Unit-4 Unsymmetrical fault Analysis

When network is unsymmetrically faulted or loaded, (or) any fault which cause unbalanced current in all the phases unbalanced/unsymmetrical fault.

This type of fault happen due to short circuit in any one (or) two phases to the ground (or) b/w the phases.

Types of unsymmetrical fault

a) Line to ground (L-G)
b) Line to line (L-L)
c) Double line to ground (L-L-G)

Causes of unsymmetrical fault

- lightning
- wind damage
- birds shorting the line
- trees falling across lines
- excessive ice (or) snow loading.

Some other causes are,

- Breaking of one or two conductors
- action of fuses and other protective devices that may not open.

$$\boxed{I_a + I_b + I_c = I_n}$$

* Cannot be solved by per phase analysis.

* Unbalanced/ unsymmetrical component $\xrightarrow{\text{decompose}}$ Balanced Component

Symmetrical components:

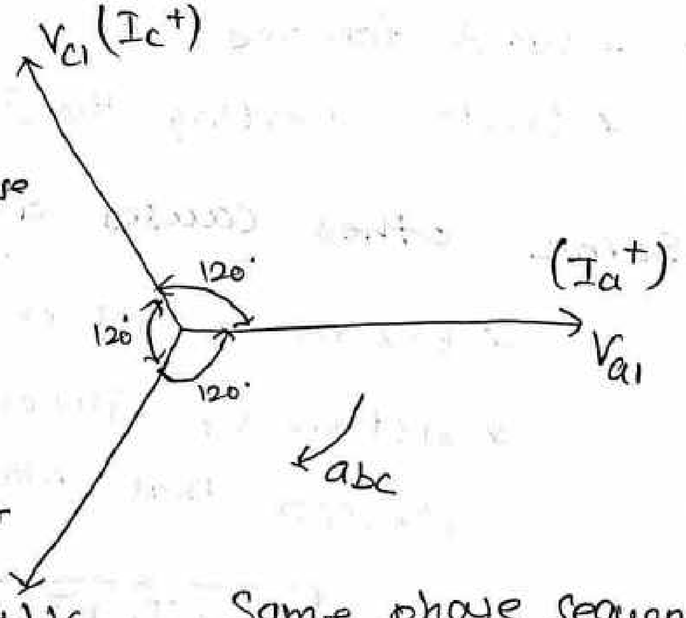
Defined as 'n' set of balanced vector which are resolved into set of 'n' balanced vector by using symmetrical component transformation.

- Positive sequence components
- Negative sequence components
- zero sequence components

Positive sequence components

"a" → an operator used to rotate vector in an anticlockwise direction. $a = 1\angle 120^\circ$

* equal in magnitude but are displaced from each other by 120° in phase.



Same phase sequence as original phasor's.

$V_{a1} = V_{a1} \angle 0^\circ$ $\qquad I_a^+ = I_a^+ \angle 0^\circ = I_a^+$

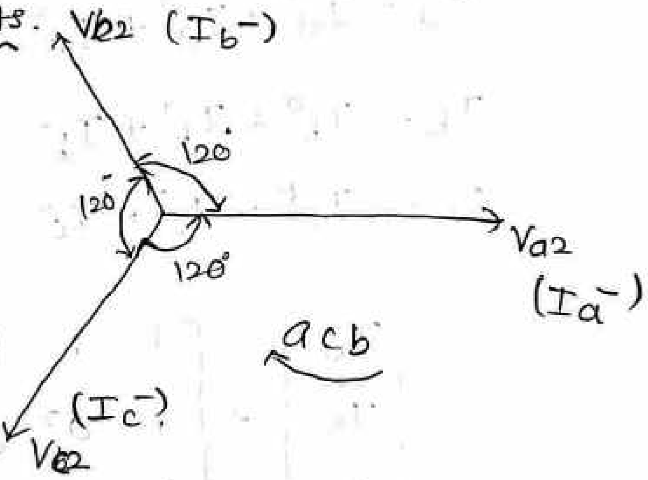
$V_{b1} = V_{a1} \angle 240^\circ$ $\qquad I_b^+ = I_a^+ \angle 240^\circ = a^2 I_a^+$

$= a^2 V_{a1}$

$V_{c1} = V_{a1} \angle 120^\circ$ $\qquad I_c^+ = I_a^+ \angle 120^\circ = a I_a^+$

$= a V_{a1}$

Negative sequence components.



$V_{a2} = V_{a2} \angle 0^\circ = V_{a2}$

$V_{b2} = V_{a2} \angle 120^\circ = a V_{a2}$

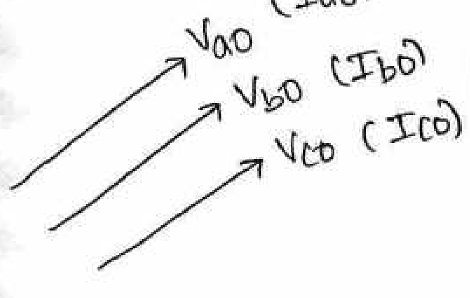
$V_{c2} = V_{a2} \angle 240^\circ = a^2 V_{a2}$

$I_a^- = I_a^- \angle 0^\circ = I_a^-$

$I_b^- = I_a^- \angle 120^\circ = a I_a^-$

$I_c^- = I_a^- \angle 240^\circ = a^2 I_a^-$

Zero sequence components



* Equal in magnitude

* does not have any phase shift.

$V_{a0} = V_{b0} = V_{c0}$

$I_a^0 = I_b^0 = I_c^0$

Identity

$a+a^2=-1$ $\qquad a = 1\angle 120^\circ$

$$a^2 = 1\angle 240 = 1\angle -120^\circ$$

$$a^3 = 1\angle 360^\circ = 1\angle 0^\circ$$

$$a = -0.5+j\,0.866$$

$$a^2 = -0.5-j\,0.866$$

Symmetrical component transformation

$$I_a = I_a^0 + I_a^+ + I_a^- = I_a^0 + I_a^+ + I_a^-$$

$$I_b = I_b^0 + I_b^+ + I_b^- = I_a^0 + a^2 I_a^+ + a I_a^-$$

$$I_c = I_c^0 + I_c^+ + I_c^- = I_a^0 + a I_a^+ + a^2 I_a^-$$

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_a^0 \\ I_a^+ \\ I_a^- \end{bmatrix} \qquad [I_P] = [T][I_S]$$

phase current $\quad$ Symmetrical component transformation Matrix $\quad$ sequence current

Symmetrical current from phase current

$$[I_S] = [T]^{-1}[I_P]$$

$$\begin{bmatrix} I_a^0 \\ I_a^+ \\ I_a^- \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}^{-1} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

$$|T| = 3(a-a^2)$$

$$T^{-1} = \frac{1}{3(a-a^2)}\begin{bmatrix} a-a^2 & a-a^2 & a-a^2 \\ a-a^2 & a^2-1 & 1-a \\ a-a^2 & 1-a & a^2-1 \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}$$

$$I_a^0 = \frac{1}{3}\left[I_a + I_b + I_c\right]$$

$$I_a^+ = \frac{1}{3}(I_a + aI_b + a^2 I_c)$$

$$I_a^- = \frac{1}{3}(I_a + a^2 I_b + aI_c)$$

$$I_a + I_b + I_c = 0$$

$$I_a^0 = \frac{1}{3} \times 0 = 0.$$

Unbalanced phase voltages in terms of Symmetrical component voltages

Sub. S/m Compments wrt 'a'.

$$V_a = V_a^0 + V_a^+ + V_a^-$$

$$V_b = V_b^0 + V_b^+ + V_c^-$$

$$V_c = V_c^0 + V_c^+ + V_c^-$$

$$V_a = V_a^0 + V_a^+ + V_a^-$$

$$V_b = V_a^0 + a^2 V_a^+ + a V_a^-$$

$$V_c = V_a^0 + a V_a^+ + a^2 V_a^-$$

In matrix form,

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_a^0 \\ V_a^+ \\ V_a^- \end{bmatrix}$$

$$[V_P] = [T][V_S]$$

$$[V^{abc}] = [T][V^{012}].$$

Symmetrical component voltage in terms of phase voltages

$$[V_S] = [T]^{-1}[V_P]$$

$$\begin{bmatrix} V_a^0 \\ V_a^+ \\ V_a^- \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}^{-1} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

$$\begin{bmatrix} V_a^0 \\ V_a^+ \\ V_a^- \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

V_S = Sequence Voltages.

V_P = phase voltages

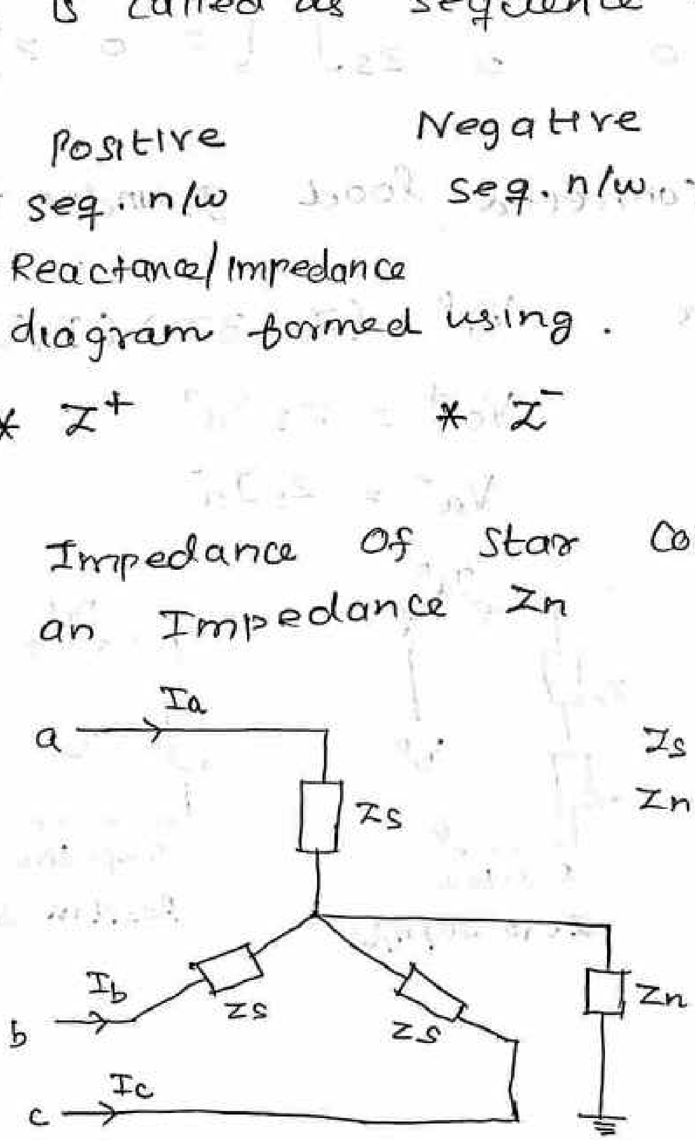
Sequence impedance: Impedance offered by the device or component to respective sequence current.

