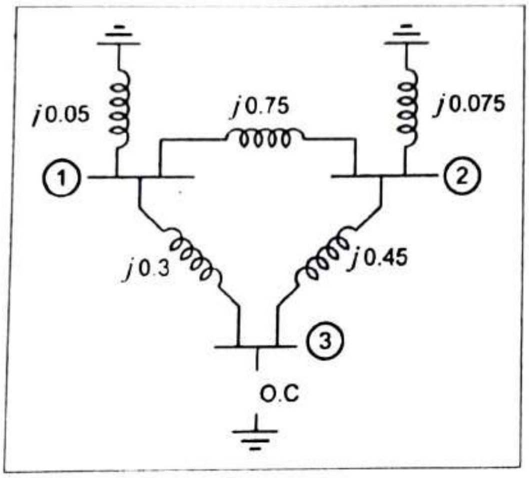
(a) Determine the fault current, the bus voltages and the line currents during the fault when a balanced three-phase fault with a fault impedance $Z_f = j0.19$ per unit occurs on bus 3.



(b) For bolted fault, determine fault current, bus voltages, line flows.

☺ ***Solution : Case (a) :*** Find Thevenin equivalent impedance.

Voltage source is Short circuited and the fault impedance open-circuited.



Converting Δ-connected network into Y connected network using the formula,

$$Z_{i0} = \frac{Z_{ij} \times Z_{i\,(j+1)}}{\sum\limits_{j=1}^{3} Z_{ij}}$$

$$Z_{10} = \frac{j0.75 \times j0.3}{j0.75 + j0.45 + j0.3}$$

$$= \frac{j0.225}{j1.5} = j0.15$$

$$Z_{20} = \frac{j0.75 \times j0.45}{j0.75 + j0.45 + j0.3} = j0.225$$

$$Z_{30} = \frac{j0.45 \times j0.3}{j0.75 + j0.45 + j0.3} = j0.09$$

Thevenin's equivalent circuit

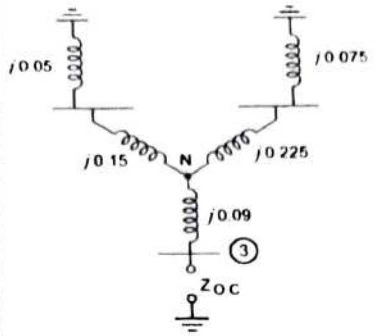


Fig. 1

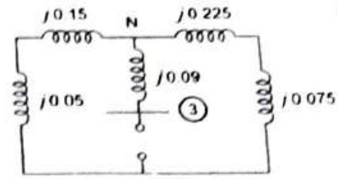


Fig. 2

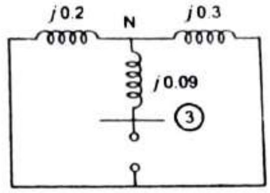


Fig. 3

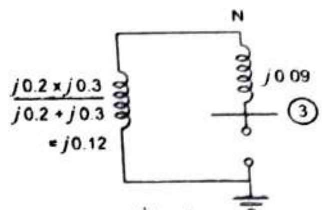


Fig. 4

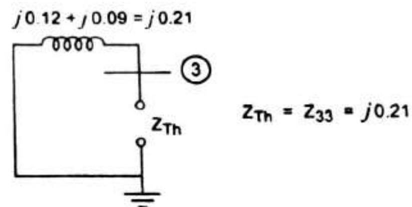
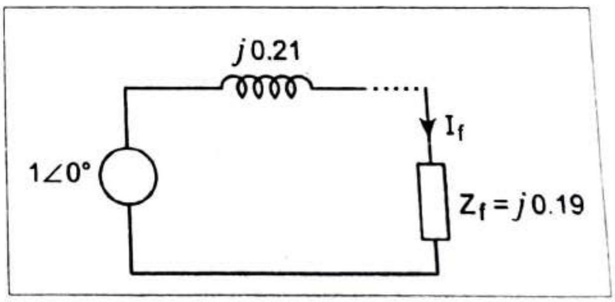


Fig. 5

$$\text{Fault current } I_f = \frac{E_{Th}}{Z_{Th} + Z_f} = \frac{1 \angle 0^\circ}{j0.21 + j0.19} = j2.5 \text{ p.u.}$$



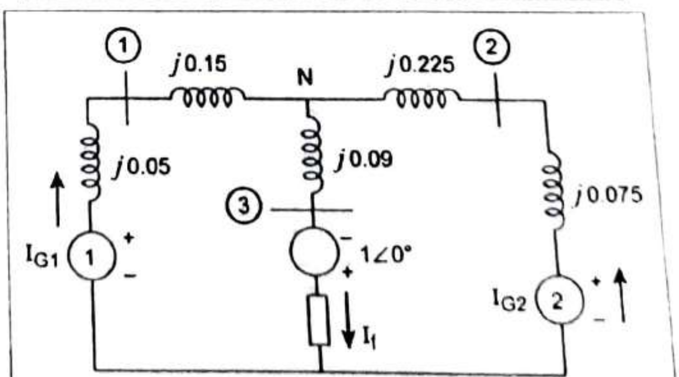


Fig.6

Current contributed by generator 1 from Fig.6,

$$I_{G1} = I_f \times \frac{j0.3}{j0.2 + j0.3} = \frac{-j2.5 \times j0.3}{j0.2 + j0.3} = -j1.5 \text{ p.u.}$$

Current contributed by generator 2,

$$I_{G2} = I_f \times \frac{j0.2}{j0.2 + j0.3} = \frac{-j2.5 \times j0.2}{j0.5} = -j1.0 \text{ p.u.}$$

Bus voltages :

Bus voltage during the fault at bus 1, $V_1^f = V_{p.f.} + \Delta V_1$

$$V_1^f = V_{p.f.} - I_{G1} \times Z_{10}$$

$$= 1 \angle 0° (-j1.5 \times j0.05) = 0.925 \text{ p.u.}$$

Bus voltage during the fault at bus 2 $= V_2^f = V_{p.f.} + \Delta V_2$

$$V_2^f = V_{p.f.} - I_{G2} \times Z_{20}$$

$$= 1 \angle 0° - (-j1.0) \times j0.075 = 0.925 \text{ p.u.}$$

Bus voltage during the fault at bus 3 $= V_3^f = V_{p.f.} + \Delta V_3$

$$= V_{p.f.} + Z_f I_f - 1 \angle 0°$$

$$= 1 \angle 0° + j0.19(-j2.5) - 1 \angle 0°$$

$$= 0.475 \text{ p.u.}$$

Line flows during the fault :

$$I_{12}^f = \frac{V_1 - V_2}{Z_{12 \text{ series}}} = \frac{0.925 - 0.925}{j0.75} = 0 \text{ p.u.}$$

$$I_{13}^f = \frac{V_1 - V_3}{Z_{13 \text{ series}}} = \frac{0.925 - 0.475}{j0.3} = -j1.5 \text{ p.u.} = 1.5 \angle -90° \text{ p.u.}$$

$$I_{32}^f = \frac{V_3 - V_2}{Z_{32 \text{ series}}} = \frac{0.475 - 0.925}{j0.45} = j1.0 \text{ p.u.} = 1.0 \angle 90° \text{ p.u.}$$

$$I_{23}^f = -I_{32}^f = -j1.0 = 1.0 \angle -90° \text{ p.u.}$$

Case (b) : For the bolted fault, $Z_f = 0$

Fault current $I_f = \frac{E_{Th}}{Z_{Th}} = \frac{1 \angle 0°}{j0.21} = -j4.76$ p.u.

Current contributed by generator 1 $= \frac{I_f \times j0.3}{j0.2 + j0.3} = -j2.857$ p.u.

Current contributed by generator 2 $= \frac{I_f \times j0.2}{j0.2 + j0.3} = -j1.904$ p.u.

Bus voltage during the fault at bus 1 $= V_{p.f.} + \Delta V_1$

$$= 1 \angle 0° + (I_{G1} Z_{10}) = 1 \angle 0° - (-j2.85) \times j0.05$$

$$= 0.8575 \text{ p.u.}$$

$$V_2^f = V_2 + \Delta V_2 = 1.0 + (I_{G2} \times Z_{20})$$

$$= 1\angle 0° - (-j1.904) \times j0.075$$
$$= 0.8572 \text{ p.u.}$$
$$V_3^f = V_3 + \Delta V_3 = 1\angle 0° - 1\angle 0° = 0$$

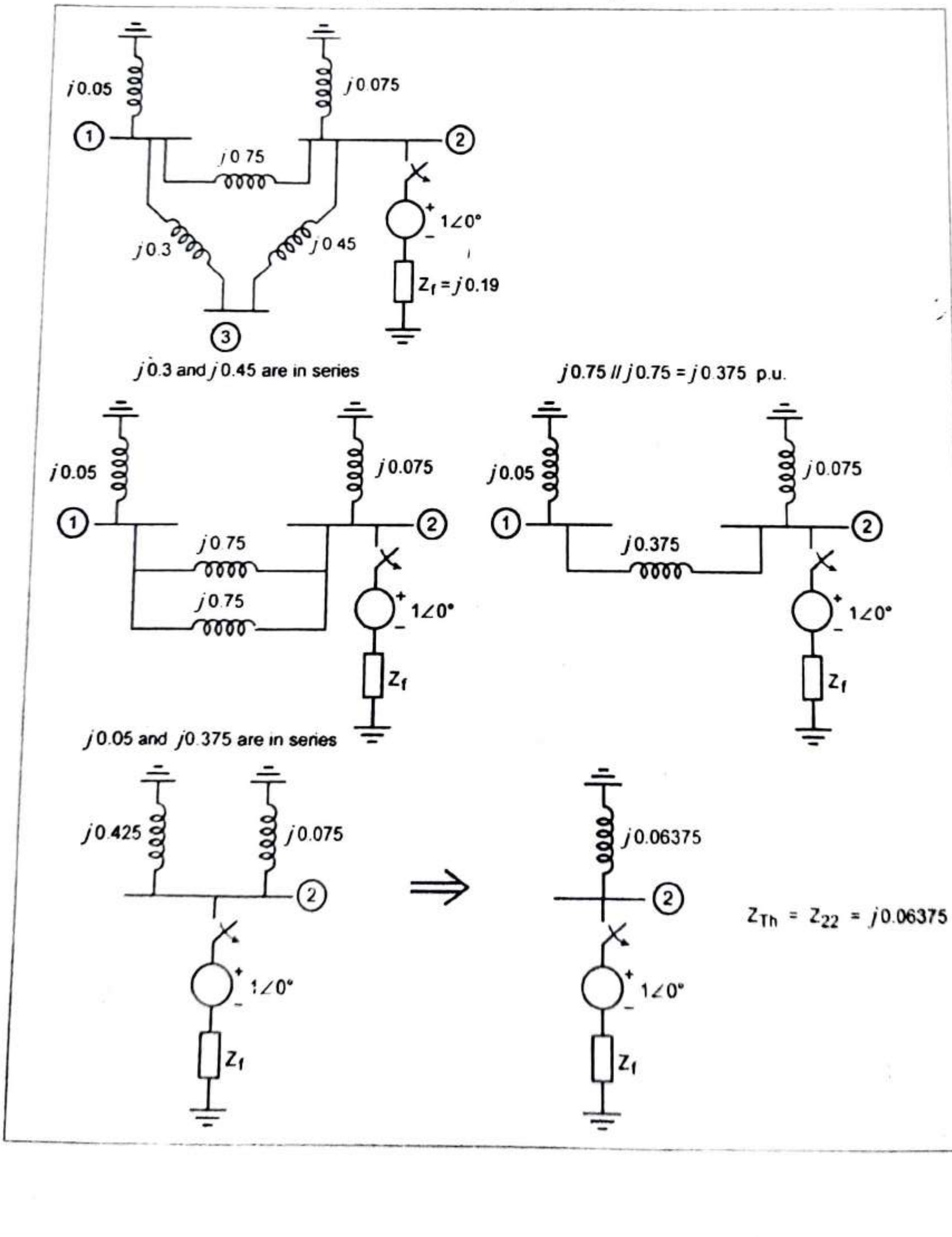
Line flows during the fault $I_{12}^f = \frac{V_1 - V_2}{Z_{12\,\text{series}}} = \frac{0.8575 - 0.8572}{j0.75} = -j0.0004$ p.u.

$$I_{13} = \frac{V_1 - V_3}{Z_{13\,\text{series}}} = \frac{0.8575 - 0}{j0.3} = -j2.858 \text{ p.u.}$$

$$I_{23} = \frac{V_2 - V_3}{Z_{13\,\text{series}}} = \frac{0.8572 - 0}{j0.45} = -j1.9049 \text{ p.u.}$$

Example 8.12 ***For the previous example 8.11, assume fault occurs at bus 2. Determine fault current, bus voltages and line voltage flows during the fault for case (a).***

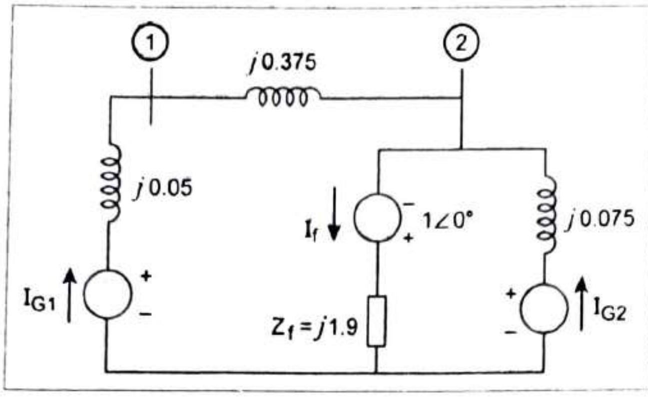
☺ ***Solution :*** Find Thevenin's equivalent impedance,



$$\text{Fault current } I_f = \frac{E_{Th}}{Z_{Th} + Z_f}$$

$$= \frac{1 \angle 0°}{j0.06375 + j0.19} = -j3.94 \text{ p.u.}$$

Current contribution by generators :



$$\text{Current contribution by generator 1, } I_{G1} = \frac{I_f \times j0.075}{j0.05 + j0.375 + j0.075}$$

$$= \frac{-j3.94 \times j0.075}{j0.5} = -j0.591 \text{ p.u.}$$

$$\text{Current contributed by generator 2, } I_{G2} = (-j3.94) \times \frac{[j0.05 + j0.375]}{j0.05 + j0.375 + j0.075}$$

$$= -j3.349 \text{ p.u.}$$

Bus voltages & Line flows :

$$V_1^f = V_{p.f.} + \Delta V_1 = 1 \angle 0° - I_{G1} \times Z_{10}$$

$$= 1 \angle 0° - (-j0.591) \times j0.05 = 0.97 \text{ p.u.}$$

$$V_2^f = V_{p.f.} + \Delta V_2 = 1 \angle 0° - I_{G2} \times Z_{20}$$

$$= 1 \angle 0° - (-j3.349) \times j0.075 = 0.7488 \text{ p.u.}$$

From Fig.1,

$$I_{12} = \frac{V_1 - V_2}{Z_{12 \text{ series}}} = \frac{0.97 - 0.7488}{j0.75} = -j0.295 \text{ p.u.}$$

$$I_{13} = I_{G1} - I_{12} = -j0.591 - (-j0.295) = -j0.296 \text{ p.u.}$$

$$V_{13} = I_{13} \times Z_{13 \text{ series}} = -j0.296 \times j0.3 = 0.089 \text{ p.u.}$$

$$V_3^f = V_1^f - V_{13} = 0.97 - 0.089 = 0.881 \text{ p.u.}$$

$$I_{23} = \frac{V_2 - V_3}{Z_{23 \text{ series}}} = \frac{0.7488 - 0.881}{j0.45} = j0.294 \text{ p.u.}$$

Transient Reactance: Ratio of induced emf on no-load and the transient symmetrical rms current.

$$X_d' = \frac{|E_g|}{|I'|} = \text{Reactance of syn. machine under transient condition}$$

Sub-transient Reactance: Ratio of included emf on no-load and the sub-transient rms current.