- * Positive sequence impedance ; offered to the flow of positive sequence current (Z^+)
- * Negative sequence impedance (Z^-) offered to the flow of negative sequence current
- * zero sequence impedance (Z^0) offered to the flow of zero sequence current.

Sequence network:- Single phase equivalent ckt of power system formed using impedance of any one sequence is called as sequence network. (SN)

Positive seq. n/w	Negative seq. n/w	Zero seq. n/w
- Reactance/Impedance diagram formed using.		
* Z^+	* Z^-	* Z^0.

Sequence Impedance of Star Connected load grounded through an Impedance Zn



Zs : self impedance

Zn = neutral impedance

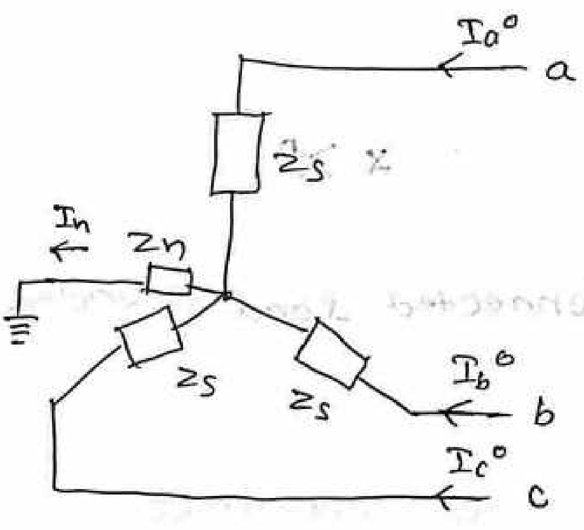
$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} Z_s+Z_n & Z_n & Z_n \\ Z_n & Z_s+Z_n & Z_n \\ Z_n & Z_n & Z_s+Z_n \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

$$[Z^{012}] = [T]^{-1} [Z^{abc}] [T]$$

$$= \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} Z_s+Z_n & Z_n & Z_n \\ Z_n & Z_s+Z_n & Z_n \\ Z_n & Z_n & Z_s+Z_n \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}$$

$$= \begin{bmatrix} Z_s+3Z_n & 0 & 0 \\ 0 & Z_s & 0 \\ 0 & 0 & Z_s \end{bmatrix} = \begin{bmatrix} Z^0 & 0 & 0 \\ 0 & Z^+ & 0 \\ 0 & 0 & Z^- \end{bmatrix}$$

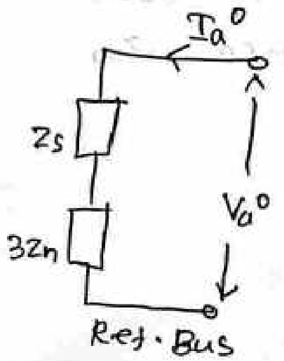
3ϕ Balanced Y connected load grounded through Zn



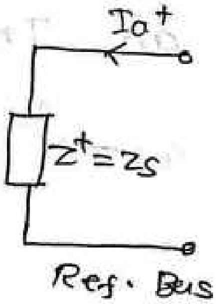
$$V_a^0 = (Z_s+3Z_n)(I_a^0)$$

$$V_a^+ = Z_s I_a^+$$

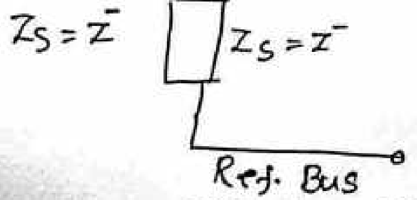
$$V_a^- = Z_s I_a^-$$



Zero seq n/w

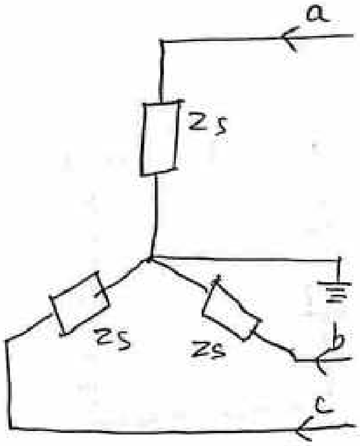


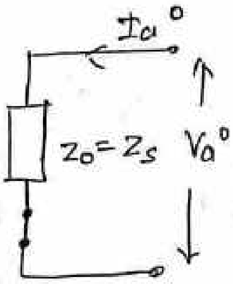
Positive seq. n/w



Negative seq/n/w

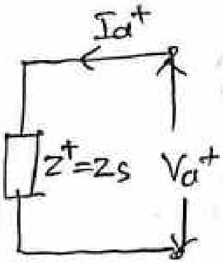
3ϕ Balanced Y connected load solidly grounded.



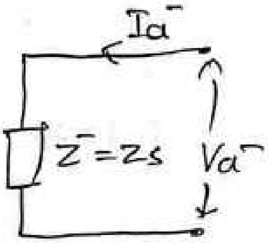


Ref · Bus

Zero SN

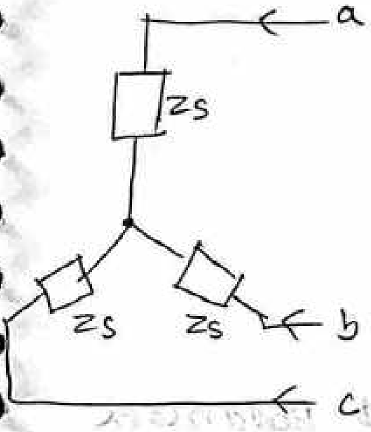


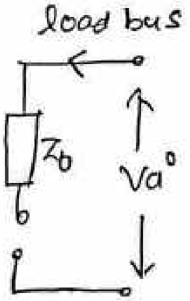
Positive SN



Negative SN

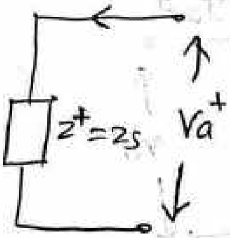
3ϕ Balanced Y connected load ungrounded



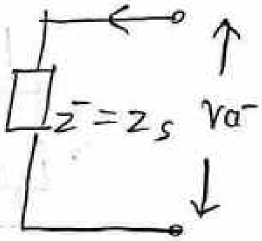


Ref Bus

Zero

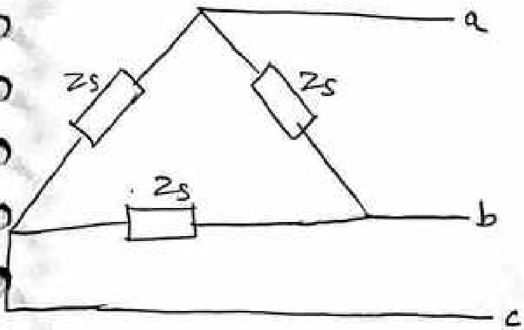


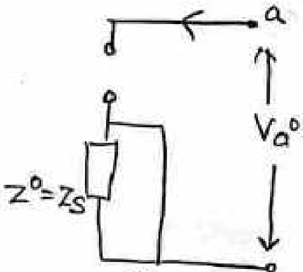
Positive



Negative

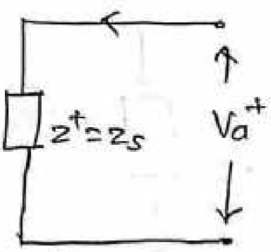
3ϕ Balanced Delta connected load



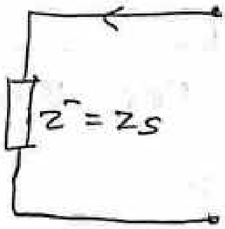


Ref · Bus

Zero

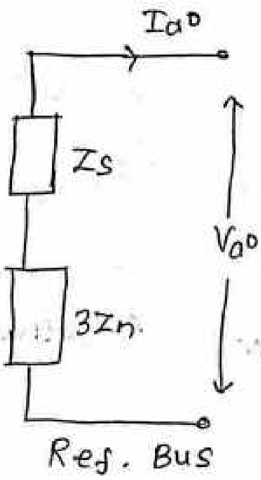


Positive

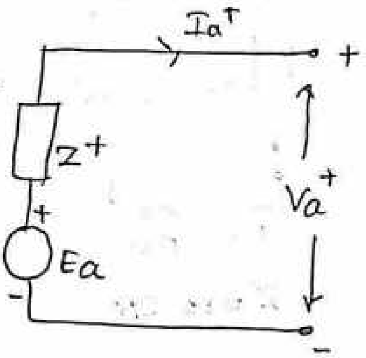


Negative

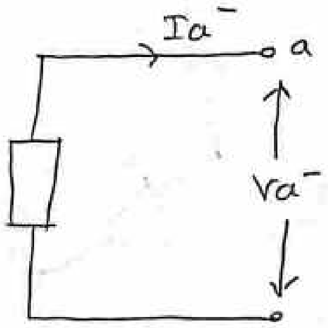
Seq. N/w of a 3ϕ Balanced generator



Zero

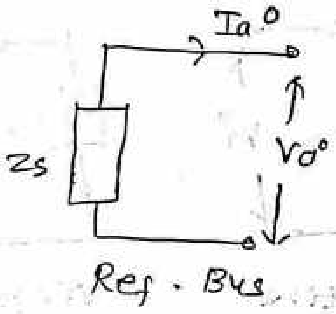


Positive

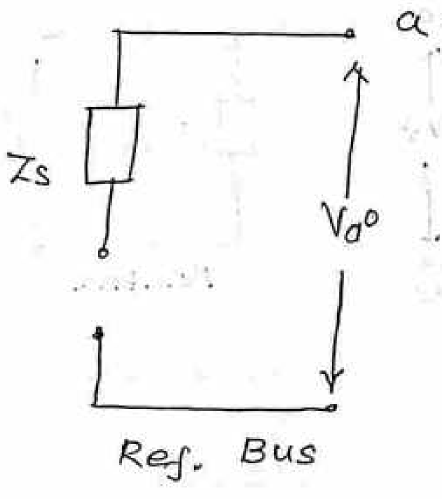


Negative

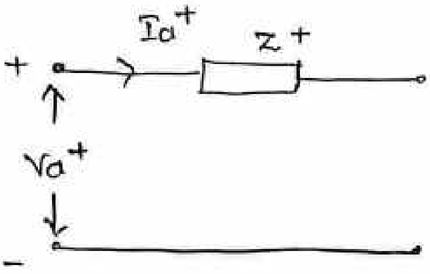
Zero sequence n/w of a Solidly grounded 3ϕ Balanced generator