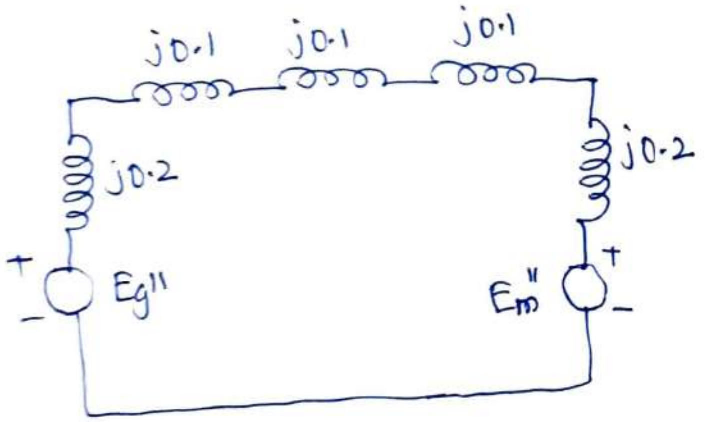
$$X_d'' = \frac{|E_g|}{|I''|} = \text{Reactance of syn. machine under subtransient conduction.}$$

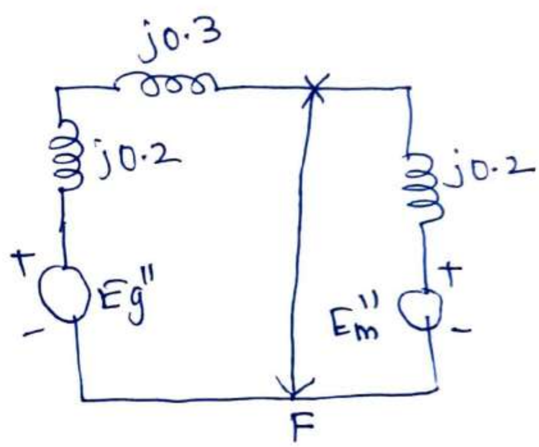
Load flow studies	Short circuit studies
* The resistances and reactance are considered.	* The resistance are neglected.
* To solve load flow analysis, the Y_{Bus} matrix is used.	* To solve SC analysis, the Z_{Bus} matrix is used.
* used to determine the exact voltages and current	* prefault voltages are assumed to be 1 pu and the prefault current can be neglected.

(4)

A synchronous generator and a synchronous motor each rated 30 MVA, 11 kV having 20% sub-transient reactance are connected through transformers and a line as shown in figure. The transformers are rated 30 MVA, 11/66 kV and 66/11 kV with leakage reactance of 10% each. The line has a reactance of 10% on a base of 30 MVA, 66 kV. The motor is drawing 20 MW at 0.8 pf leading and a terminal voltage of 10.6 kV when a symmetrical 3ϕ fault occurs at the motor terminals. Find the subtransient current in the generator and motor.

Base MVA = 30   Base kV = 11 kV.





Terminal voltage = 10.6 kV

prefault voltage $V_{pf} = V^{0} = \dfrac{\text{Actual voltage}}{\text{Base voltage}}$

$= \dfrac{10.6}{11} = 0.9636$ pu

Load in p.u $= \dfrac{\text{Actual MW}}{\text{Base MVA} \times \text{pf}} = \dfrac{20}{30 \times 0.8} = 0.8333$ pu

Prefault current I^{0} (I_L) $= \dfrac{\text{MVA in pu}}{\text{kV in pu}} = \dfrac{0.8333}{0.9636} \angle \cos^{-1} 0.8$

$= 0.865 \angle 36.87$

Using Thevenin's theorem, $Z_{th} = \frac{j0.2 \times j0.5}{j0.2 + j0.5}$

$= j0.1428$

p.u value of prefault voltage

$V_{p.f} = E_{th} = \frac{10.6\,kV}{11} = 0.9636\,pu$

Fault current $I_f = \frac{E_{th}}{Z_{th}} = \frac{0.9636}{j0.1428}$

$= -j6.745\,pu$

Base current $I_b = \frac{MVA_b}{\sqrt{3}\,kV_b} = \frac{30}{\sqrt{3} \times 11} = 1.5746\,KA$

$I_f = -j6.745 \times 1.5746 = -j10.62\,KA$

$I_{uf}'' = \frac{I_f \times j0.2}{j0.5 + j0.2} = -j1.9272\,pu$

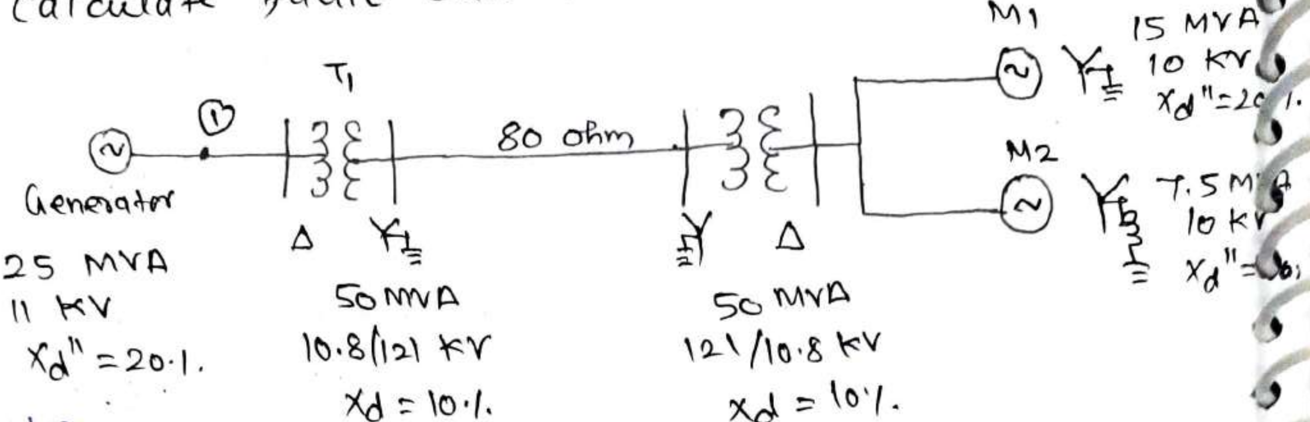
$I_{mf}'' = I_f \times \frac{j0.5}{j0.5 + j0.2} = -j4.818\,pu$

$I_g'' = I_{uf}'' + I_L = 0.692 - j1.408$

$I_m'' = I_{mf}'' - I_L = 0.692 - j5.337.$

25 MVA, 11 kV 3ϕ generator has a sub-transient reactance of 20%. The generator supplies two motors over a transmission line with transformers at both ends as shown in fig. The motors have rated input of 15 and 7.5 MVA both 10 kV and $x'' = 20\%$. The 3ϕ transformers are both rated 50 MVA, 10.8/121 kV with Δ-Y connection with leakage reactance of 10% each. The series reactance of the line is 80 Ω. Draw the

impedance diagram of the s/m with reactances marked in p.u when symmetrical fault occurs at bus 2 and calculate fault current.



Generator

$$Z_{p.u(new)} = Z_{p.u(gn)} \times \left[\frac{KV_{b,given}}{KV_{b,new}}\right]^2 \times \frac{MVA_{b,new}}{MVA_{b,given}}$$

$$= j0.2 \times \left(\frac{11}{11}\right)^2 \times \left(\frac{25}{25}\right) = j0.2 \text{ pu}$$

Transformer 1

$$Z_{p.u(new)} = j0.1 \times \left[\frac{10.8}{11}\right]^2 \times \left[\frac{25}{50}\right] = j0.0482 \text{ pu}$$

Transmission line

$$KV_{b,new} = KV_{b,old} \times \left(\frac{H.T\ rating}{L.T\ rating}\right) = 11 \times \frac{121}{10.8} = 123.24 \text{ KV}$$

$$Z_{p.u(new)} = \frac{Z_{actual}}{Z_{Base}} = \frac{j80}{KV_b^2} \times MVA_b$$

$$= \frac{j80 \times 25}{(123.24)^2} = j0.1316 \text{ pu}$$

Transformer 2

$$Z_{p.u(new)} = j0.1 \times \left[\frac{121}{123.24}\right]^2 \times \left[\frac{25}{50}\right] = j0.482$$

Motor 1

$$KV_{b,new} = KV_{b,old} \times \left[\frac{LT\ rating}{HT\ rating}\right]$$

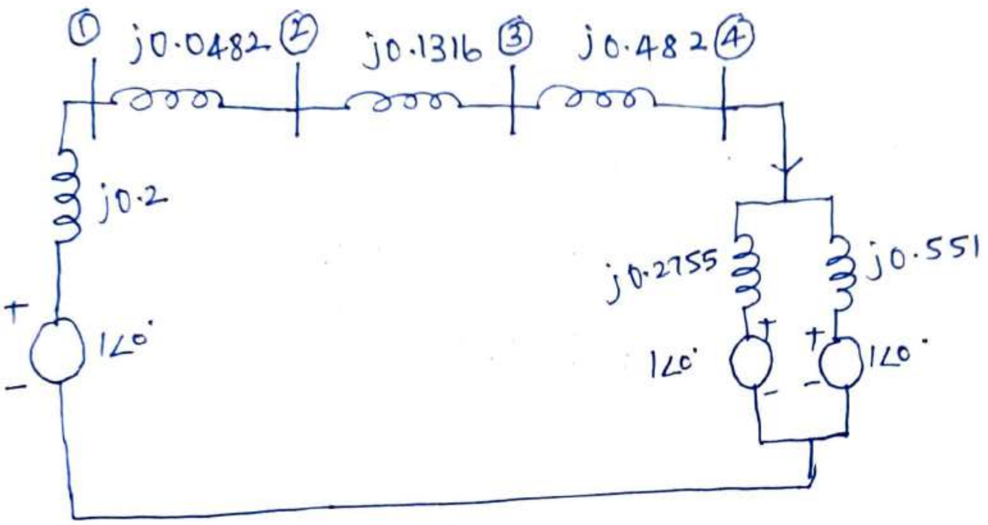
$$= 123.24 \times \left[\frac{10.8}{121}\right] = 11\ KV$$

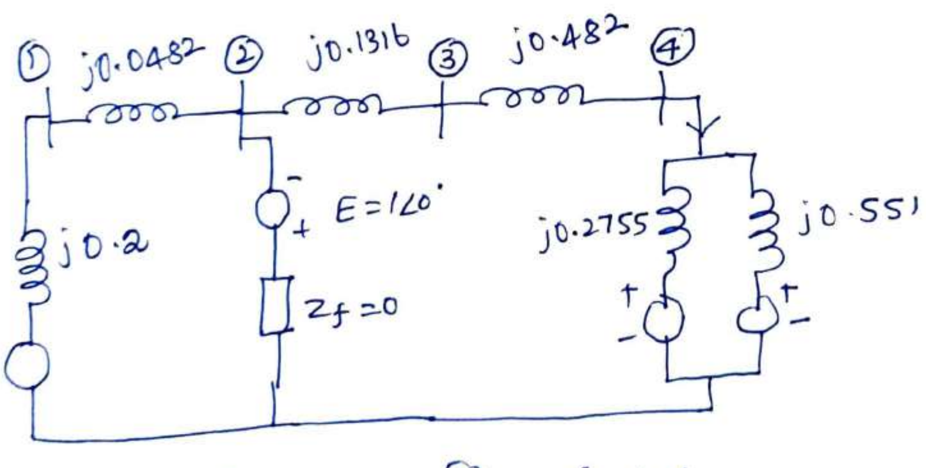
$$Z_{pu(new)} = j0.2 \times \left[\frac{10}{11}\right]^2 \left[\frac{25}{11}\right] = j0.2755\ pu$$

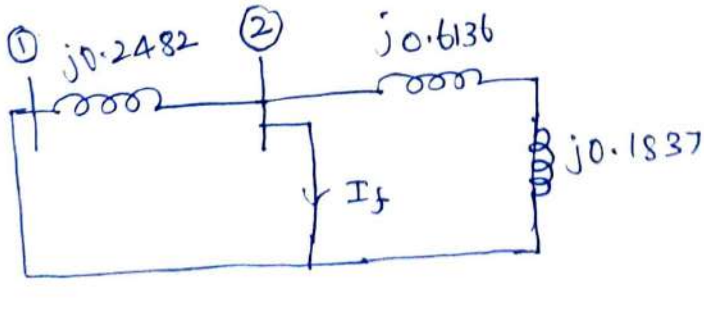
Motor 2

$$Z_{pu(new)} = j0.2 \times \left[\frac{10}{11}\right]^2 \times \left[\frac{25}{50}\right]$$

$$= j0.551\ pu$$







$$Z_{th} = \frac{j0.2482 \times [j0.6136 + j0.1837]}{j0.2482 + j0.6136 + j0.1837}$$

$$= j0.1893 \text{ pu}$$

$$I_f = \frac{E_{th}}{Z_{th} + Z_f} = \frac{1\angle 0^\circ}{j0.1893} = -j5.28$$

$$= 5.28\angle -90^\circ \text{ pu}$$

Base current $I_b = \frac{MVA_b}{\sqrt{3}\ KV_b} = \frac{25 \times 10^6}{\sqrt{3} \times 123.24 \times 10^3}$