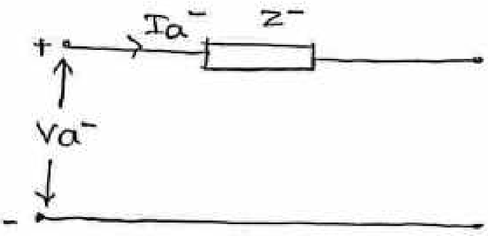


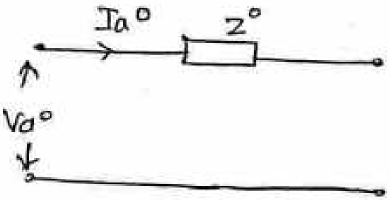
Zero sequence N/w of a ungrounded 3ϕ Balanced generator



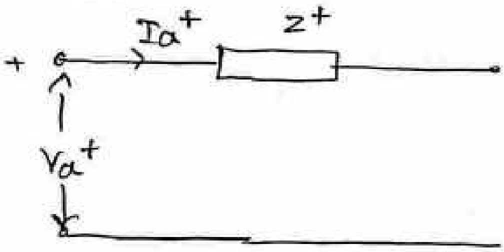
Sequence impedance and networks of a transmission line:

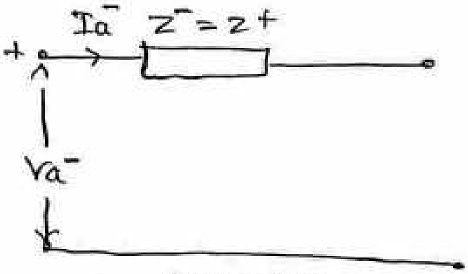

Positive


Negative


Zero

Sequence Impedance and sequence Network of transformer:


Positive


Negative

Zero sequence impedance of transformer

The eq. ckt for the zero sequence Impedance depends on:

(i) winding Connection
(ii) whether neutral grounded or not

① Transformer Y-Y connected with Both neutral grounded

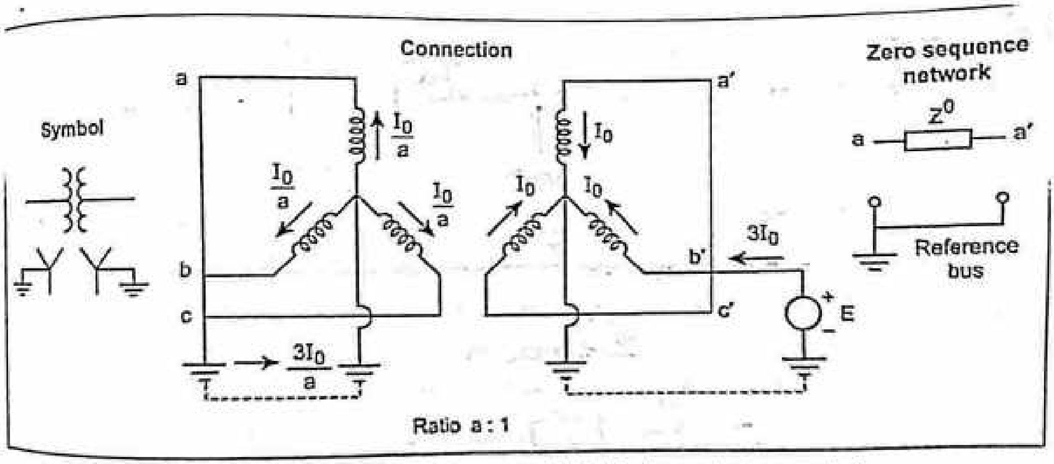


Fig. 9.17. Transformer Y-Y connected with both neutral grounded

② Transformer Y-Y connected, one Neutral grounded.

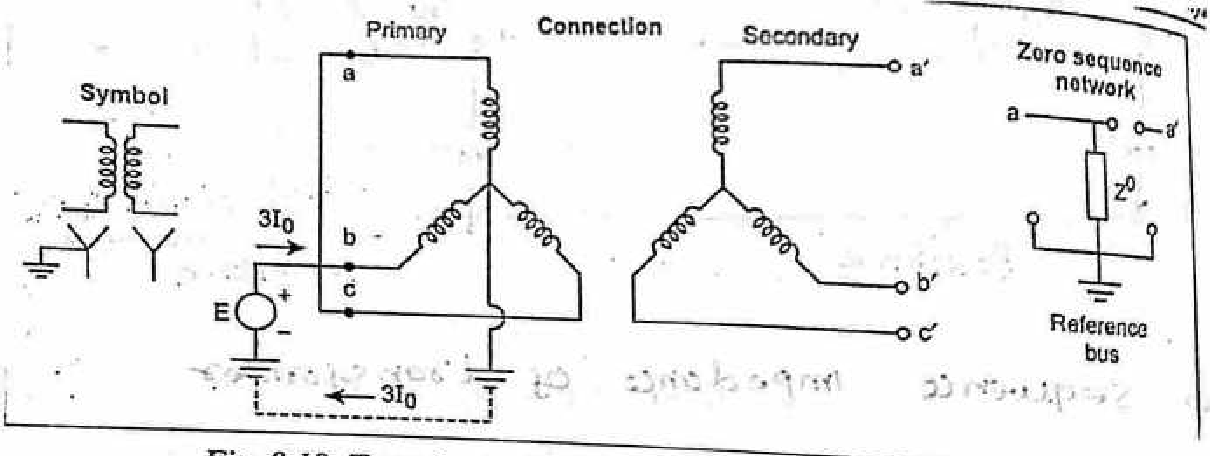


Fig. 9.18. Transformer Y-Y connected, one neutral grounded

③ Transformer Y-Δ connected neutral solidly grounded

ow because of circulating curren

connected primary. The zero sequence network is as shown

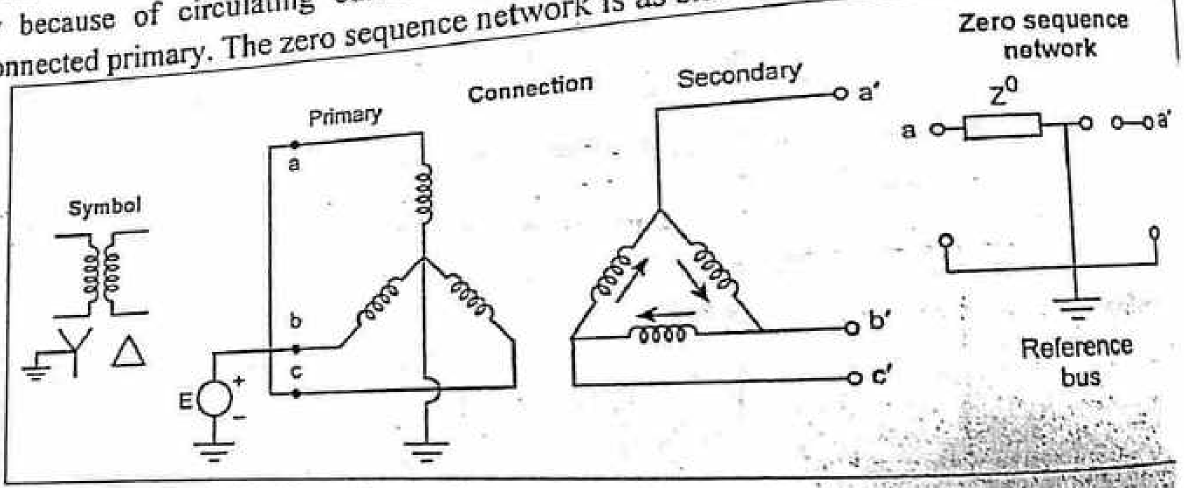


Fig. 9.19. Transformer Y-Δ connected, neutral solidly grounded

④ Transformer Y-Δ connected with Isolated neutral

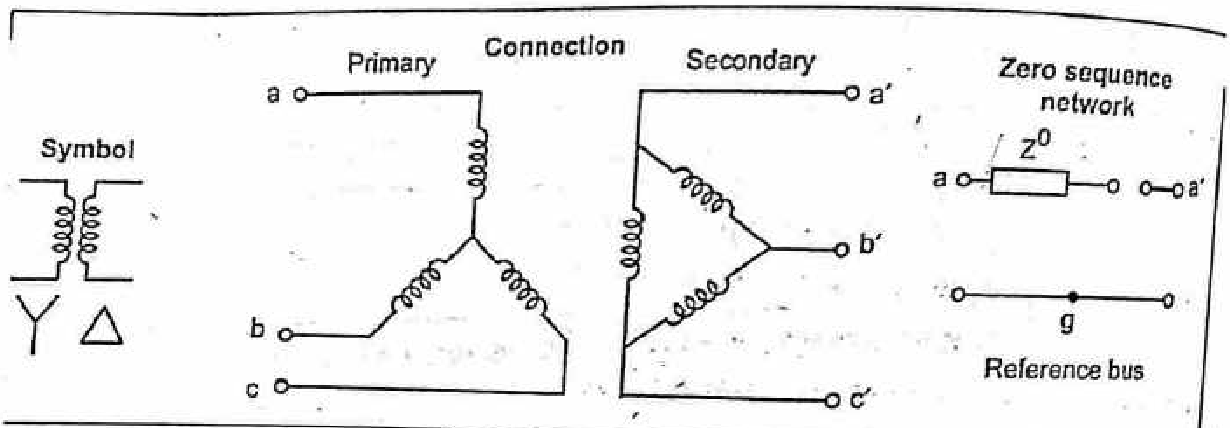


Fig. 9.20. Transformer Y-Δ connected with isolated neutral

⑤

Transformer Δ – Δ connected

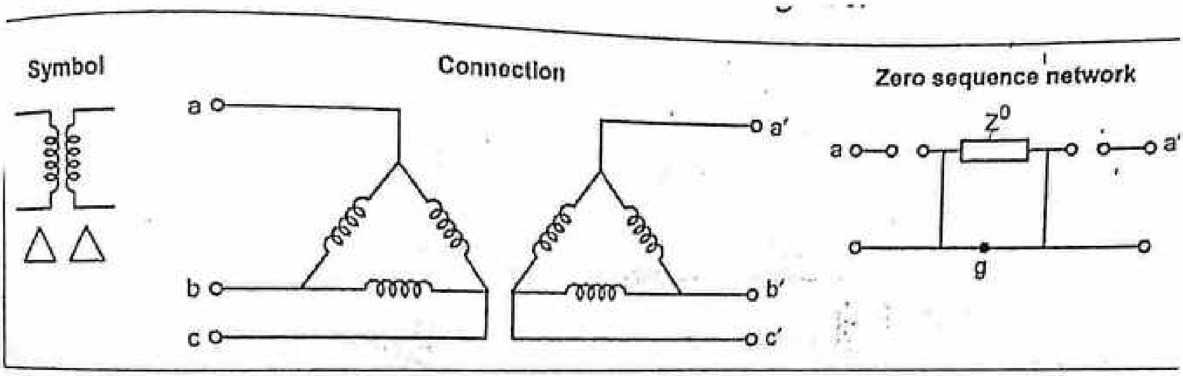


Fig. 9.21. Transformer Δ-Δ connected

⑥

Transformer Y-Y ungrounded connection

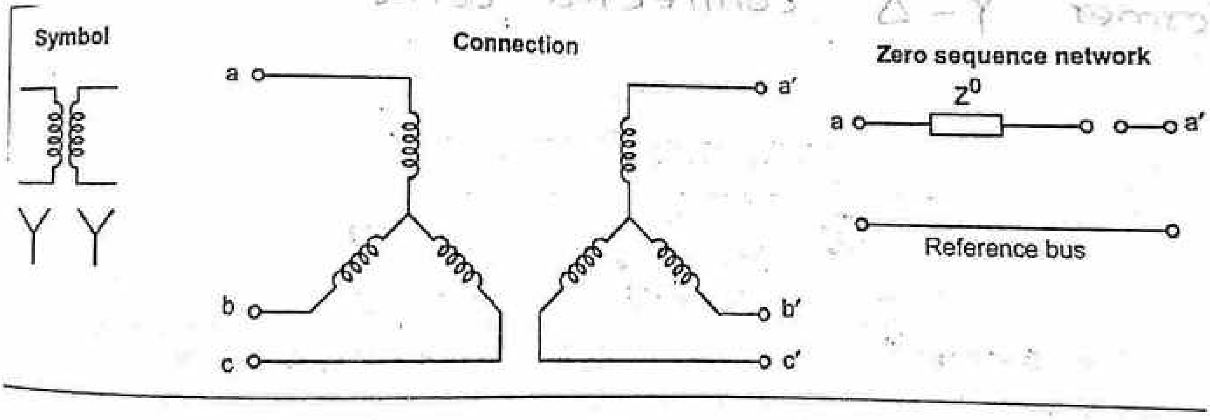


Fig. 9.22. Transformer Y-Y ungrounded connection

Unsymmetrical fault analysis / Unbalanced faults

* when the network is unsymmetrically faulted or loaded, neither the phase currents nor the phase voltage will possess three phase symmetry.

* Unsymmetrical fault occurs → Unbalanced current flow in s/m.

* Use s/mil components to analyze unsym. faults.
* use thevenin's theorem (or) Bus Impedance matrix to determine positive, negative and zero sequence impedance.

Types of symmetrical faults

(a) Line to ground (L-G) 70 %.

(b) Line to line fault (L-L) 15 %.

(c) Double line to ground fault (L-L-G) 10 %

(d) open conductor fault. 5 %.

Causes

Lightning, wind damage, trees falling across lines vehicles colliding with towers or poles.

Short circuit analysis of unbalanced System (low order system)

(a) Draw the positive, negative and zero sequence network with their appropriate description.

(b) Choice of type of fault and location of fault and mathematical description for the particular type of fault.

(c) Use thevenin's theorem or bus impedance matrix determine the solution of the network equation. Determine: fault current, post fault current, post fault voltages are found at the point of fault. and also calculate the all bus voltages and line flows.

* Single line to Ground fault
* line to line fault
* Double line to ground fault

] Refer book "Power S/m analysis" - Jeraldin Ahila

Unit Name: Unsymmetrical fault analysis - unbalanced faults.

the point of fault, all the bus

10.3. SINGLE LINE-TO-GROUND FAULT (L-G FAULT)

The single line to ground fault, the most common type, is caused by lightning or by conductors making contact with grounded structures. Fig.10.1 shows a three phase generator with neutral grounded through impedance Z_n.

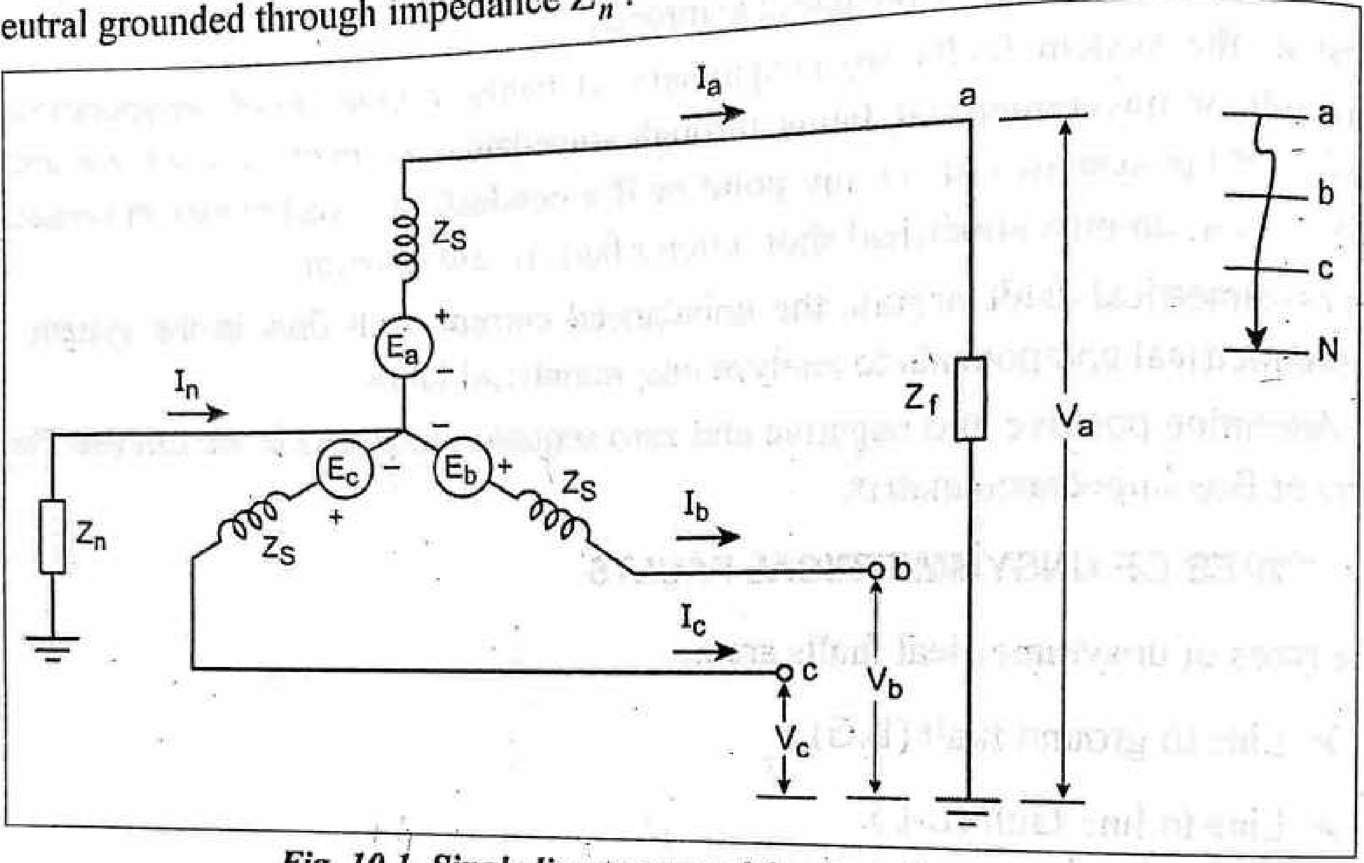


Fig. 10.1. Single line to ground fault at phase 'a'

Suppose a line-to-ground fault occurs on phase 'a' connected to ground through impedance Z_f.