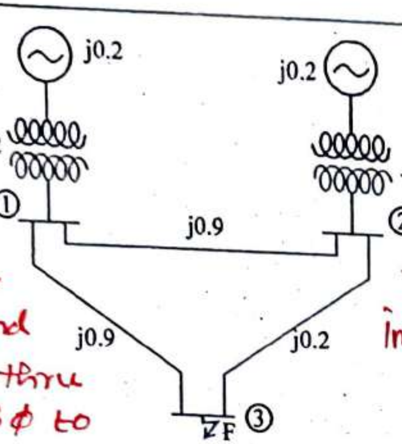
$$= 117.119 \text{ Amps}$$

Actual fault current $= I_f \times I_{base}$

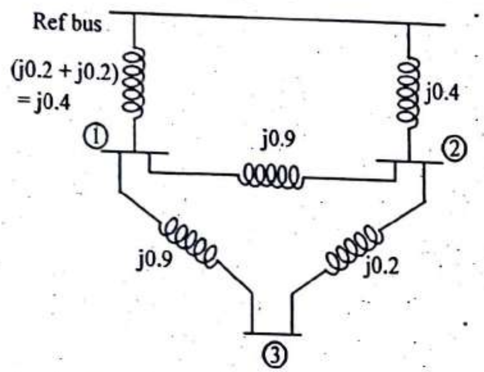
$$= 5.28 \times 117.119$$

$$= 618.388 \text{ Amp}$$

Question : Take Base as 100 MVA, Determine the fault current, fault voltages at all the buses and fault current thru the lines, if a 3ϕ to ground fault has occured at Bus 3 through a fault impedance = j0.15 pu

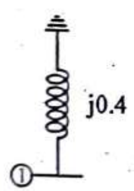


Solution :



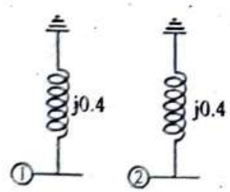
The equivalent diagram becomes

Step 1 : Adding an new bus (1) to reference bus (Type - I)



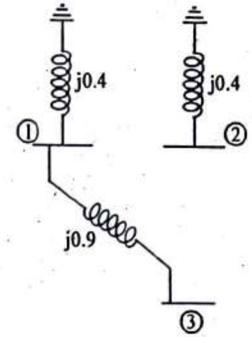
$$Z_{Bus} = [j\,0.4]$$

Step 2 : Adding bus (2) to reference bus (Type - I)



$$Z_{Bus} = \begin{bmatrix} j0.4 & 0 \\ 0 & j0.4 \end{bmatrix}$$

Step 3 : Adding a new bus (3) to old bus (1) Type II



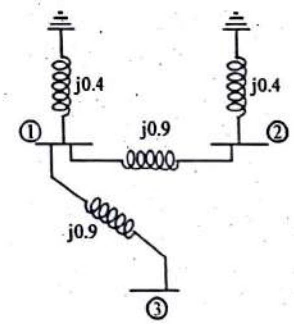
$$Z_{Bus} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{bmatrix} 1 & 2 & 1 \\ j0.4 & 0 & j0.4 \\ 0 & j0.4 & 0 \\ j0.4 & 0 & j1.3 \end{bmatrix}$$

$$\Downarrow$$

$$= Z_{11} + Z_S$$
$$= j\,0.4 + j\,0.9$$
$$= j\,1.3$$

Step 4 : Adding an impedance j0.9 between two old buses (2) and (1)

i = 2
k = 1
2 – 1



$$Z_{Bus} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4^{2-1} \end{array} \begin{bmatrix} 1 & 2 & 3 & 4^{1-2} \\ j0.4 & 0 & j0.4 & -j0.4 \\ 0 & j0.4 & 0 & j0.4 \\ j0.4 & 0 & j1.3 & -j0.4 \\ -j0.4 & j0.4 & -j0.4 & j1.7 \end{bmatrix}$$

$$\Downarrow$$

$$= Z_{ii} + Z_{kk} + Z_s - 2Z_{ik}$$
$$= Z_{22} + Z_{11} + Z_s - 2\,Z_{21}$$
$$= j\,0.4 + j0.4 + j0.9 - 2\,(0)$$
$$= j\,1.7$$

P = 4 the node to be eliminate.

By kron's reduction rechnique,

$$Z_{bus,new} = Z_{bus,old} - \frac{1}{j1.7} \begin{bmatrix} -j0.4 \\ j0.4 \\ -j0.4 \end{bmatrix} [-j0.4 \;\; j0.4 - j0.4]$$

$$= \begin{bmatrix} j0.4 & 0 & j0.4 \\ 0 & j0.4 & 0 \\ j0.4 & 0 & j1.3 \end{bmatrix} - \frac{1}{j1.7} \begin{bmatrix} -0.16 & +0.16 & -0.16 \\ 0.16 & -0.16 & 0.16 \\ -0.16 & 0.16 & -0.16 \end{bmatrix}$$

$$= \begin{bmatrix} j0.4 & 0 & j0.4 \\ 0 & j0.4 & 0 \\ j0.4 & 0 & j1.3 \end{bmatrix} - \begin{bmatrix} j0.094 & -j0.094 & j0.094 \\ -j0.094 & +j0.094 & -j0.094 \\ j0.094 & -j0.094 & j0.094 \end{bmatrix}$$

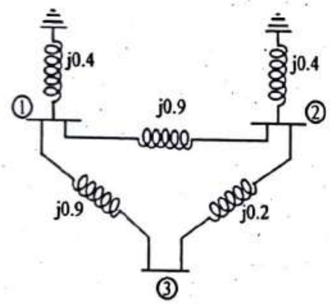
$$= \begin{bmatrix} j0.306 & j0.094 & j0.306 \\ j0.094 & j0.306 & j0.094 \\ j0.306 & j0.094 & j1.206 \end{bmatrix}$$

Step 5 : Adding an impedance j0.2 between two old buses (3) and (2)

i = 3

k = 2

3 – 2



$$Z_{bus} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 3.2 \end{array} \begin{bmatrix} 1 & 2 & 3 & 3.2 \\ j0.306 & j0.094 & j0.306 & j0.212 \\ j0.094 & j0.306 & j0.094 & -j0.212 \\ j0.306 & j0.094 & j1.206 & j1.112 \\ j0.212 & -j0.212 & j1.112 & j1.524 \end{bmatrix}$$

$$\Downarrow$$

$$= Z_{33} + Z_{22} - Z_{32} + Z_5$$

$$= j\,1.206 + j\,0.306 - 2\,(j\,0.094 + j\,0.2$$

$$= j\,1.524$$

$$Z_{Bus,new} = Z_{bus,old} - \frac{1}{j1.524}\begin{bmatrix} j0.212 \\ -j0.212 \\ j1.112 \end{bmatrix}[j0.212 - j0.212 + j1.112]$$

$$= \begin{bmatrix} j0.306 & j0.094 & j0.306 \\ j0.094 & j0.306 & j0.094 \\ j0.306 & j0.094 & j1.206 \end{bmatrix} \frac{1}{j1.52}\begin{bmatrix} -0.04 & 0.04 & -0.235 \\ 0.04 & -0.04 & 0.235 \\ -0.235 & 0.235 & -1.236 \end{bmatrix}$$

$$= \begin{bmatrix} j0.306 & j0.094 & j0.306 \\ j0.094 & j0.306 & j0.094 \\ j0.306 & j0.094 & j1.206 \end{bmatrix} - \begin{bmatrix} j0.026 & -j0.026 & j0.154 \\ -j0.026 & +j0.026 & -j0.154 \\ j0.154 & -j0.154 & +j0.813 \end{bmatrix}$$

$$Z_{bus,new} = \begin{bmatrix} j0.28 & j0.12 & j0.152 \\ j0.12 & j0.28 & j0.248 \\ j0.152 & j0.248 & j0.393 \end{bmatrix}$$

Fault occurs at bus 3 :

$$I_k(f) = \frac{V_k(0)}{Z_{kk} + Z_f}$$

$$I_3(f) = \frac{V_3(0)}{Z_{33} + Z_f} \quad \text{(fault current)}$$

$$z_f = j\,0.15 \text{ (Given)}$$

$$= \frac{110^\circ}{j0.393 + j0.15}$$

$$= \frac{110^\circ}{j0.543} = -j1.84 = 1.84\angle -90^\circ\ pu$$

$$\begin{bmatrix} \Delta V_1 \\ \Delta V_2 \\ \Delta V_3 \end{bmatrix} = \begin{bmatrix} j0.28 & j0.12 & j0.152 \\ j0.12 & j0.28 & j0.248 \\ j0.152 & j0.248 & j0.393 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ +j1.84 \end{bmatrix}$$

$$\begin{bmatrix} \Delta V_1 \\ \Delta V_2 \\ \Delta V_3 \end{bmatrix} = \begin{bmatrix} -0.279 \\ -0.456 \\ -0.723 \end{bmatrix}$$

Fault voltages at all the buses

$$V_i(f) = V_i(0) + \Delta V_i \text{ (General)}$$

$$v_1(f) = V_1(0) + \Delta V_1$$

$$= 1 - 0.279$$

$$\boxed{V_1(f) = 0.721 \text{ pu}}$$

$$V_2(f) = V_2(0) + \Delta V_2$$

$$= 1 - 0.456$$

$$\boxed{V_2(f) = 0.54 \text{ pu}}$$

$$V_3(f) = V_3(0) + \Delta V_3$$

$$= 1 - 0.723$$

$$\boxed{V_3(f) = 0.277 \text{ pu}}$$

Fault current through the lines (ie) line currents

$$\text{In general } I_{ij}(f) = \frac{V_i(f) - V_j(f)}{Z_{bij}}$$

$$I_{12}(F) = \frac{0.721 - 0.54}{j0.9} = -j0.2\,pu$$

$$I_{23}(F) = \frac{V_2(f) - V_3(f)}{Z_{b13}}$$

$$= \frac{0.54 - 0.277}{j0.2} = -j1.3216\,pu$$

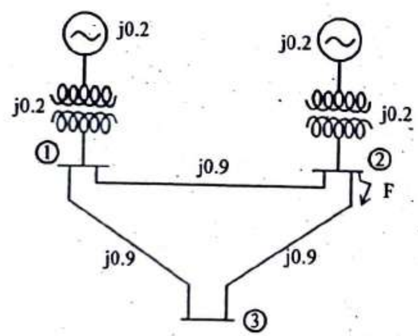
$$I_{13}(F) = \frac{V_1(f) - V_3(f)}{Z_{b13}}$$

$$= \frac{0.721 - 0.277}{j0.9} = -j0.49\,pu$$

Example: 3.21

Determine the fault current, fault voltages at all buses and fault current through the lines. Consider the fault is occuring at 2nd bus. Take $Z_f = 0.15\,pu$

Solution:



$$Z_{bus} = \begin{bmatrix} j0.288 & j0.112 & j0.2 \\ j0.112 & j0.288 & j0.2 \\ j0.2 & j0.2 & j0.65 \end{bmatrix}$$

i. Fault current

$$I_2(f) = \frac{V_2(0)}{Z_{22} + Z_f} = \frac{1\angle 0^\circ}{j0.228 + j0.15} = \frac{1}{j0.378}$$

$$I_2(f) = -j\,2.645\text{ pu}$$

ii. Fault voltages at all buses

$$\begin{bmatrix} \Delta V_1 \\ \Delta V_2 \\ \Delta V_3 \end{bmatrix} = [Z_{bus}] \begin{bmatrix} 0 \\ j2.645 \\ 0 \end{bmatrix} = \begin{bmatrix} -0.296 \\ -0.761 \\ -0.529 \end{bmatrix}$$

$$V_1(f) = V_1(0) + \Delta V_1 = 1 - 0.296 = 0.704\text{ pu}$$

$$V_2(f) = V_2(0) + \Delta V_2 = 1 - 0.761 = 0.239\text{ pu}$$

$$V_3(f) = V_3(0) + \Delta V_3 = 1 - 0.529 = 0.471\text{ pu}$$

iii. Line currents

$$I_{12} = -j\,0.516\text{ pu}$$

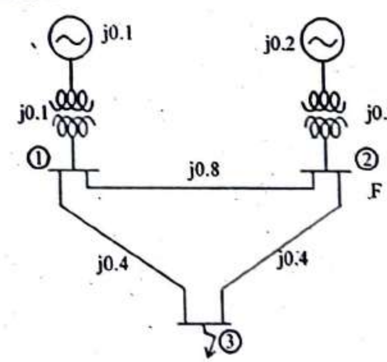
$$I_{13} = -j\,0.258\text{ pu}$$

$$I_{23} = +j\,0.257\text{ pu}$$

Example: 3.22

The single line diagram of a sample 3 bus system is shown in figure. Find the fault current and fault voltages at all the buses when a 3ϕ SC fault occurs @ bus 3 through a fault impedance $Z_f = 0.16\,pu$.

Solution :



$$Z_{bus} = \begin{bmatrix} j0.16 & j0.08 & j0.12 \\ j0.08 & j0.24 & j0.16 \\ j0.12 & j0.16 & j0.34 \end{bmatrix}$$

i. **Fault current :**

$$I_3(f) = \frac{V_3(0)}{Z_{33}+Z_f} = \frac{1}{j0.34+j0.16} = -j2$$

ii. **Fault voltages at all the2 buses**

at Bus 1 : $V_1(f) = V_1(0) - Z_{13} I_3(f)$

$= 1\angle 0^0 - (j\,0.12)(-j2) = 1 - 0.24$

$V_1(f) = 0.76$ pu

at Bus 2 : $V_2(f) = V_2(0) - Z_{23} I_3(f)$

$= 1\angle 0^o - (j\,0.16)(-j2) = 1 - 0.32$

$V_2(f) = 0.68$ pu

at Bus 3 : $V_3(f) = V_3(0) - Z_{33} I_3(f)$

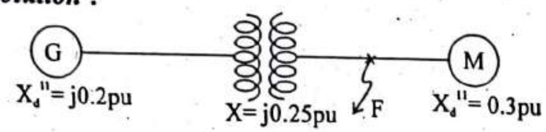
$= 1\angle 0^o - j\,(0.34)(-j2) = 1 - 0.68$

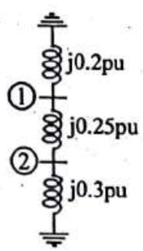
$V_3(f) = 0.32$ pu

Example: 3.23

Find the fault current above for the ps shown.

☺ ***Solution :***



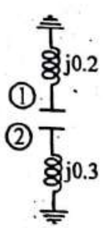


Form the Z_{bus} by building Algorithm

Step 1 : Adding a new bus (1) to reference

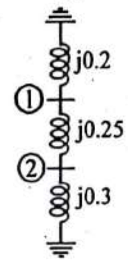
$$Z_{bus} = [j0.2]$$

Step 2 : Adding a new bus (2) to reference



$$Z_{bus} = \begin{bmatrix} j0.2 & 0 \\ 0 & j0.3 \end{bmatrix}$$

Step 3 : Adding an impedance j0.25 between two old buses (1) & (2)



i = 2

k = 1

(2 – 1)

$$Z_{bus} = \begin{bmatrix} j0.2 & 0 & j0.2 \\ 0 & j0.3 & -j0.3 \\ j0.2 & -j0.3 & j0.75 \end{bmatrix}$$

By kron's reduction technique

$$Z_{11,new} = j(0.2) - \frac{j(0.2)^2}{j0.75} = j0.147$$

$$Z_{22,new} = j0.3 - \frac{j(-0.3)^2}{j0.75} = j0.18$$

$$Z_{12,new} = Z_{21,new} = j(0) - \frac{j(0.2)-j0.3}{j0.75} = j0.08$$

$$\therefore Z_{Bus} = \begin{bmatrix} j0.147 & j0.08 \\ j0.08 & j0.18 \end{bmatrix}$$

The fault current is $Z_f = 0$

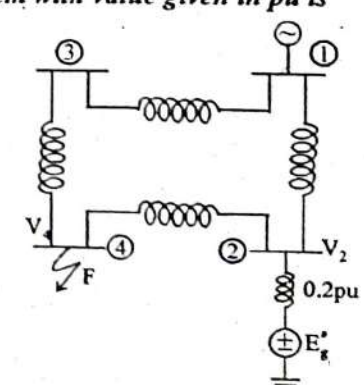
$$I_2(f) = \frac{V_2(0)}{Z_{22}+Z_f} = \frac{1\angle 0^o}{j0.18} = -j5.56$$

$I_f = 5.56\angle -90^o$ pu

Example: 3.24

The bus impedance matrix of 4 bus system with value given in pu is

$$Z_{bus} = \begin{bmatrix} 0.15 & 0.08 & 0.04 & 0.07 \\ 0.08 & 0.15 & 0.06 & 0.09 \\ 0.04 & 0.06 & 0.13 & 0.05 \\ 0.07 & 0.09 & 0.05 & 0.12 \end{bmatrix}$$



In this system generaters are connected at bus (1) and (2) and their subtransient reactance are induded while finding Z_{bus}. If pre-fault current is neglected. Find the subtransient current in pu if 3 ϕ faults occurs at bus 4. Assume pre fault voltager ats 1 pu. If X_d'' of G in bus 2 is 0.2 pu. Find I_f'' supplied by the generater.

Solution :

$$I_k(f) = \frac{V_k(0)}{Z_{kk}+Z_f}$$

$$I_4(f) = \frac{V_4(0)}{Z_{44}+Z_f} = \frac{1\angle 0^\circ}{j0.12} = -j8.3333\ pu$$

$$I_g'' = \frac{E_g - V_2(f)}{X_d''}$$

$$= 1\angle 0^\circ - 0.2503$$

$$I_g'' = -j3.7485$$

$$V_2(f) = V_2(0) - Z_{24} I_4(f)$$
$$= 1 - (j\,0.09)(-j\,8.333)$$

$$\boxed{V_2(f) = 0.2503\ \text{pu}}$$

Example: 3.25

The bus impedance matrix of 4 bus system with values in pu is given by

$$Z_{bus} = \begin{bmatrix} j0.2 & j0.05 & j0.071 & j0.085 \\ j0.05 & j0.25 & j0.013 & j0.072 \\ j0.071 & j0.013 & j0.44 & j0.06 \\ j0.085 & j0.072 & j0.06 & j0.45 \end{bmatrix}$$ ***If a 3 ϕ fault occurs at bus 1***

when there is no load, find the subtransient current in the fault and voltage at all buses. Also find the subtransient current supplied by the generator connected to bus 2 by taking the subtransient reactance of generator as j 0.2 pu. The prefault voltage is 1 pu.

Solution :

Faults occurs at bus 1

$$\therefore I_1(f) = \frac{V_1(0)}{Z_{11}+Z_f} = \frac{1\angle 0^\circ}{j0.2} = 5\angle -90^\circ\ pu$$

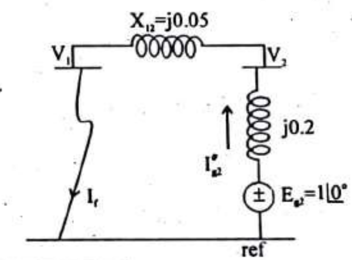
Voltages at all buses

at Bus 1 : $V_1(f) = V_1(0) - Z_{11} I_1(f)$
$$= 1\angle 0^\circ - (j\,0.2)(-j5) = 0$$

at Bus 2 : $V_2(f) = V_2(0) - Z_{21} I_1(f)$
$$= 1\angle 0^\circ - (j\,0.05)(-5\angle -90^\circ) = 0.75\ \text{pu}$$

at Bus 3 : $V_3(f) = V_3(f) = V_3(0) - Z_{31} I_1(f)$
$$= 1\angle 0^\circ - (j\,0.071)(-5\angle -90^\circ) = 0.645\ \text{pu}$$

at Bus 4 : $V_4(f) = V_4(0) - Z_{41} I_1(f)$
$$= 1\angle 0^\circ - (j\,0.085)(-5\angle -90^\circ) = 0.575\ \text{pu}$$



$$\therefore I_{g2}'' = \frac{E_{g2}''}{j0.2}$$

$$= \frac{1\angle 0^\circ - 0.75\angle 0^\circ}{0.2\angle 90^\circ}$$

$$\boxed{I_{g2}'' = 1.25\angle -90^\circ\ pu}$$

Example: 3.26

The bus impedance matrix of four bus system with values in pu is given by

$$Z_{bus} = j\begin{bmatrix} 0.1488 & 0.0651 & 0.0864 & 0.0978 \\ 0.0651 & 0.1554 & 0.0799 & 0.0967 \\ 0.0864 & 0.0798 & 0.1341 & 0.1058 \\ 0.0978 & 0.0967 & 0.1058 & 0.1566 \end{bmatrix}$$ ***If a 3ϕ fault occurs at bus – 1***

when there is no – load. Find the subtransient current in the fault and voltager at all buses. Also find the subtransient current supplied by the generator connected to bus - 2 by taking the subtransient reactance of generator as j 0.2 pu. The prefault voltage is 1 pu.

Solution :

Let $I_1(f)$ (or) I_f'' be the subtransient fault current injected to bus - 1 due to a 3 ϕ fault

$$I''_f = \frac{V_1(0)}{Z_{11}} = \frac{1\angle 0^\circ}{j0.1488} = 6.7204 \angle -90^\circ \ pu$$

Let V_1, V_2, V_3 & V_4 be the voltages at all buses

$$V_1(f) = V_1(0) - Z_{11} I''_f = 0$$

$$V_2(f) = V_1(0) - Z_{21} I''_f = 0.5625 \text{ pu}$$

$$V_3(f) = V_1(0) - Z_{31} I''_f = 1\angle 0^\circ - 6.7204 \angle 90^\circ \times 0.0864 \angle 90^\circ$$

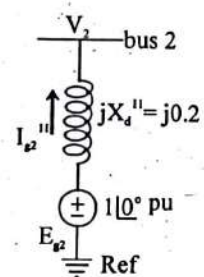
$$= 1 - 0.5806 = 0.4194 \text{ pu}$$

$$V_4(f) = V_1(0) - Z_{41} I''_f$$

$$= 1\angle 0^\circ - (6.7204 \angle 90^\circ \times 0.0978 \angle 90^\circ) = 0.327 \text{ pu}$$

From figure

The subtract fault current delivered by gen at bus – 2



$$I_{g2}'' = \frac{E_{g2}'' - V_2}{jX_{d2}''}$$

$$= \frac{1\angle 0^\circ - 0.5625}{j0.2}$$

$$= 2.1875 \angle -90^\circ \text{ pu}$$

3.6 Short Circuit Capacity (SCC) of a bus

✓ SCC is defined as the product of the magnitude of prefault bus voltage and post fault current.

✓ The size and dimension of busbars are determined by SC MVA

✓ Interrupting capacity of CB is also determined by SCC.

$$SCC = \sqrt{3}\, V_{k(L-L)}\, I_k(f) \times 10^{-3} \text{ MVA} \quad \ldots (1)$$

(for bus k)

$V_{k(L-L)} \rightarrow$ line to line voltage at bus k

$I_k(f) \rightarrow$ fault current at bus k

Symmetrical 3ϕ fault current in pu

$$I_k(f) \text{ in pu} = \frac{V_k(0)}{X_{kk}}$$

$V_{k(0)}$ – prefault voltage at bus 'k'

Base current

$$I_B = \frac{S_B \times 10^3}{\sqrt{3}\, KV_b}$$

$$\therefore I_k(f)_{act} = I_k(f) \text{ pu} * I_B$$

$$I_k(f)_{act} = \frac{V_k(0)}{X_{kk}} * \frac{S_B \times 10^3}{\sqrt{3}\, KV_b} \quad \ldots (2)$$

Sub (2) in (1)st equation, we get

$$SCC = \sqrt{3}\, V_{k(L-L)} \frac{V_k(0)}{X_{kk}} * \frac{S_B \times 10^3}{\sqrt{3} KV_b} \times 10^{-3}$$

$$= \frac{V_k(0)}{X_{kk}} S_B \frac{V_{k(L-L)} \times 10^3}{KV_b} \times 10^{-3}$$

if $V_{k(L-L)} = KV_b$

Then $SCC = \frac{V_k(0)}{X_{kk}} S_B \qquad V_k(0) = 1$ pu

$$SCC = \frac{1}{X_{kk}} S_B$$

$$\boxed{SCC = \frac{S_B}{X_{kk}}}$$

Example: 3.27

A generator connected through a 5 cycle CB to a transforms is rated as 100 MVA, 18 KV with reactances Xd'' = 20%, Xd' = 25%, Xd = 110%. It is operated on no load at rated voltage when a three phase faults occurs between breaker and transformer then

Calculate

(a) SC current in CB

(b) the initial sym rms current in CB

(c) the maximum possible dc component of SC in the breaker

(d) the current to be interrupted by the breaker

(e) the interrupting MVA.

☺ ***Solution :***

$$\text{Base current } I_B = \frac{MVA_b}{\sqrt{3}\, KV_b} \times 1000 = \frac{100 \times 1000}{\sqrt{3} \times 18}$$

$$= 3207 \text{ A}$$

Note :

Speed of operation	Multiplication factor
8 cycles or more	1
5 cycles	1.1
3 cycles	1.2
2 cycles	1.4
1 1/2 cycles	1.5

(a) To find SC currents in CB

Subtransient I_f in gen, $I_g'' = \frac{E_g''}{jX_d''}$

$$= \frac{1\angle 0^\circ}{j0.2} = 5\angle -90^\circ \ pu$$

Transient I_f in gen, $I_g^1 = \frac{Eg^1}{jX_d^1}$

$$= \frac{1\angle 0^\circ}{j0.25} = 4\angle -90^\circ \ pu$$

Steady state I_f in gen, $I_g = \frac{E_g}{jX_d}$

$$= \frac{1\angle 0^\circ}{j1.1} = 0.909\angle -90^\circ \ pu$$

The max momentry S.C current is obtained by multiplying $|I_g''|$ by 1.6

$\therefore$ Max mom S.C current = $1.6 * |I_g''| = 1.6 \times j\,5$

$= j\,8$ pu

(b) To find the initial sym rms current

I_g'' = Initial symmetric rms current

$\therefore$ Initial sym rms current = $5\angle -90^\circ$ (or) $-j5$

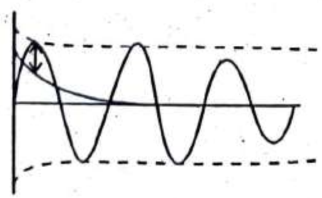
(c) To find the maximum possible dc component

Max possible DC component

= Different between maximun momentry and initial symmetric SC current

$= -j8 + j5$

$= -j3$ pu



(d) To find the interrupting current

Interrupting current = Transient $I_f \times$ M.F

(for 5 cycles CB, MF = 1.1)

$= j\,4 \times 1.1 = 4.4 \times I_b$

$= j\,4.4$ pu

(e) To find interrupting MVA

Interrupting MVA = pu value of * pu value * MVA_b pre fault voltage of I_f

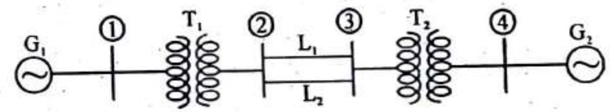
$= 1\angle 0^\circ * -j\,4.4 * 100$

$= -j\,440$ MVA

I_{nt} MVA = 440 $\angle -90^\circ$ MVA

Example: 3.28

A symmetrical fault occurs on bus 4 of system shown in figure. Determine the fault current, post fault voltager and line currents.