Assuming the generator is initially on no load, the conductions at the fault bus 'K' are expressed by the following relations.

$$\left.\begin{aligned} V_a &= Z_f I_a \\ I_b &= I_c = 0 \\ I_f &= I_a \end{aligned}\right\} \qquad \text{... (10.1)}$$

Symmetrical components of currents are

$$\begin{bmatrix} I_a^0 \\ I_a^+ \\ I_a^- \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \qquad \text{... (10.2)}$$

Substitute for $I_b = I_c = 0$, the symmetrical components of currents are

$$\begin{bmatrix} I_a^0 \\ I_a^+ \\ I_a^- \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} I_a \\ 0 \\ 0 \end{bmatrix} \qquad \text{... (10.3)}$$

From the equation (10.3), we find that

$$I_a^0 = \frac{I_a}{3}$$

$$I_a^+ = \frac{I_a}{3}$$

$$I_a^- = \frac{I_a}{3} = \frac{I_f}{3}$$

i.e.,
$$I_a^+ = I_a^- = I_a^0 = \frac{I_a}{3} = \frac{I_f}{3} \qquad \text{... (10.4)}$$

From sequence networks of the generator, the symmetrical voltages are given by

$$\begin{bmatrix} V_a^0 \\ V_a^+ \\ V_a^- \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z_{KK}^0 & 0 & 0 \\ 0 & Z_{KK}^+ & 0 \\ 0 & 0 & Z_{KK}^- \end{bmatrix}\begin{bmatrix} I_a^0 \\ I_a^+ \\ I_a^- \end{bmatrix}$$

$$\left.\begin{aligned} V_a^0 &= -Z_{KK}^0 \; I_a^0 = -Z_{KK}^0 \; I_a^+ \\ V_a^+ &= E_a - Z_{KK}^+ \; I_a^+ \\ V_a^- &= -Z_{KK}^- \; I_a^- = -Z_{KK}^- \; I_a^+ \end{aligned}\right\} \qquad \text{... (10.5)}$$

The phase voltages are given by

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}\begin{bmatrix} V_a^0 \\ V_a^+ \\ V_a^- \end{bmatrix} \qquad \text{... (10.6)}$$

From equation (10.6), $V_a = V_a^0 + V_a^+ + V_a^-$

From the condition $V_a = Z_f I_a$

$$\therefore \; V_a^0 + V_a^+ + V_a^- = Z_f I_a \qquad \text{... (10.7)}$$

Substitute symmetrical components of voltages from equation (10.5), we get

$$-Z_{KK}^0 \; I_a^+ + E_a - Z_{KK}^+ \; I_a^+ + (-Z_{KK}^- \; I_a^+) = Z_f I_a$$

$$E_a - I_a^+ \; [Z_{KK}^0 + Z_{KK}^+ + Z_{KK}^-] = Z_f \times 3 \, I_a^+$$

$$I_a^+ \, [Z_{KK}^0 + Z_{KK}^+ + Z_{KK}^- + 3\,Z_f] = E_a$$

$$I_a^+ = \frac{E_a}{Z_{KK}^0 + Z_{KK}^+ + Z_{KK}^- + 3\,Z_f} \quad \cdots (10.8)$$

The fault current is

$$\boxed{I_f = I_a = 3\,I_a^+ = \frac{3\,E_a}{Z_{KK}^0 + Z_{KK}^+ + Z_{KK}^- + 3\,Z_f}} \quad \cdots (10.9)$$

On substituting symmetrical components of currents in equation (10.5) and (10.6), the symmetrical components of voltages and phase voltages at the fault point are obtained.

Sequence Network

From equation (10.4) and (10.5), the positive sequence, negative sequence and zero sequence networks are connected in series as shown in Fig.10.2. Thus, for LG faults, the Thevenin impedance to the fault point is obtained for each sequence network and are connected in series.

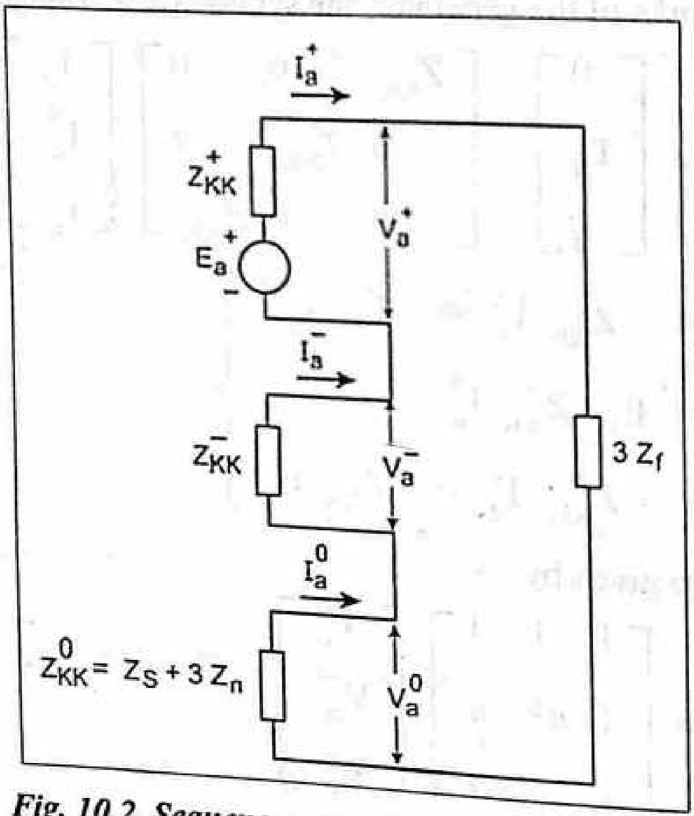


Fig. 10.2. Sequence network for LG fault with Z_f

Mostly $Z_{KK}^+ = Z_{KK}^-$

- If the generator is solidly grounded, $Z_n = 0$ and for bolted faults (or direct short circuit fault) or solid fault, $Z_f = 0$.
- If the neutral of the generator is ungrounded, the zero sequence network is open circuited.

$$\therefore \; I_a^+ = I_a^- = I_a^0 = 0$$

$$\text{and} \quad I_f = 0$$

10.3.1. DIRECT SHORT CIRCUIT OR BOLTED FAULT

Fig.10.3 shows the direct short circuit line to ground fault occurs at phase '*a*'.

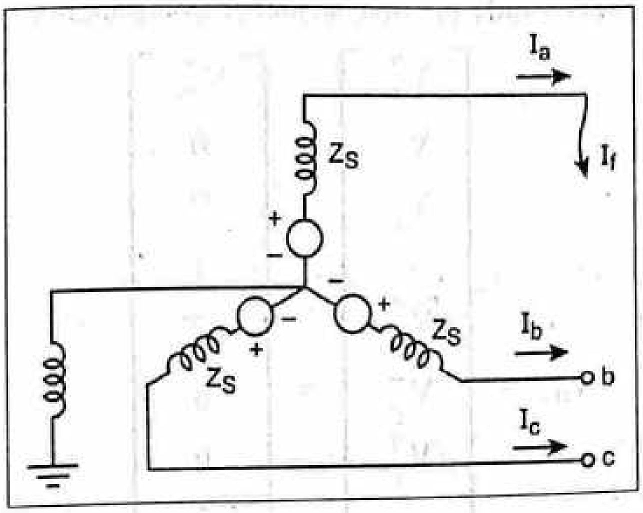


Fig. 10.3. Direct short circuit LG fault at phase 'a'

Fault impedance $Z_f = 0$

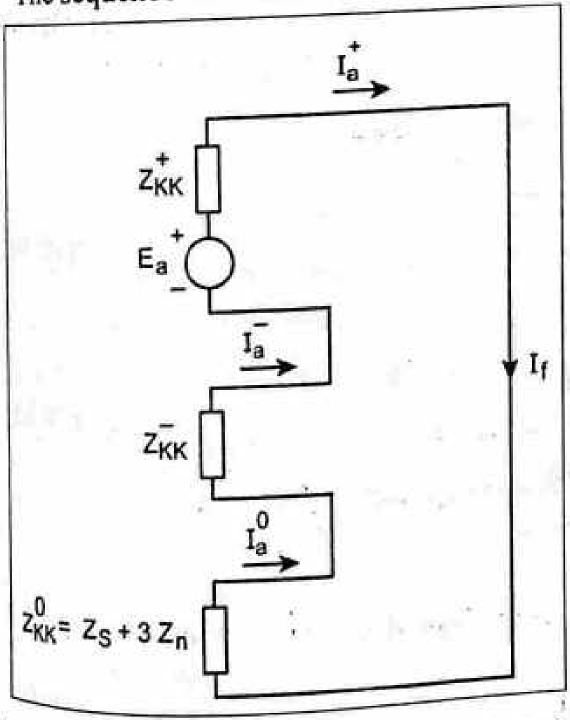
The conditions of the fault at bus K are

$$V_a = 0$$

$$I_b = I_c = 0$$

$$I_f = I_a$$

The sequence network for direct short circuit LG fault is as shown in Fig.10.4.



$$I_a^+ = I_a^- = I_a^0 = I_f \qquad \text{... (10.10)}$$

$$I_f = \frac{E_a}{Z_{KK}^+ + Z_{KK}^- + Z_{KK}^0} \qquad \text{... (10.11)}$$

Fig. 10.4. Sequence network for direct short circuit LG fault

Prefault Sequence Voltages

Since the fault is assumed to occur when the prefault system is under balanced condition, all prefault bus voltages contain only positive sequence components.

$$V_{0s} = \begin{bmatrix} V_1^+ \\ V_1^- \\ V_1^0 \\ \vdots \\ V_q^+ \\ V_q^- \\ V_q^0 \\ \vdots \\ V_n^+ \\ V_n^- \\ V_n^0 \end{bmatrix} = \begin{bmatrix} V_{10}^+ \\ 0 \\ 0 \\ \vdots \\ V_{q0}^+ \\ 0 \\ 0 \\ \vdots \\ V_{n0}^+ \\ 0 \\ 0 \end{bmatrix} \quad \ldots (10.12)$$

Post-fault Voltages

The post fault positive sequence bus voltages are

$$V_f^+ = V_0^+ + Z_{iK}^+ \; I_f \quad \ldots (10.13)$$

Since the fault current is injected at bus K,

$$I_f = \begin{bmatrix} 0 \\ 0 \\ -I_f^+ \\ 0 \\ 0 \end{bmatrix} \quad \ldots (10.14)$$

$$\therefore \; V_f^+ = V_0^+ - Z_{iK}^+ \; I_f^+ \quad \ldots (10.15)$$

The positive sequence post fault bus voltages are

$$\begin{aligned} V_{f1}^+ &= V_0^+ - Z_{1K}^+ \; I_f^+ \\ &\vdots \\ V_{fK}^+ &= V_0^+ - Z_{KK}^+ \; I_f^+ \\ &\vdots \\ V_{fn}^+ &= V_0^+ - Z_{nK}^+ \; I_f^+ \end{aligned} \quad \ldots (10.16)$$

The post fault negative sequence bus voltages are

$$\begin{aligned} V_{f1}^{-} &= -Z_{1K}^{-}\ I_f^{-} \\ &\vdots \\ V_{fK}^{-} &= -Z_{KK}^{-}\ I_f^{-} \\ &\vdots \\ V_{fn}^{-} &= -Z_{nK}^{-}\ I_f^{-} \end{aligned} \qquad \ldots (10.17)$$

The post fault zero sequence bus voltages are

$$\begin{aligned} V_{f1}^{0} &= -Z_{1K}^{0}\ I_f^{0} \\ &\vdots \\ V_{fK}^{0} &= -Z_{KK}^{0}\ I_f^{0} \\ &\vdots \\ V_{fn}^{0} &= -Z_{nK}^{0}\ I_f^{0} \end{aligned} \qquad \ldots (10.18)$$