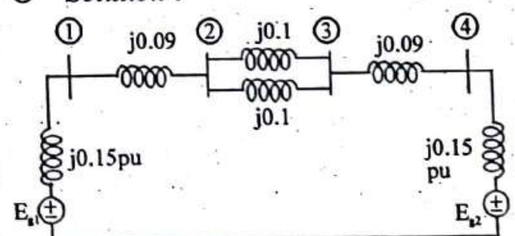


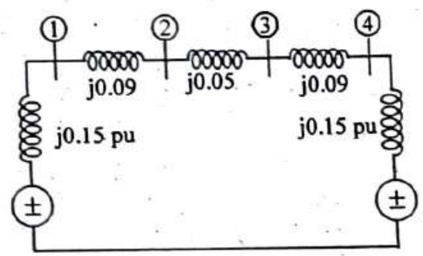
G_1, G_2 : 100 MVA, 20 KV, X = 15%

Transformer T_1 and T_2 : X = 9%

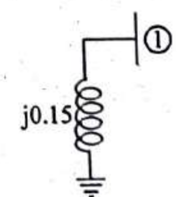
Line L_1 and L_2 : X = 10%

Solution :



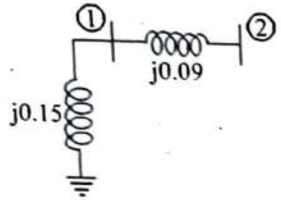


Step 1 :



$Z_{bus} = [j\,0.15]$

Step 2 :

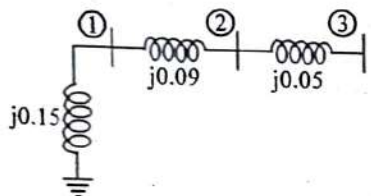


$$Z_{bus} = \begin{matrix} & 1 & 2 \\ 1 & j0.15 & j0.15 \\ 2 & j0.15 & j0.24 \end{matrix}$$

$$\Downarrow$$

$$= Z_{11} + Z_S$$
$$= j0.15 + j0.09$$
$$= j\,0.24$$

Step 3 :



$$Z_{bus} = \begin{bmatrix} j0.15 & j0.15 & j0.15 \\ j0.15 & j0.24 & j0.24 \\ j0.15 & j0.24 & j0.29 \end{bmatrix}$$

$$\Downarrow$$

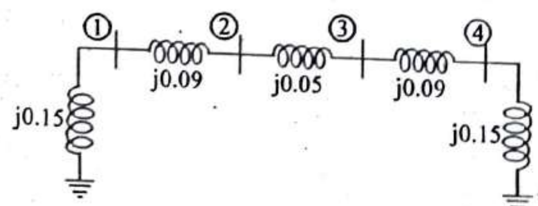
$$= Z_{22} + Z_s$$
$$= j\,0.24 + j\,0.05$$
$$= j0.29$$

Step 4 :

$$Z_{bus} = \begin{bmatrix} j0.15 & j0.15 & j0.15 & j0.15 \\ j0.15 & j0.24 & j0.24 & j0.24 \\ j0.15 & j0.24 & j0.29 & j0.29 \\ j0.15 & j0.24 & j0.29 & j0.38 \end{bmatrix}$$

$$\Downarrow$$

$$= j\,0.29 + j\,0.09$$
$$= j\,0.38$$



Step 5 :

$$Z_{bus} = \begin{bmatrix} j0.15 & j0.15 & j0.15 & j0.15 & j0.15 \\ j0.15 & j0.24 & j0.24 & j0.24 & j0.24 \\ j0.15 & j0.24 & j0.29 & j0.29 & j0.29 \\ j0.15 & j0.24 & j0.29 & j0.38 & j0.38 \\ j0.15 & j0.24 & j0.29 & j0.38 & j0.53 \end{bmatrix}$$

Eliminate node 5 using kron's reduction tech

$$\therefore Z_{bus,new} = \begin{bmatrix} j0.1075 & j0.172 & j0.068 & j0.0424 \\ j0.172 & j0.1313 & j0.108 & j0.068 \\ j0.068 & j0.108 & j0.1313 & j0.0821 \\ j0.0424 & j0.068 & j0.0821 & j0.1075 \end{bmatrix}$$

The fault current is

$$I_f = \frac{V_k(0)}{Z_{44} + Z_f} = \frac{1\angle 0^\circ}{j0.1075} = -j9.3023$$

$$= 9.3023\angle -90^\circ \text{ pu}$$

Post fault voltages at all the buses

$$V_1(f) = V_1(0) - Z_{14} I_4(f)$$
$$= 1\angle 0^\circ - j\,0.042 \times 9.3023\angle -90^\circ = 0.6056 \text{ pu}$$

$$V_2(f) = V_2(0) - Z_{24} I_4(f)$$
$$= 1\angle 0^\circ - j\,0.068 \times 9.3023\ \angle -90^\circ = 0.3686 \text{ pu}$$

$$V_3(f) = V_3(0) - Z_{34} I_4(f)$$
$$= 1\ \angle 0^\circ - j\,0.082 \times 9.3023\ \angle -90^\circ = 0.2374 \text{ pu}$$

$$V_4(f) = V_4(0) - Z_{44} I_4(f)$$
$$= 110^\circ - j\,0.1073 \times 9.3023\ \angle -90^\circ = 0 \text{ pu}$$

Post fault line currents

$$I_{ij}(f) = \frac{V_i(f) - V_j(f)}{Z_{bij}}$$

$$I_{12}(f) = \frac{V_1(f) - V_2(f)}{Z_{12(b)}}$$

$$= \frac{0.6056 - 0.3686}{j0.09} = 2.634\ pu$$

$$I_{23}(f) = \frac{V_2(f) - V_3(f)}{Z_{23(b)}}$$

$$= \frac{0.3686 - 0.2374}{j0.05} = 2.63\ pu$$

$$I_{34}(f) = \frac{V_3(f) - V_4(f)}{Z_{b34}}$$

$$= \frac{0.2374 - 0}{j0.09} = 2.637\ pu$$

Example: 3.29

Two synchronous motors are connected to the bus of a large system through a transmission line as shown in figure.

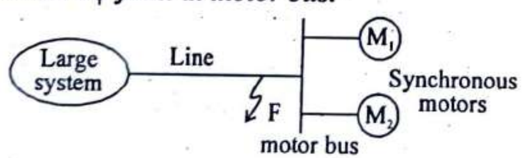
The ratings of the various components are :

Motor each : 1 MVA, 440 V, 0.1 pu transtent reactance

Line : 0.05 Ω reactance

Large system : SC MVA at its bus at 440 v is 8.

When the motors are operated at 400 v, calculate the S.C current (symmetrical fed into a 3 φ fault at motor bus.



Solution :

Choore a base values $MVA_b = 1$ MVA

$$KV_b = 440/1000 = 0.44\ KV$$

Tr line :

$$Z_b = \frac{KV_b^2}{MVA_b} = \frac{0.44^2}{1} = 0.1986\ pu$$

$$pu\ Im\ p = \frac{\text{Actual value}}{\text{Base value}} = \frac{0.05}{0.1936} = 0.2582\ pu$$

Motors :

✓ The motor is operated at 400 V. Hence the voltage at motor bus is 400 V

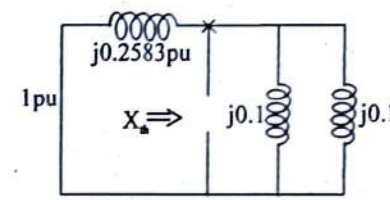
✓ Actual value of prefault voltage = 400 V

$$\text{pu value of prefault voltage} = \frac{Actual}{Base}$$

$$= \frac{400}{440} = 0.909\ pu$$

$$\text{pu value of voltage of infinite bus} = \frac{440}{440} = 1\ pu$$

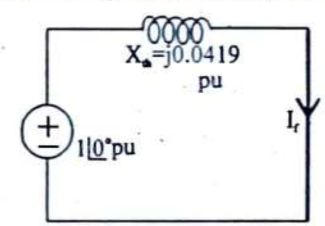
To find X^{th}



$$\frac{1}{X^{th}} = \frac{1}{j0.2583} + \frac{1}{j0.1} + \frac{1}{j0.1}$$

$$\boxed{X^{th} = j\ 0.0419\ pu}$$

The theverin's equation of the system



$$\text{The pu value of } I_f = \frac{V^{th}}{X^{th}}$$

$$= \frac{0.909}{j0.0419} = j21.6969\ pu$$

$$\boxed{I_{f,pu} = 21.6969\angle -90^\circ\ pu}$$

$$\text{Base current, } I_b = \frac{MVA_b}{\sqrt{3} \times KV_b} \times 1000$$

$$I_b = \frac{1 \times 1000}{\sqrt{3} \times 0.44} = 1312.16\ A$$

$\therefore$ Actual value of $I_f = I_f$, pu * I_b

$= 21.6969 * 1312.16$

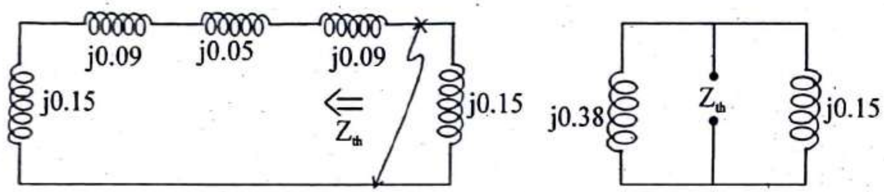
$$I_{f',act} = 28.47 \angle -90° \text{ KA}$$

Example: 3.30

Repeat problem no 28 by therenin's theorem method. (fault occurs at Bus 4)

☺ ***Solution :***

The reactance diagram is as



$$X^{th} = j\,0.38 \| j\,0.15$$

$$= j\,0.1075$$

(In problem No 3.28, $Z_{44} = j0.1075$)

The fault current

$$I_4(f) = \frac{V^{th}}{Z^{th} + Z_f}$$

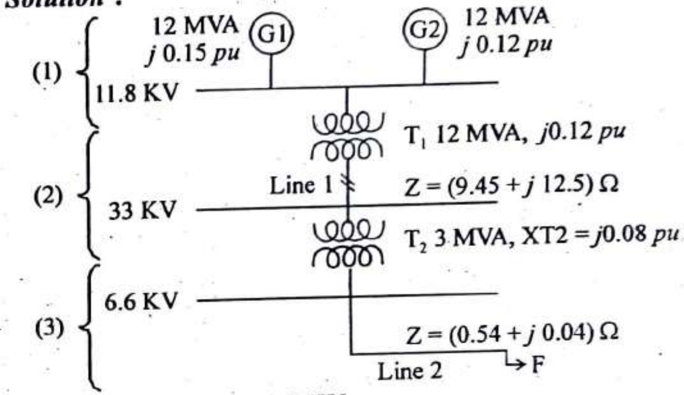
$$= \frac{1\angle 0°}{j0.1075+0} = -j9.298$$

$$I_4(f) = 9.298 \angle -90° \; pu$$

Example: 3.31

For the radial network shown in fig three phase fault occurs at point F. Determine the fault current and the line voltage at 11.8 KV bus under fault condition. ***(Nov, 2006)***

☺ ***Solution :***



Choose a base KV as 11.8 KV

Base MVA as 12 MVA

For generator 1

$$X_{pu\,new} = X_{pu,old} \times \left[\frac{KV_{b,old}}{KV_{b,new}}\right]^2 \times \left[\frac{MVA_{b,new}}{MVA_{b,old}}\right]$$

$$= j0.15 \times \left[\frac{11.8}{11.8}\right]^2 \times \left[\frac{12}{12}\right]$$

$$= j0.15\,pu$$

For generator 2

$$X_{pu\,new} = j0.12 \times \left[\frac{11.8}{11.8}\right]^2 \times \left[\frac{12}{12}\right]$$

$$= j0.12\,pu$$

For Transformer T_1

$$X_{pu\,new} = j0.12 \times \left[\frac{11.8}{11.8}\right]^2 \times \left[\frac{12}{12}\right]$$

$$= j0.12\,pu$$

For Line 1

$$X_{pu\,new} = \frac{X_{actual}}{X_{base}}$$

$$= \frac{9.45 + j12.6}{90.75}$$

$$= 0.1041 + j0.1388\,pu$$

$$X_b = \frac{KV_b^2}{MVA_b}$$

$$= \frac{33^2}{12}$$

$$= 90.75\,\Omega$$

For Transformer T_2

$$X_{pu\,new} = X_{pu,old} \times \left[\frac{KV_{b,old}}{KV_{b,new}}\right]^2 \times \left[\frac{MVA_{b,new}}{MVA_{b,old}}\right]$$

$$= j0.8 \times \left[\frac{6.6}{6.6}\right]^2 \times \left[\frac{12}{3}\right]$$

$$= j0.32\,pu$$

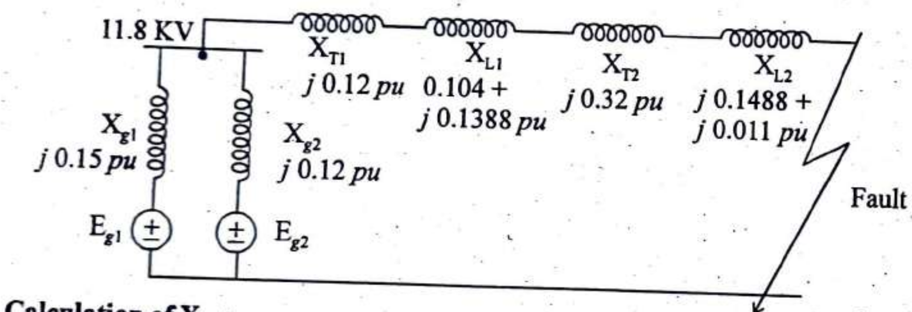
For Line 2

$$X_{pu\,new} = \frac{X_{actual}}{X_{base}} \qquad X_{base} = \frac{KV_b^2}{MVA_b} = \frac{6.6^2}{12} = 3.63\,\Omega$$

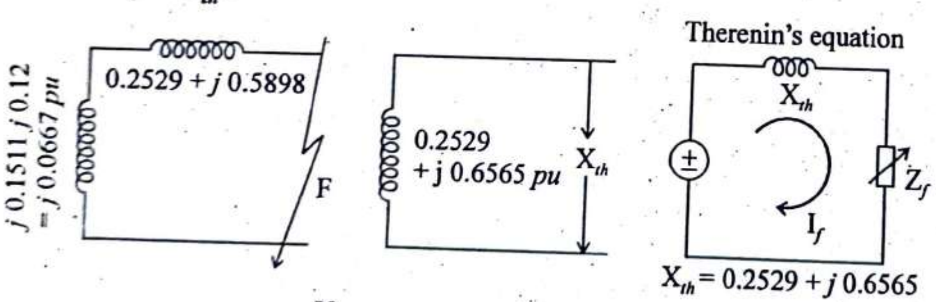
$$= \frac{0.54 + j0.04}{3.63}$$

$$= 0.1488 + j0.011\,pu$$

The reactance diagram is as follows :



Calculation of X_{th} :



@ fault side

$$\text{Fault current, } I_{f,pu} = \frac{V_{th}}{Z_{th} + Z_f} \qquad I_b = \frac{MVA_b}{\sqrt{3} \times KV_b}$$

$$= \frac{1\angle 0^\circ}{0.2529 + j0.6565}$$

$$= \frac{1\angle 0^\circ}{0.7035 + \angle 68.93^\circ} \qquad = \frac{12 \times 10^6}{\sqrt{3} \times 6.6 \times 10^3}$$

$$= 1.4215\angle -68.93\,pu \qquad = 1049.727\text{ Amps}$$

$$I_{f,actual} = |I_{f,pu}| * I_b$$

$$= 1.4215\angle -68.93 * 1049.727$$

$$= 1492.188\angle -68.93^\circ\text{ Amps}$$

Determine fault voltage @ 11.8 KV bus

$$\left.\begin{array}{r}\text{Total impedana from}\\ \text{11.8 KV to faulted point 'F'}\end{array}\right\} Z = j0.12 + 0.1041 + j0.1388 + j0.32 + 0.1488 + j0.011$$

$$= 0.2529 + j0.5898$$

$$= 0.6417\angle 66.79^\circ\,pu$$

$$V_{pu} = Z * I_{f,pu}$$

$$= 0.6417\angle 66.79^\circ * 1.4215\angle -68.93^\circ$$

$$= 0.9122\angle -2.14^\circ\,pu$$

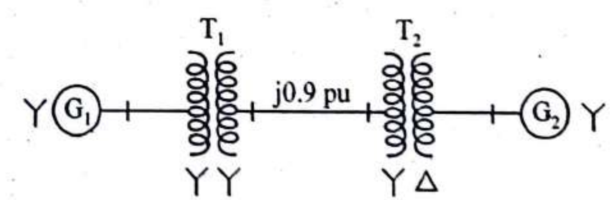
$$V_{actual} = V\,pu * \text{Base voltage}$$

$$= 0.9122 * 11.8$$

$$\boxed{V_{actual} = 10.76\text{ KV}}$$

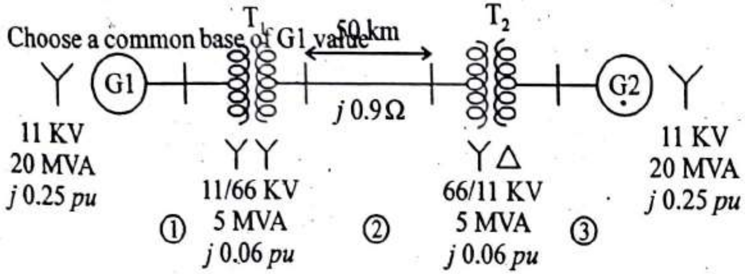
Example: 3.32

Generator G1 and G2 are identical and rated 11 KV, 20 MVA and have a transient reactance of 0.25 pu at own MVA base. The transformer T1 and T2 are also identical and are rated 11/66KV, 5MVA and have a reactance of 0.06 pu to their own MVA base. A 50km long transmission line is connected between the two generators. Calculate three phase fault current, when fault occurs at middle of the line as shown in figure. ***(Nov, 2015)***



☺ ***Solution :***

Choose a common base of G1 value



$$MVA_{b,\,new} = 20\ MVA$$

$$KV_{b,\,new} = 11\ KV$$

For Generator G1:

$$X_{pu\,new} = X_{pu,old} \times \left[\frac{KV_{b,old}}{KV_{b,new}}\right]^2 \times \left[\frac{MVA_{b,new}}{MVA_{b,old}}\right]$$

$$= j0.25 \times \left[\frac{11}{11}\right]^2 \times \left[\frac{20}{20}\right]$$

$$= j0.25\,pu$$

For Transformer T_1:

$$X_{pu\,new} = j0.6 \times \left[\frac{11}{11}\right]^2 \times \left[\frac{20}{5}\right]$$

$$= j0.24\,pu$$

$X_{pu,\,new}$ for Transformer line (for 25 km length)

$$X_{pu\,new} = \frac{X_{actual}}{X_{base}} \qquad X_b = \frac{KV_b^2}{MVA_b}$$

$$= \frac{j0.9 \times 25}{217.8} \qquad = \frac{66^2}{20}$$

$$= j0.1033\,pu \qquad = 217.8\ \Omega$$

For Remaining 25 km length

$$X_{pu\,new} = j0.1033\,pu$$

For Transformer T_2:

$$X_{pu\,new} = X_{pu,old} \times \left[\frac{KV_{b,old}}{KV_{b,new}}\right]^2 \times \left[\frac{MVA_{b,new}}{MVA_{b,old}}\right]$$

$$= j0.06 \times \left[\frac{66}{66}\right]^2 \times \left[\frac{20}{5}\right]$$

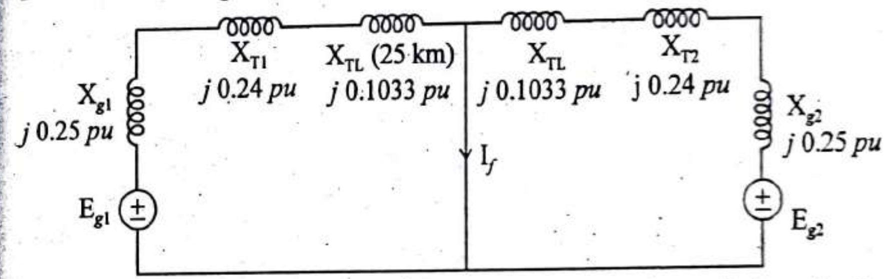
$$= j0.24\,pu$$

For Generator G2:

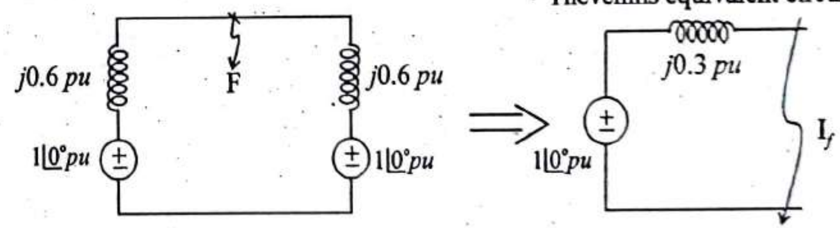
$$X_{pu\,new} = j0.25 \times \left[\frac{11}{11}\right]^2 \times \left[\frac{20}{20}\right]$$

$$= j0.25\,pu$$

The reactance diagram becomes



Thevenins equivalent circuit



$$\text{Fault current, } I_f = \frac{V_{th}}{X_{th}}$$

$$= \frac{1\angle 0^\circ}{j0.3} = -j3.3\,pu$$

$$\boxed{I_{f,pu} = 3.3\angle{-90^\circ}\ pu}$$

$$\text{Base current, } I_b = \frac{MVA_b \times 1000}{\sqrt{3} \times KV_b}$$

$$= \frac{20 \times 10^3}{\sqrt{3} \times 66}$$

$$= 175\ \text{Amps}$$

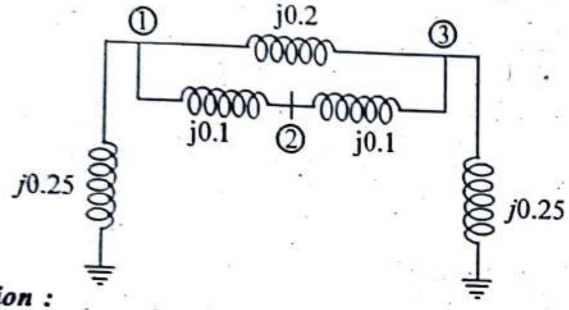
$\therefore$ Actual fault current, $I_{f,act} = |I_{f,pu}| * I_b$

$= 3.3 * 175$

$\boxed{I_{f,act} = 577.4 \text{ Amps}}$

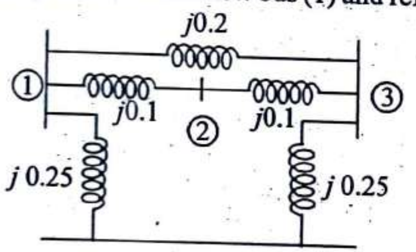
Example: 3.33

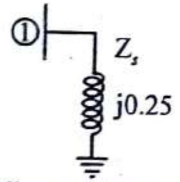
Form the bus impedance matrix for the network. Shown by building algorithms.



Solution :
(Redrawing the above network)

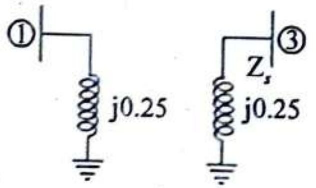
Step 1: Adding an element $j\,0.25$ between new bus (1) and reference. (Type – I)





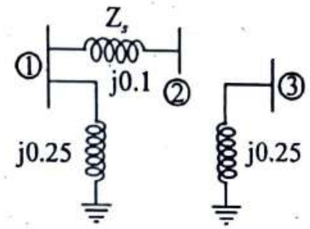
$$Z_{bus} = [\,j0.25]_{1\times1}$$

Step 2: Adding an element $j\,0.25$ between new bus (3) and reference. (Type – I)



$$Z_{bus} = \begin{bmatrix} j0.25 & 0 \\ 0 & j0.25 \end{bmatrix}_{2\times2}$$

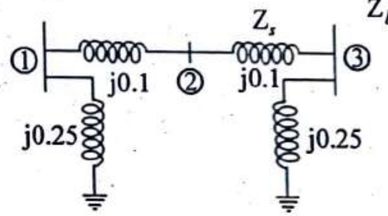
Step 3: Adding an new bus (2) to old bus (1) (Type – II)



$$Z_{bus} = \begin{bmatrix} j0.25 & 0 & j0.25 \\ 0 & j0.25 & 0 \\ j0.25 & 0 & j0.35 \end{bmatrix}$$

$$\begin{aligned} &= Z_{kk} + Z_s \\ &= Z_{11} + Z_s \\ &= j0.25 + j0.1 \\ &= j0.35 \end{aligned}$$

Step 4: Adding an element $j\,0.1$ between two old buses (2) and (3) (Type – IV)



$$Z_{bus} = \begin{bmatrix} j0.25 & 0 & j0.25 & -j0.25 \\ 0 & j0.25 & 0 & j0.25 \\ j0.25 & 0 & j0.35 & -j0.35 \\ -j0.25 & j0.25 & -j0.35 & j0.7 \end{bmatrix}$$

$$\begin{aligned} &= Z_{22} + Z_{33} - 2Z_{23} + Z_s \\ &= j0.25 + j0.35 - 2(0) + j0.1 \\ &= j0.7 \end{aligned}$$

Apply Kron's reduction technique, because no new node is added.

We know that

$$Z_{ij(new)} = Z_{ij(old)} - \frac{Z_{ip}Z_{pj}}{Z_{pp}} \qquad \boxed{p = 4}$$

$$Z_{11(new)} = Z_{11(old)} - \frac{Z_{14}Z_{41}}{Z_{44}} = j0.25 - \frac{(-j0.25)(-j0.25)}{j0.7} = j0.1607$$

$$Z_{12(new)} = Z_{12(old)} - \frac{Z_{14}Z_{42}}{Z_{44}} = j0 - \frac{(-j0.25)(j0.25)}{j0.7} = j0.0893$$

$$Z_{13(new)} = Z_{13(old)} - \frac{Z_{14}Z_{43}}{Z_{44}} = j0.25 - \frac{(-j0.25)(-j0.35)}{j0.7} = j0.125$$

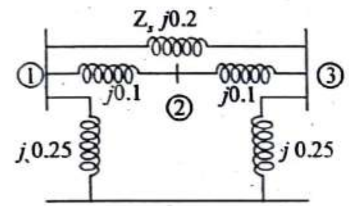
$$Z_{22(new)} = Z_{22(old)} - \frac{Z_{24}Z_{42}}{Z_{44}} = j0.25 - \frac{(-j0.25)(j0.25)}{j0.7} = j0.1607$$

$$Z_{23(new)} = Z_{23(old)} - \frac{Z_{24}Z_{43}}{Z_{44}} = 0 - \frac{(j0.25)(-j0.35)}{j0.7} = +j0.125$$

$$Z_{33(new)} = Z_{33(old)} - \frac{Z_{34}Z_{43}}{Z_{44}} = j0.35 - \frac{(-j0.35)(-j0.35)}{j0.7}$$
$$= j0.35 - j0.75 = j0.175$$

$$\therefore Z_{bus} = \begin{bmatrix} j0.1607 & j0.0893 & j0.125 \\ j0.0893 & j0.1607 & j0.125 \\ j0.125 & j0.125 & j0.175 \end{bmatrix}_{3\times3}$$

Step 5: Adding an element j 0.2 between two old buses (1) and (3)



$$Z_{bus} = \left[\begin{array}{ccc:c} j0.1607 & j0.0893 & j0.125 & -j0.036 \\ j0.0893 & j0.1607 & j0.125 & j0.036 \\ j0.125 & j0.125 & j0.175 & j0.05 \\ \hdashline -j0.036 & j0.036 & j0.05 & j0.2857 \end{array}\right]$$

$$\Downarrow$$
$$= Z_{11} + Z_{33} - 2Z_{13} + Z_s$$
$$= j0.1607 + j0.175 - 2(j0.125) + j0.2$$
$$= j0.2857$$

Apply Kron's reduction technique, because no new node is added.

Here $p = 4$

$$Z_{ij(new)} = Z_{ij(old)} - \frac{Z_{ip}Z_{pj}}{Z_{pp}}$$

$$Z_{11(new)} = Z_{11(old)} - \frac{Z_{14}Z_{41}}{Z_{44}} = j0.1607 - \frac{(-j0.036)(-j0.036)}{j0.2857} = j0.1562$$

$$Z_{12(new)} = Z_{12(old)} - \frac{Z_{14}Z_{42}}{Z_{44}} = j0.0893 - \frac{(-j0.036)(j0.036)}{j0.2857} = j0.0938$$

$$Z_{13(new)} = Z_{13(old)} - \frac{Z_{14}Z_{43}}{Z_{44}} = j0.125 - \frac{(-j0.036)(j0.05)}{j0.2857} = j0.1313$$

$$Z_{22(new)} = Z_{22(old)} - \frac{Z_{24}Z_{42}}{Z_{44}} = j0.1607 - \frac{(j0.036)(j0.036)}{j0.2857} = j0.1562$$

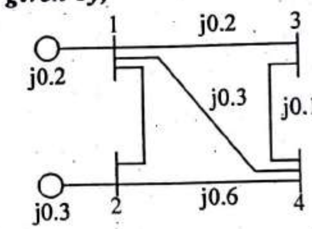
$$Z_{23(new)} = Z_{23(old)} - \frac{Z_{24}Z_{43}}{Z_{44}} = j0.125 - \frac{(j0.036)(j0.05)}{j0.2857} = j0.1187$$

$$Z_{33(new)} = Z_{33(old)} - \frac{Z_{34}Z_{43}}{Z_{44}} = j0.175 - \frac{(j0.125)(j0.05)}{j0.2857} = j0.1531$$

$$\therefore Z_{bus} = \begin{bmatrix} j0.1562 & j0.0938 & j0.1313 \\ j0.0938 & j0.1562 & j0.1187 \\ j0.1313 & j0.1187 & j0.1531 \end{bmatrix}$$

Example: 3.34

The per unit bus impedance matrix for the power system in figure below, is given by,



$$Z_{bus} = j\begin{bmatrix} 0.150 & 0.075 & 0.140 & 0.135 \\ 0.075 & 0.1875 & 0.090 & 0.0975 \\ 0.140 & 0.090 & 0.2533 & 0.210 \\ 0.135 & 0.0975 & 0.210 & 0.2475 \end{bmatrix}$$

A three-phase fault occurs at bus 4 through a fault impedance of $Z_f = j$ 0.0025 pu using bus impedance matrix. Calculate the fault current, bus voltages and line currents during the fault.