Sequence Line Currents

$$\begin{aligned} \text{Positive sequence current } I_{ij}^{+} &= \frac{V_{fi}^{+} - V_{fj}^{+}}{Z_{ij}^{+}} \\ \text{Negative sequence current } I_{ij}^{-} &= \frac{V_{fi}^{-} - V_{fj}^{-}}{Z_{ij}^{-}} \\ \text{Zero sequence current } I_{ij}^{0} &= \frac{V_{fi}^{0} - V_{fj}^{0}}{Z_{ij}^{0}} \end{aligned} \qquad \ldots (10.19)$$

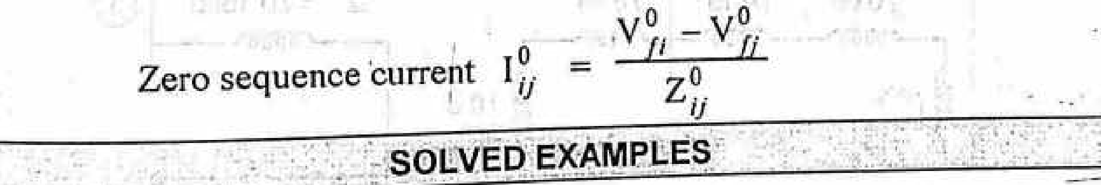
SOLVED EXAMPLES

Example 10.1(a) *Two synchronous machines are connected through three-phase transformers to the transmission line as given in Fig. The ratings and reactances of the machines and transformers are :*

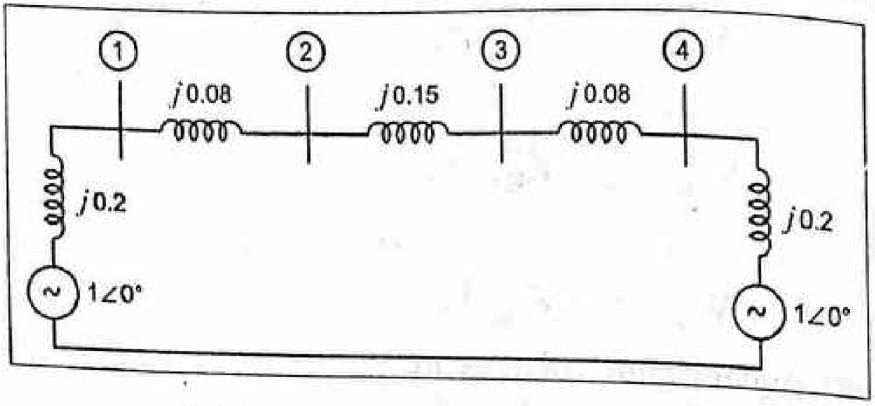
Machines 1 and 2 : 100 MVA ; 20 KV ; $X_d'' = X_1 = X_2 = 20\%$; $X_0 = 4\%$; $X_n = 5\%$

Transformers T_1 *and* T_2 *: 100 MVA ; 20 Y / 345 Y KV ;* $X = 8\%$

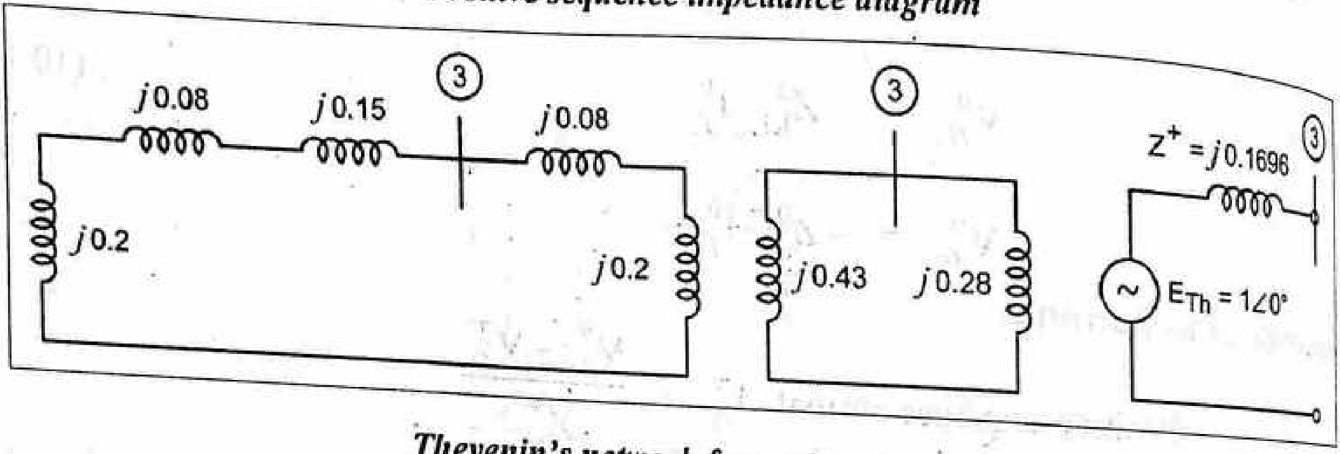
Both transformers are solidly grounded on two sides on a chosen base of 100 MVA, 345 KV in the transmission line circuit. The line reactances are $X_1 = X_2 = 15\%$ *and* $X_0 = 50\%$. *The system is operating at nominal voltage without prefault currents when a bolted* ($Z_f = 0$) *single line-to-ground fault occurs on phase 'a' at bus 3. Determine the subtransient current to ground at the fault.*



☺ **Solution : *Positive sequence Thevenin equivalent viewed from bus (3) :***

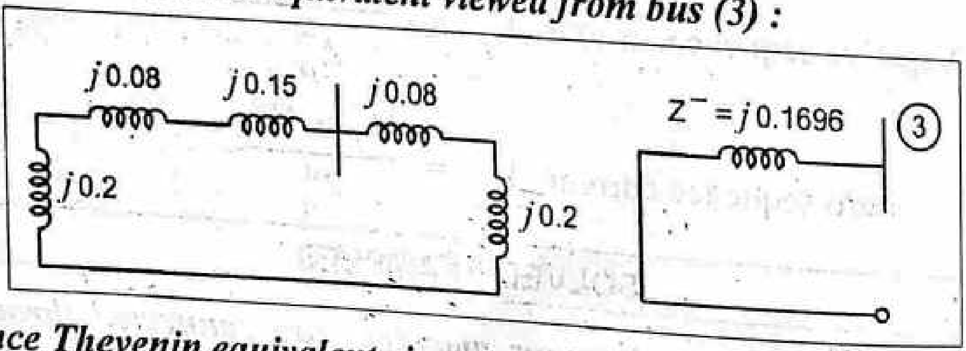


Positive sequence impedance diagram

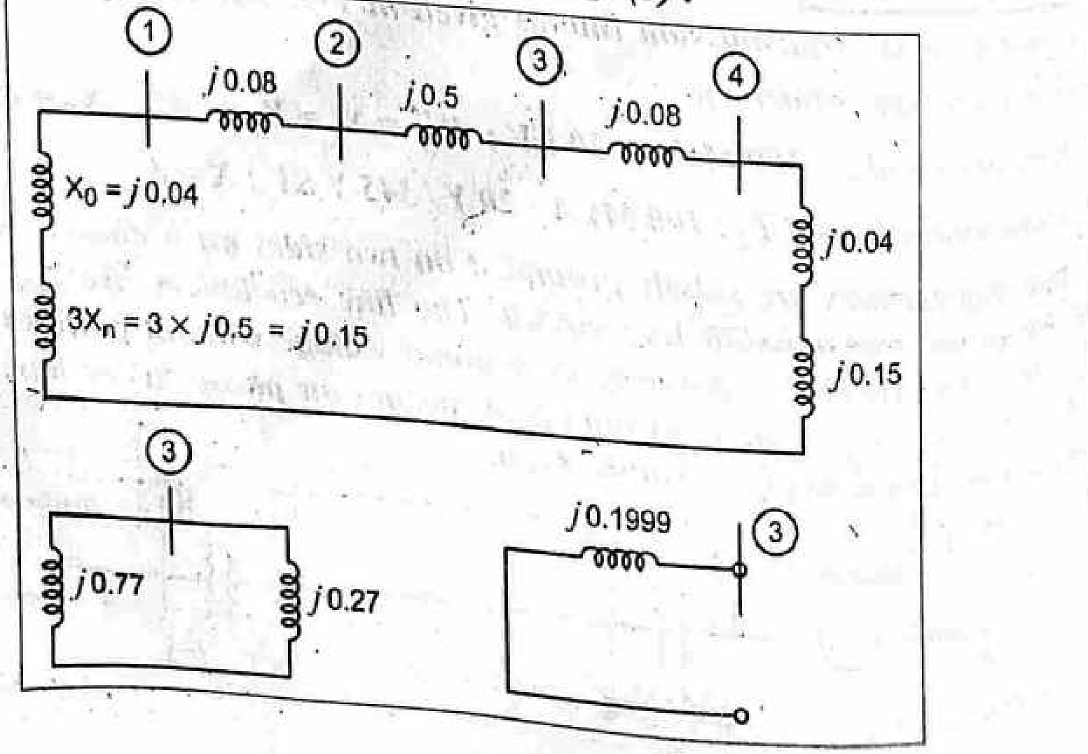


Thevenin's network for positive sequence

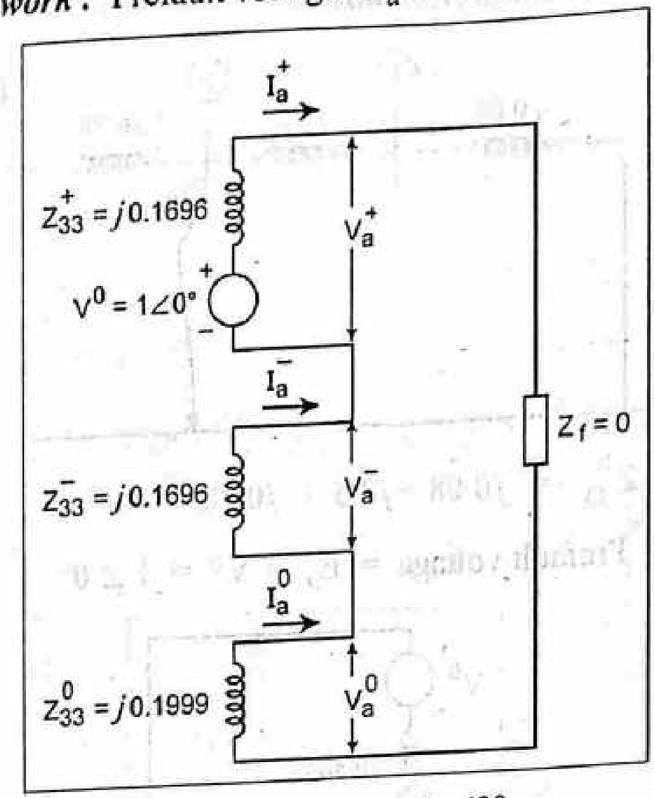
Negative sequence Thevenin equivalent viewed from bus (3) :