☺ ***Solution :***

For a fault at bus 4 with fault impedance Z_f = j 0.0225 *pu*, the current is

$$I_4(F) = \frac{V_4(0)}{Z_{44} + Z_f} = \frac{1.0}{j0.2475 + j0.0025} = -j4 \ pu$$

The bus voltages during the fault are,

$$V_1(F) = V_1(0) - Z_{14}I_4(F) = 1.0 - (j0.135)(-j4) = 0.46 \ pu$$
$$V_2(F) = V_2(0) - Z_{24}I_4(F) = 1.0 - (j0.0975)(-j4) = 0.61 \ pu$$
$$V_3(F) = V_3(0) - Z_{34}I_4(F) = 1.0 - (j0.210)(-j4) = 0.16 \ pu$$
$$V_4(F) = V_4(0) - Z_{44}I_4(F) = 1.0 - (j0.2475)(-j4) = 0.01 \ pu$$

The short circuit current's in the lines are,

$$I_{13}(F) = \frac{V_1(F) - V_3(F)}{Z_{13}} = \frac{0.46 - 0.16}{j0.2} = -j1.5 \ pu$$

$$I_{14}(F) = \frac{V_1(F) - V_4(F)}{Z_{14}} = \frac{0.46 - 0.01}{j0.3} = -j1.5\ pu$$

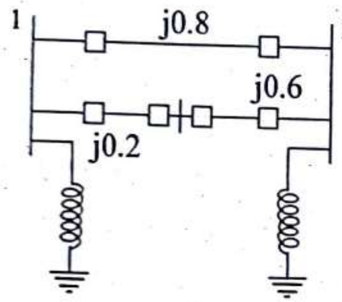
$$I_{21}(F) = \frac{V_2(F) - V_1(F)}{Z_{21}} = \frac{0.61 - 0.46}{j0.5} = -j0.3\ pu$$

$$I_{24}(F) = \frac{V_2(F) - V_4(F)}{Z_{24}} = \frac{0.61 - 0.01}{j0.6} = -j1.0\ pu$$

$$I_{34}(F) = \frac{V_3(F) - V_4(F)}{Z_{34}} = \frac{0.61 - 0.01}{j0.10} = -j1.5\ pu$$

Example: 3.35

The bus impedance matrix for the network shown below, is given by



$$Z_{bus} = j\begin{bmatrix} 0.300 & 0.200 & 0.275 \\ 0.200 & 0.400 & 0.250 \\ 0.275 & 0.250 & 0.41875 \end{bmatrix}$$

There is line outage and line from bus 1 to 2 is removed. Using the method of building algorithm determine the new bus impedance matrix.

Solution :

The removal of the line 1 – 2 is equivalent to connecting a link having an impedance equal to the negated value of the original impedance. Therefore, we add link $z_{12} = -j0.8$ between node $q = 2$ and node $p = 1$.

Add link between node $p = 1$ to node $q = 2$, $Z_{12} = -j0.8$

$$Z_{bus} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{12}-Z_{11} \\ Z_{21} & Z_{22} & Z_{23} & Z_{22}-Z_{21} \\ Z_{31} & Z_{32} & Z_{33} & Z_{32}-Z_{31} \\ Z_{21}-Z_{11} & Z_{22}-Z_{12} & Z_{23}-Z_{13} & Z_{44} \end{bmatrix}$$

$$Z_{44} = Z_{pp} + Z_{qq} + Z_{pq} - 2Z_{pq}$$

$$Z_{44} = Z_{11} + Z_{22} + Z_{12} - 2Z_{12}$$

$$= j0.3 + j0.4 - j0.8 - 2(j0.2)$$

$$Z_{44} = -j0.5$$

$$Z_{bus}^{(1)} = \begin{bmatrix} j0.300 & j0.200 & j0.275 & -j0.100 \\ j0.200 & j0.400 & j0.25 & j0.200 \\ j0.275 & j0.250 & j0.41875 & -j0.025 \\ -j0.100 & j0.200 & j0.02500 & -j0.5 \end{bmatrix}$$

$$\frac{\Delta Z \Delta Z^T}{Z_4} = \frac{1}{-j0.5}\begin{bmatrix} -j0.100 \\ j0.200 \\ -j0.025 \end{bmatrix}[-j0.10 \quad j0.20 \quad -j0.025]$$

$$= \begin{bmatrix} -j0.020 & j0.040 & -j0.0050 \\ j0.040 & -j0.080 & j0.0100 \\ -j0.005 & j0.010 & -j0.0013 \end{bmatrix}$$

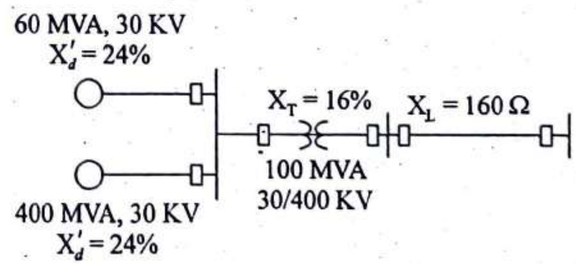
New bus impedance matrix

$$Z_{bus} = \begin{bmatrix} 0.300 & 0.200 & 0.275 \\ 0.200 & 0.400 & 0.250 \\ 0.275 & 0.250 & 0.41875 \end{bmatrix} - \begin{bmatrix} -j0.020 & j0.040 & -j0.0050 \\ j0.040 & -j0.080 & j0.010 \\ -j0.005 & j0.010 & -j0.0013 \end{bmatrix}$$

$$Z_{bus} = \begin{bmatrix} j0.320 & j0.160 & j0.280 \\ j0.160 & j0.480 & j0.240 \\ j0.280 & j0.240 & j0.420 \end{bmatrix}$$

Example: 3.36

The system shown in figure below is initially on no load with generators operating at their rated voltage with their emfs in phase. The rating of the generators and the transformers and their respective percent reachances are marked on the diagram. All resistances are neglected. The line impedance is j 160 Ω. A three phase balanced fault occurs at the receiving end of the transmission line. determine the short-circuit current and short circuit MVA.



☺ **Solution :**

$$\text{The base impedance for line is } (Z_B) = \frac{(400)^2}{100} = 1,600\ \Omega$$

$$\text{Base current } (I_B) = \frac{100,000}{\sqrt{3} \times 400} = 144.3375\,\text{A}$$

The *pu* reactances on a common 100 MVA base are,

$$X'_{d(G1)} = \frac{100}{60}(0.24) = 0.4\ pu$$

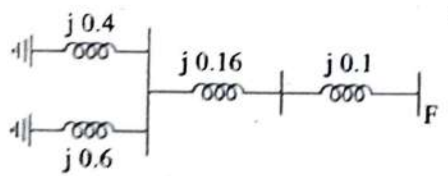
$$X'_{d(G2)} = \frac{100}{40}(0.24) = 0.6\ pu$$

$$X_t = \frac{100}{100}(0.16) = 0.16\ pu$$

$$X_{Line} = \frac{160}{1600} = 0.1\ pu$$

The reactance diagram is given below,

Impedance to the point of fault is,



$$X = j\frac{(0.4)(0.6)}{0.4 + 0.6} + j0.16 + j0.1 = j0.5\,pu$$

$$\text{Fault current } I_f = \frac{1}{j0.5} = 2\angle{-90^o}\ pu = (144.3375)\left(2\angle{-90^o}\right)$$

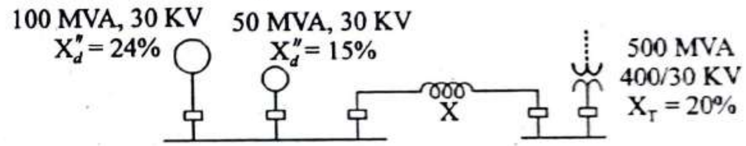
$$= 288.675\angle{-90^o}\ \text{A}$$

$$\text{SCMVA} = \sqrt{3}\,V_{LL}\ I(F) \times 10^{-3}\ \text{MVA}$$

$$= \sqrt{3}\ (400)\ (288.675)\ (10^{-3}) = 200\,\text{MVA}$$

Example: 3.37

The system shown in figure below shows on existing plant consisting of a generator of 100 MVA, 30 KV with 20% sub transient reactance and a generator of 50 MVA, 30KV with 15% sub transient reactance, connected in parallel to a 30 KV bus bar. The 30 KV bus bar feeds a transmission line via the circuit breaker C which is rated at 1250 MVA. A grid supply is connected to a station bus bar through a 500 MVA, 400/30 KV transformer with 20% reactance. Determine the reactance of a current limiting reactor in ohm to be connected between the grid system and existing bus bar such that the SCMVA of the breaker C does not exceed.



☺ **Solution :**

$$\text{The base impedance for line is } (Z_B) = \frac{(30)^2}{100} = 9\Omega$$

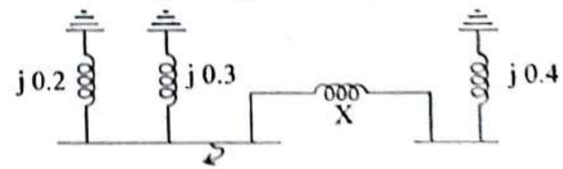
The *pu* reactances on a common 100 MVA base are,

$$X''_{d(G1)} = \frac{100}{100}(0.2) = 0.2\ pu$$

$$X''_{d(G2)} = \frac{100}{50}(0.15) = 0.3\ pu$$

$$X_t = \frac{100}{100}(0.2) = 0.04\ pu$$

The reactance diagram is given below,



Reactance to the point of fault is,

$$X_f = \frac{S_B}{\text{SCMVA}} = \frac{100}{1250} = 0.08\ pu$$

Parallel reactance of the generator is

$$X = i\frac{(0.2)(0.3)}{0.2 + 0.3} = j0.12\ pu$$

Reactance to the point of fault is,

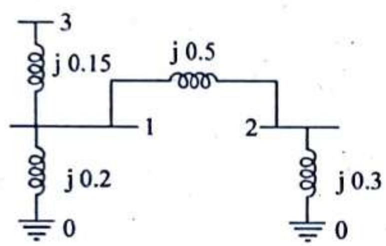
$$\frac{(0.12j)(X + 0.04j)}{0.12j + (X + 0.04)} = 0.08$$

Solving for X, we get X = 0.2 *pu*

$$X_{\Omega} = (X) \times Z_B = (0.2)(9) = 1.8\ \Omega$$

Example: 3.3 8

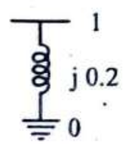
Using building algorithm find the bus impedance matrix for the network shown in fig below.



Solution :

Step 1:

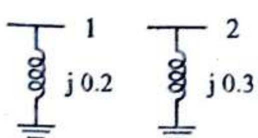
Adding an element $Z_{10} = j\,0.2$ between reference node (0) and new node $q = 1$



$$Z_{bus} = [j0.2]$$

Step 2:

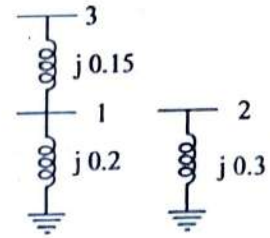
Adding an element $Z_{20} = j\,0.3$ between reference node (0) and new node $q = 2$ (Type 1)



$$Z_{bus} = \begin{bmatrix} j0.2 & 0 \\ 0 & j0.3 \end{bmatrix}$$

Step 3:

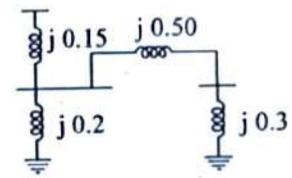
Adding an element $Z_{13} = j\,0.15$ between the new node $q = 3$ and the old node $p = 1$ (Type 2)



$$Z_{bus} = \begin{bmatrix} j0.2 & 0 & Z_{11} \\ 0 & j0.3 & Z_{21} \\ Z_{11} & Z_{12} & Z_{11}+Z_{13} \end{bmatrix} = \begin{bmatrix} j0.2 & 0 & j0.2 \\ 0 & j0.3 & 0 \\ j0.2 & 0 & j0.35 \end{bmatrix}$$

Step 4:

Adding an element $Z_{12} = j\,0.5$ between the new node $q = 2$ and the old node $p = 1$ (Type 4)



$$Z_{44} = Z_{11} + Z_{22} + Z_{12} - 2Z_{12}$$
$$= j0.2 + j0.3 + j0.5 - 2(0) = j1.0$$

By kron's reduction

$$Z_{ij(new)} = Z_{ij(new)} - \frac{Z_{ip}Z_{pj}}{Z_{pp}}$$

$$Z_{11(new)} = j0.2 - \frac{(-j0.2)(-j0.2)}{j1.0} = j0.16$$

$$Z_{12(new)} = 0 - \frac{(-j0.2)(j0.3)}{j1.0} = j0.06$$

$$Z_{13(new)} = j0.2 - \frac{(-j0.2)(-j0.2)}{j1.0} = j0.16$$

$$Z_{21(new)} = 0 - \frac{(j0.3)(-j0.2)}{j1.0} = j0.06$$

$$Z_{22(new)} = j0.3 - \frac{(j0.3)(j0.3)}{j1.0} = j0.21$$

$$Z_{23(new)} = 0 - \frac{(j0.3)(-j0.2)}{j1.0} = j0.06$$

$$Z_{31(new)} = j0.2 - \frac{(-j0.2)(-j0.2)}{j1.0} = j0.16$$

$$Z_{32(new)} = 0 - \frac{(-j0.2)(+j0.3)}{j1.0} = j0.06$$

$$Z_{33(new)} = j0.35 - \frac{(-j0.2)(-j0.2)}{j1.0} = j0.31$$

$$Z_{bus} = \begin{bmatrix} j0.16 & j0.06 & j0.16 \\ j0.06 & j0.21 & j0.06 \\ j0.16 & j0.06 & j0.31 \end{bmatrix}$$

Example: 3.39

A 3ϕ, 50 Hz generator is rated at 500 MVA, 20 KV with $X_d'' = 0.2$ pu. It supplies pure resistive load of 400 MW at 20 KV. The load is connected directly across the terminals of the generator. If all the three phases of the load are short circuit simultaneously, find the initial symmetrical rms current in the generator in per unit on a base of 500 MVA, 20 KV.