Zero sequence Thevenin equivalent viewed from bus (3) :



Draw Sequence Network : Prefault voltage $E_a = V^0 = 1 \angle 0°$



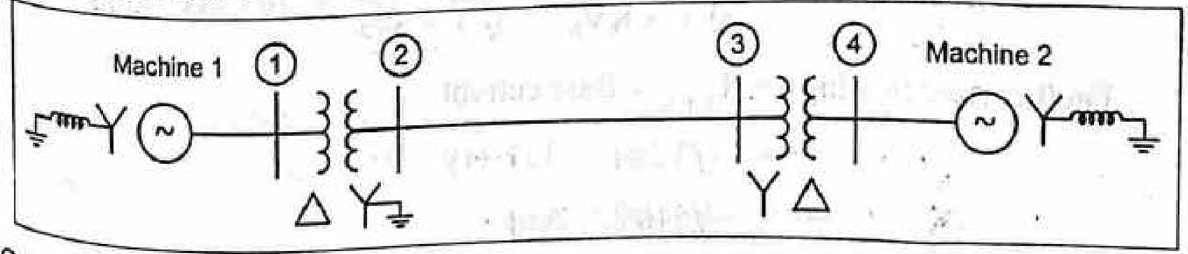
From Fig.,
$$I_a^+ = I_a^- = I_a^0 = \frac{1 \angle 0°}{Z_{33}^+ + Z_{33}^- + Z_{33}^0}$$
$$= \frac{1 \angle 0°}{j0.1696 + j0.1696 + j0.1999} = -j1.8549 \text{ p.u}$$

$$\text{Fault current in p.u} = 3 \times I_a^+$$
$$= 3 \times -j1.854 = -j5.5648 \text{ p.u}$$

Base current at the fault point or secondary side of first transformer/line
$$= \frac{MVA_b}{\sqrt{3} \times KV_b} = \frac{100 \times 10^3}{\sqrt{3} \times 345} = 167.348 \text{ Amp}$$

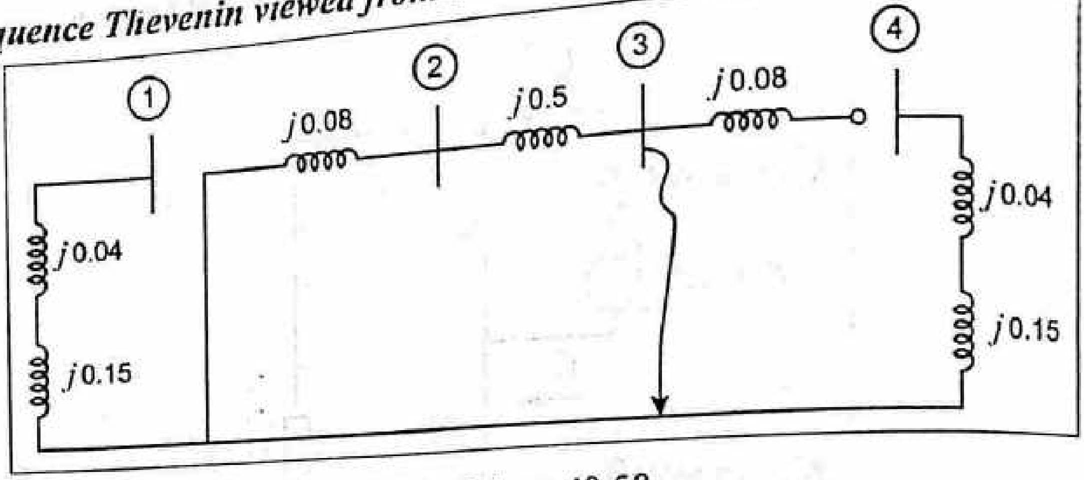
$$\text{Fault current in Amp} = I_{f\text{ p.u.}} \times \text{Base current}$$
$$= -j5.5648 \times 167.348$$
$$= -j931.2576 = 931.2576 \angle 270° \text{ Amp}$$

Example 10.1(b) ***For the Example 10.1(a), compute the fault current for Fig., when LG fault occurs at bus 3.***



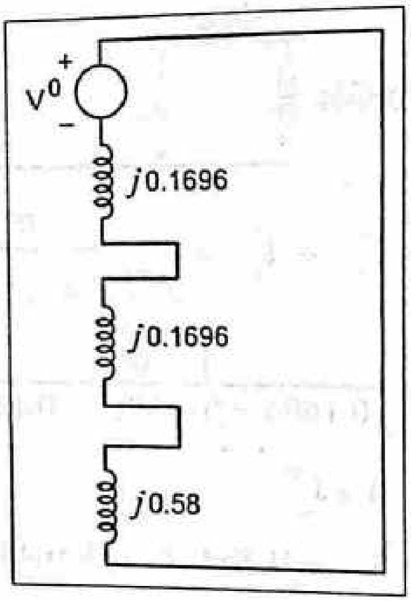
Solution : $Z_{33}^+ = j0.1696, \ Z_{33}^- = j0.1696$

Zero sequence Thevenin viewed from bus 3 :



$$Z_{33}^{0} = j0.08 + j0.5 = j0.58$$

Sequence network : Prefault voltage $= E_a = V^0 = 1 \angle 0°$



$$I_a^+ = I_a^- = I_a^0 = \frac{V^0}{Z_{33}^+ + Z_{33}^- + Z_{33}^0}$$

$$= \frac{1 \angle 0°}{j0.1696 + j0.1696 + j0.58} = -j1.088 \text{ p.u.}$$

Fault current in p.u. $= 3 \times I_a^+$

$$= 3 \times -j1.088 = -j3.264 \text{ p.u.}$$

Base current $= \dfrac{MVA_b}{\sqrt{3} \times KV_b} = \dfrac{100 \times 10^3}{\sqrt{3} \times 345} = 167.348$ Amp

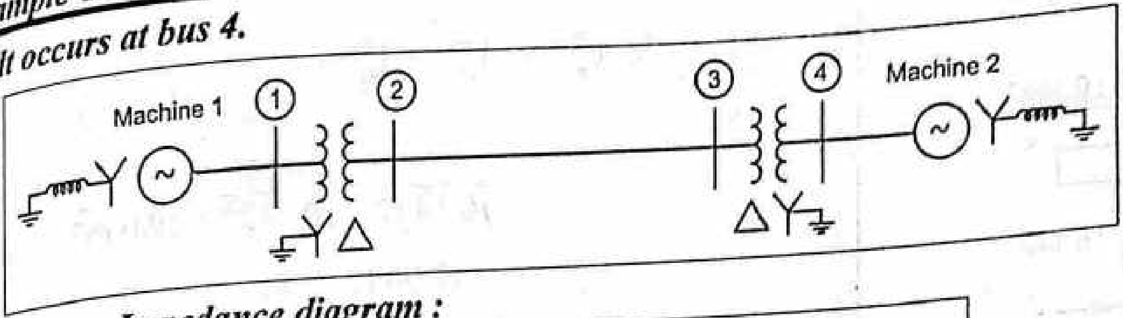
Fault current in Amp $= I_{f\text{ p.u.}} \times$ Base current

$$= -j3.264 \times 167.348$$

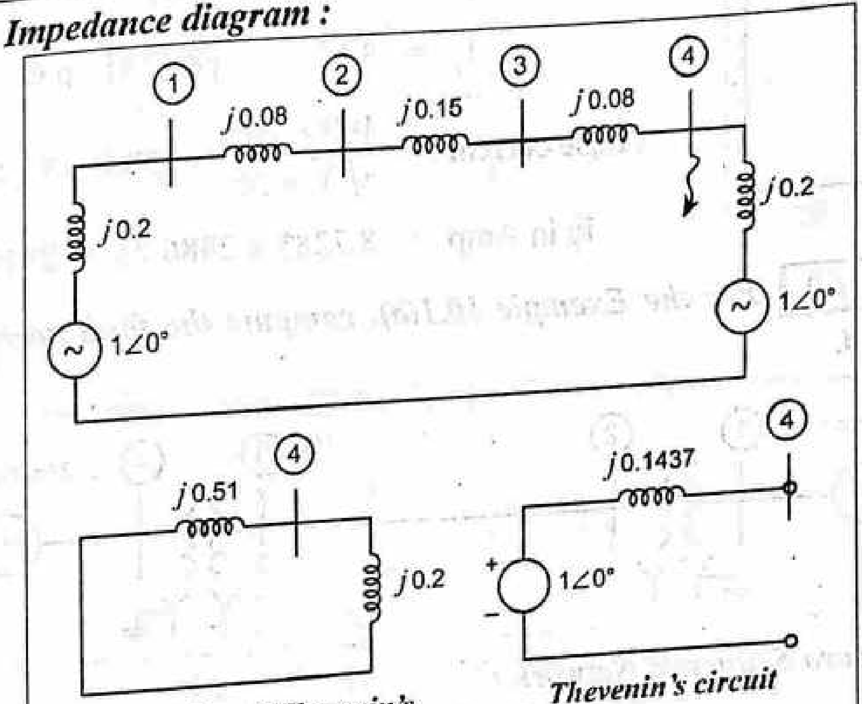
$$= -j546.22 \text{ Amp}$$

$$= 546.22 \angle 270° \text{ Amp}$$

Example 10.1(c) **For Example 10.1(a), determine fault current for the Fig., when L-G fault occurs at bus 4.**