☺ ***Solution :***

Let the Base KVA = 50 MVA

Base KV = 20 KV

Load = 400 MW @ unity prefault (load is pure resistive)

$$= \frac{400}{500} = 0.8\angle 0^\circ \ pu$$

Terminal voltage, V_o = 20 KV

$$V_o = \frac{20}{20} = 1 pu$$

Prefault current, $I_o = \frac{Load}{V_o} = \frac{0.8\angle 0^\circ}{1} = 0.8\angle 0^\circ \ pu$

Voltage behind sub transient reactance for the generator

$$E_g'' = V_o + jI_o \times d''$$

$$= (1 + j0) + j0.8\angle 0^\circ \times 0.2$$

$$= 1 + j0.16 \ (or) \ 1.01272 \angle 9.09^\circ \ pu$$

Initial symmetrical rms current in the generator on a base of 500 MVA, 20 KV

$$I'' = \frac{E_g''}{X_d''} = \frac{1.01272\angle 9.09^\circ}{0.2\angle 90^\circ}$$

$$\boxed{I'' = 0.50636 \angle -80.9^\circ \ pu}$$

Example: 3.40

A synchronous generator rated 500 KVA, 400 V, 0.1 pu sub transient reactance is supplying a passive load of 400 KW @ 0.8 lagging pf. Calculate the initial symmetrical rms current for a 3ϕ fault at generator terminals.

☺ ***Solution :***

Let KVA_b = 500 KVA

Base voltage = 400 V

Load = 400 KW @ 0.8 lagging *pf*

$$= \frac{400/0.8}{500} = 1 pu, \quad 0.8 \text{ lagging } pf$$

Prefault current, $I_o = \frac{1}{1.0}\angle \cos^{-1} pf$

$$= \frac{1}{1.0}\angle \cos^{-1} 0.8$$

$$= 1.0 \angle -36.87^\circ \ pu$$

Prefault voltage, V_o = 400 V (or) $1\angle 0^\circ$ *pu*

Voltage behind sub transient reactance for the generator

$$E_g'' = V_o + jI_o \times d''$$

$$= (1 + j0) + j(0.1) \times (0.8 - j0.6)$$

$$= (1.06 + j0.08) \ pu$$

Initial fault current, $I'' = \frac{E_g''}{X_d''} = \frac{1.06 + j0.08}{j0.1}$

$$= (0.8 - j10.6) \ pu$$

$$= 10.63 \angle -85.68^\circ \ pu$$

Base current, $I_B = \frac{500 \times 1000}{\sqrt{3} \times 400} = 721.7$ Amps

Initial symmetrical fault current

$$= 721.7 * 10.63\angle -85.68^\circ$$

$$= 7672 \angle -85.68^\circ \text{ Amps}$$

Example: 3.41

The pu bus impedance matrix for the power system is given below

$$Z_{bus} = j\begin{bmatrix} 0.240 & 0.140 & 0.200 & 0.200 \\ 0.140 & 0.2275 & 0.175 & 0.175 \\ 0.200 & 0.175 & 0.310 & 0.310 \\ 0.200 & 0.175 & 0.310 & 0.500 \end{bmatrix}$$

(a) A bolted three-phase fault occurs at bus 4. Using the bus impedance matrix calculate the fault current, bus voltages and line currents during fault.

(b) Repeat (a) for a 3ϕ fault at bus 2 with a fault impedance of $Z_f = j\,0.0225$.

Solution :

For a solid fault at bus 4 the fault current is $I_4(F) = \dfrac{V_4(0)}{Z_{44}} = \dfrac{1.0}{j0.5} = -j2\,pu$

The bus voltages during the fault are

$$V_1(F) = V_1(0) - Z_{14} I_4(F) = 1.0 - (j2.000)(-j2) = 0.6\ pu$$

$$V_2(F) = V_2(0) - Z_{24} I_4(F) = 1.0 - (j0.175)(-j2) = 0.65\ pu$$

$$V_3(F) = V_3(0) - Z_{34} I_4(F) = 1.0 - (j0.310)(-j2) = 0.38\ pu$$

$$V_4(F) = V_4(0) - Z_{44} I_4(F) = 1.0 - (j0.500)(-j2) = 0\ pu$$

The short circuit currents in the lines are

$$I_{21}(F) = \frac{V_2(F) - V_1(F)}{Z_{12}} = \frac{0.65 - 0.6}{j0.5} = -j0.1\ pu$$

$$I_{13}(F) = \frac{V_1(F) - V_3(F)}{Z_{13}} = \frac{0.60 - 0.38}{j0.2} = -j1.1\ pu$$

$$I_{23}(F) = \frac{V_2(F) - V_3(F)}{Z_{23}} = \frac{0.65 - 0.38}{j0.3} = -j0.9\ pu$$

$$I_{34}(F) = \frac{V_3(F) - V_4(F)}{Z_{34}} = \frac{0.38 - 0}{j0.19} = -j2\ pu$$

(b) For a fault at bus 2 with $Z_f = j\,0.0225\,pu$. The fault current is

$$I_2(F) = \frac{V_2(F)}{V_{22} + Z_f} = \frac{1\angle 0^\circ}{j0.2275 + j0.0225} = -j4\ pu$$

The bus voltages during the fault are

$$V_1(F) = V_1(0) - Z_{12} I_2(F) = 1.0 - (j0.140)(-j4) = 0.44\ pu$$

$$V_2(F) = V_2(0) - Z_{22} I_2(F) = 1.0 - (j0.2275)(-j4) = 0.09\ pu$$

$$V_3(F) = V_3(0) - Z_{32} I_2(F) = 1.0 - (j0.175)(-j4) = 0.30\ pu$$

$$V_4(F) = V_4(0) - Z_{42} I_2(F) = 1.0 - (j0.175)(-j4) = 0.30\ pu$$

The short circuit currents in the lines are

$$I_{12}(F) = \frac{V_1(F) - V_2(F)}{Z_{12}} = \frac{0.44 - 0.09}{j0.5} = -j0.7\ pu$$

$$I_{13}(F) = \frac{V_1(F) - V_3(F)}{Z_{13}} = \frac{0.44 - 0.30}{j0.2} = -j0.7\ pu$$

$$I_{32}(F) = \frac{V_3(F) - V_2(F)}{Z_{32}} = \frac{0.30 - 0.09}{j0.3} = -j0.7\ pu$$

$$I_{34}(F) = \frac{V_3(F) - V_4(F)}{Z_{34}} = \frac{0.30 - 0.30}{j0.19} = 0\ pu$$

REVIEW QUESTIONS

PART – B

1. Two synchronous motors are connected to the bus of a large system through a short transmission line as shown. The ratings of the various components are: Motor each: 1MVA, 440V, 0.1p.u reactance. Line:0.05 Ω reactance. Large system S.C MVA at 440V bus is 8.0. When two motors are in operation at 440V, calculate the S.C current (symmetrical) fed into a 3 phase fault at the motors.

2. A small generating station has a bus bar divided into three sections. Each section is connected to a tie-bar with reactors each rated at 5MVA, 0.1p.u reactance. A generator of 8 MVA rating and 0.15 p.u reactance is connected to each section of the bus bar. Determine the S.C capacity of the breaker if a 3 phase fault takes place on one of the sections of the bus bar.

3. An alternator and a synchronous motor each rated for 50 MVA, 13.2 KV having sub transient of 20% are connected through a transmission link of reactance 10% on the base of machine ratings. The motor acts as a load of 30 MW at 0.8 p.f lead and terminal voltage 12.5 KV when a 3 phase fault takes place at the motor terminals. Determine the sub transient current in the alternator, the motor and the fault.

4. The per unit impedance matrix of a four bus power system shown in figure below,

$$Z_{Bus} = \begin{bmatrix} j0.15 & j0.075 & j0.14 & j0.135 \\ j0.075 & j0.1875 & j0.09 & j0.0975 \\ j0.14 & j0.09 & j0.2533 & j0.21 \\ j0.135 & j0.0975 & j0.21 & j0.2475 \end{bmatrix}$$

Calculate the fault current for a solid three symmetrical fault at bus 4. Also calculate the post fault bus voltages and line currents.

5. Explain symmetrical fault analysis using Z-bus matrix with neat flow chart. (May 2011)(Nov 2012)(May 2013)

6. A 3 ϕ 5MVA 6.6 KV alternator with a reactance of 8% is connected to a feeder of series impedance 0.12 + j0.48 Ω/Km. The transformer is rated at 3 MVA 6.6kV/33 KV and has a reactance of 5%. Determine the fault current supplied by the generator operating under no load with a voltage of 6.9 KV, when a 3 ϕ symmetrical fault occurs at a point 15 Km along the feeder. (May 2012, Nov, 2016).

7. The bus impedance matrix of 4-bus system with values in p.u is given by,

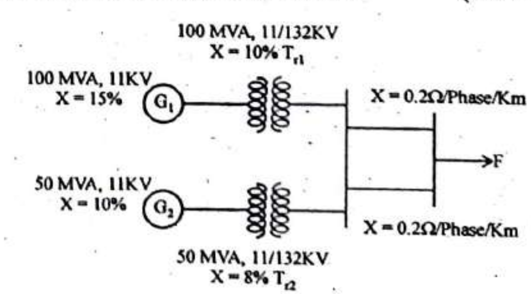
$$Z_{Bus} = j\begin{bmatrix} 0.15 & 0.08 & 0.04 & 0.07 \\ 0.08 & 0.15 & 0.06 & 0.09 \\ 0.04 & 0.06 & 0.13 & 0.05 \\ 0.07 & 0.09 & 0.05 & 0.12 \end{bmatrix}$$

In this system generator are connected to buses 1 and 2 and their sub transient reactances included when finding Z_{Bus}. If pre-fault current is neglected, find sub transient current in p.u in the fault for a 3-ph fault voltage as 1.p.u. If the sub transient reactance of generator in Bus 2 is 0.2p.u., find the sub transient fault current supplied by generator. (May 2012)

8. A synchronous generator and motor are rated 30 MVA, 13.2 KV and both have sub transient reactance of 20%. The line connecting them has reactance of 10% on the base of machine ratings. The motor is drawing 20,000 KW at 0.8 p.f leading and terminal voltage of 12.8 KV when a symmetrical 3 phase fault occurs at the motor terminals. Find the sub transient current in the generator, motor and fault by using internal voltages of machines. (May 2013)(Nov 2015)

9. A 11 KV, 100 MVA alternators having a sub-transient reactance of 0.25 p.u is supplying a 50 MVA motor having a sub-transient reactance of 0.2 p.u through a transmission line. The line reactance is 0.05 pu on a base of 100 MVA. Motor is drawing 40 MW at 0.8 power factor leading with a terminal voltage of 10.95 KV when a 3-phase fault occurs at the generator terminals. Calculate the total current in the generator and motor under fault conditions. (Nov 2013)(May 2011)

10. The figure shows a generating station feeding a 132 KV system. Determine the total fault current, fault level and fault current supplied by each alternator for a 3 ϕ fault at the receiving end bus. The line is 200 Km long. Take a base of 100 MVA, 11 KV for LV side and 132 KV for HT side. (Nov 2013) (May 2016)



11. A generator is connected through afive cycle circuit breaker to a transformer is rated 100 MVA,18 KV with reactances $X_d'' = 20\%$, $X_d' = 25\%$ and $X_d = 110\%$. It is operated on no-load and at rated voltage. When a 3-phase fault occurs between the breaker and the transformer, find,

   (i) Short circuit current in circuit breaker;

   (ii) The initial symmetrical rms current in the circuit breaker

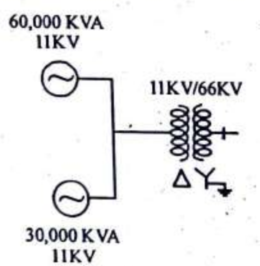
   (iii) The maximum possible dc component of the short circuit current in the breaker;

   (iv) The current to be interrupted by the breaker;

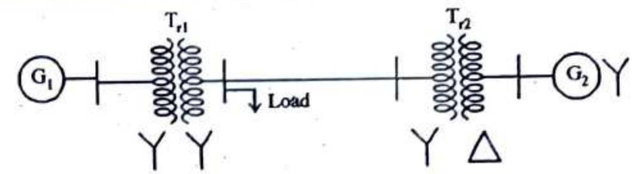
   (v) The interrupting MVA (May 2014)

12. With the help of a detailed algorithm, Explain how a symmetrical fault can be analysed using Z_{Bus}. (May 2014)

13. Two generators are connected in parallel to the low voltage side of a 3 ϕ delta star transformer as shown in figure. Generator 1 is rated 60,000KVA, 11KV. Generator 2 is rated 30,000 KVA, 11Kv. Each generator has a sub transient reactance of $X_d'' = 25\%$. The transformer is rated 90,000 KVA at 11kv Δ/66kv- Y with a reactance of 10%. Before a fault occurred, the voltage is unloaded and there is no circulating current between the generators. Find the sub transient current in each generator when a three phase fault occurs on the hv side of the transformer. (May 2015)

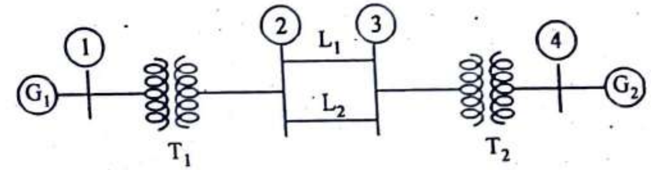


14. Generator G_1 and G_2 are identical and rated 11Kv, 20 MVA and have a transient reactance of 0.25pu at own MVA base. The transformer T_1 and T_2 are also identical and are rated 11/66KV, 5MVA and have a reactance of 0.06pu tp their own MVA base. A 50km long transmission line is connected between the two generators. Calculate three phase fault current, when fault occurs at middle of the line as shown in figure. (Nov 2015)



15. A Symmetrical fault occurs at bus 4 for the system shown in Fig .Determine the fault current using Z_{bus} Building algorithm. (May 2016)

G_1, G_2: 100 MVA, 20 KV, $X^+ = 15\%$; Transformer: $X_{leakage} = 9\%$; L1,L2: $X^+ = 10\%$



16. For the radial network shown in fig three phase fault occurs at point F. Determine the fault current and the line voltage at 11.8 KV bus under fault condition. (Nov, 2016)