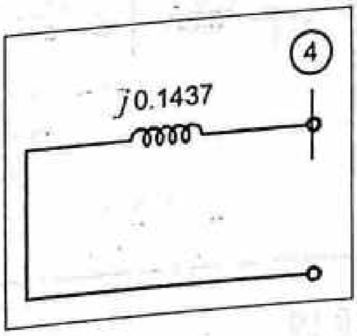
☺ **Solution : Impedance diagram :**



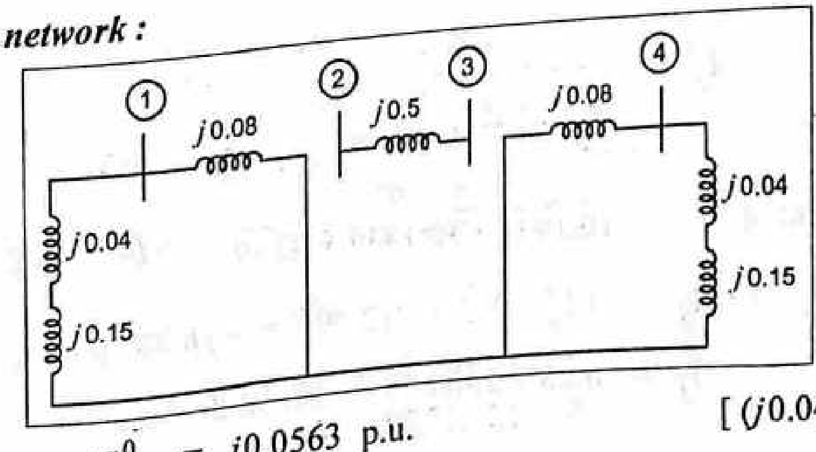
Determination of Thevenin's equivalent impedance

Thevenin's circuit

Negative sequence network :

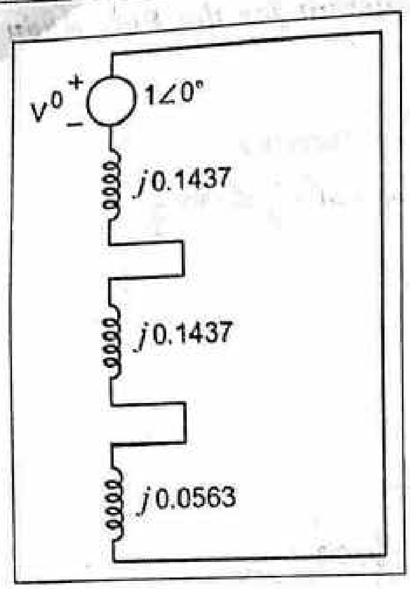


Zero sequence network :



[(j0.04 + j0.15) // j0.08]

$Z_{44}^{0} = j0.0563$ p.u.



$$\text{Prefault voltage} = E_a = V^0 = 1\angle 0°$$

$$I_a^+ = I_a^- = I_a^0 = \frac{V^0}{Z_{44}^+ + Z_{44}^- + Z_{44}^0}$$

$$= \frac{1\angle 0°}{j0.1437 + j0.1437 + j0.0563}$$

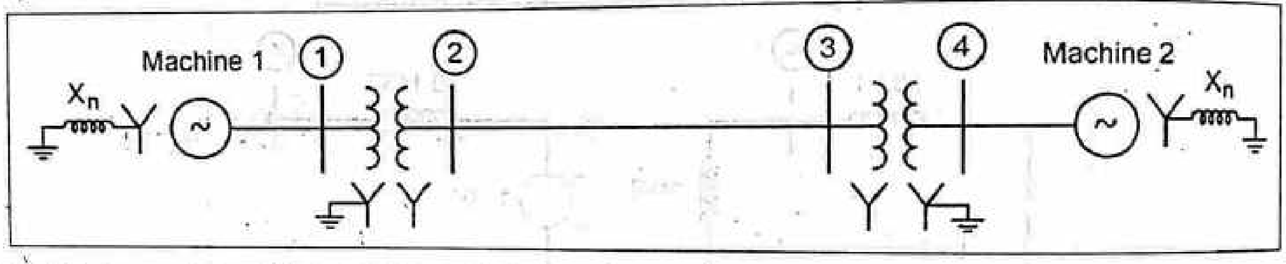
$$= -j2.9095 \text{ p.u.}$$

$$I_f = 3 I_a^+ = -j8.7285 \text{ p.u.}$$

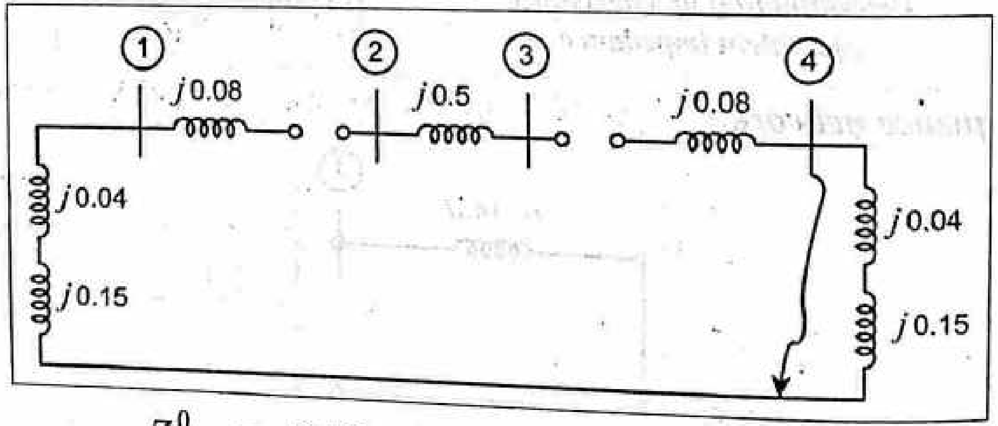
$$\text{Base current} = \frac{100\times 10^3}{\sqrt{3}\times 20} = 2886.75 \text{ Amp}$$

$$I_f \text{ in Amp} = 8.7285 \times 2886.75 = 25197 \text{ Amp}$$

Example 10.1(d) ***For the Example 10.1(a), compute the fault current for the Fig., when fault at bus 4.***



☺ ***Solution : Zero Sequence Network :***



$$Z_{44}^0 = j0.19$$

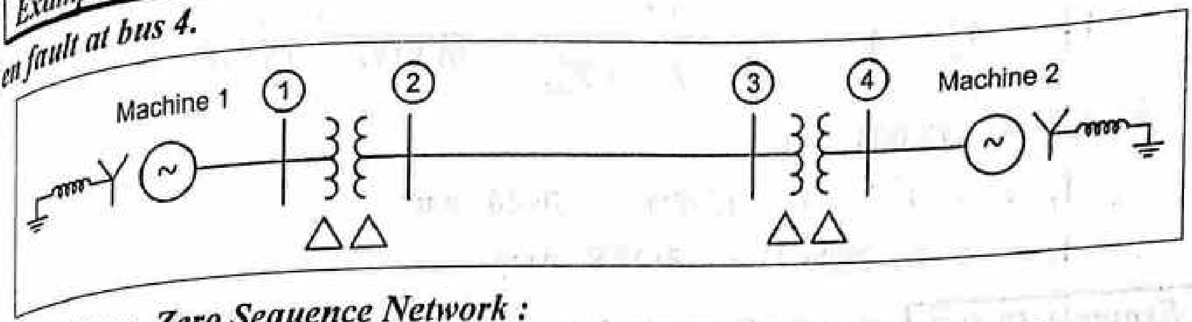
$$I_a^+ = \frac{V^0}{Z_{44}^+ + Z_{44}^- + Z_{44}^0}$$

$$= \frac{1\angle 0°}{j0.1437 + j0.1437 + j0.19} = -j2.094 \text{ p.u.}$$

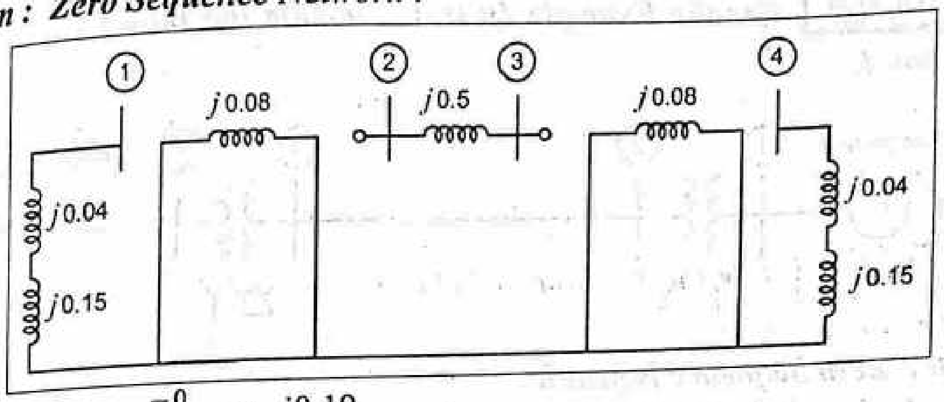
$$I_f = 3 I_a^+ = 3\times -j2.094 = -j6.28 \text{ p.u.}$$

$$I_f = 6.28 \times 2886.75 = 18128.8 \text{ Amp}$$

Example 10.1(e) **For the Example 10.1(a), compute the fault current for the Fig., when fault at bus 4.**



Solution : ***Zero Sequence Network :***



$$Z_{44}^{0} = j0.19$$

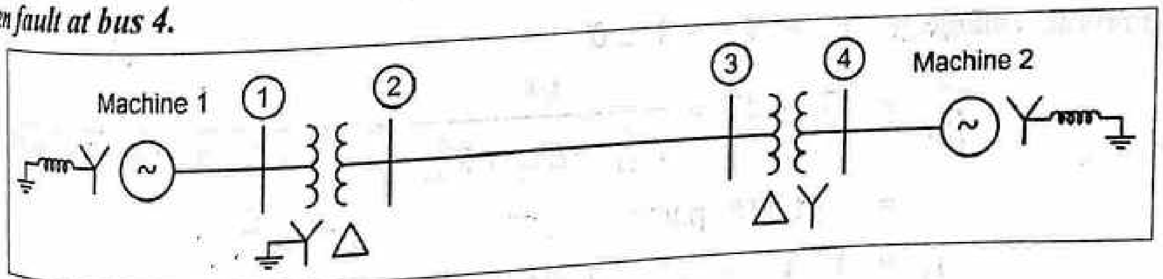
$$I_a^{+} = \frac{V^0}{Z_{44}^{+} + Z_{44}^{-} + Z_{44}^{0}} = \frac{1 \angle 0^\circ}{j0.1437 + j0.1437 + j0.19}$$

$$= -j2.094 \text{ p.u.}$$

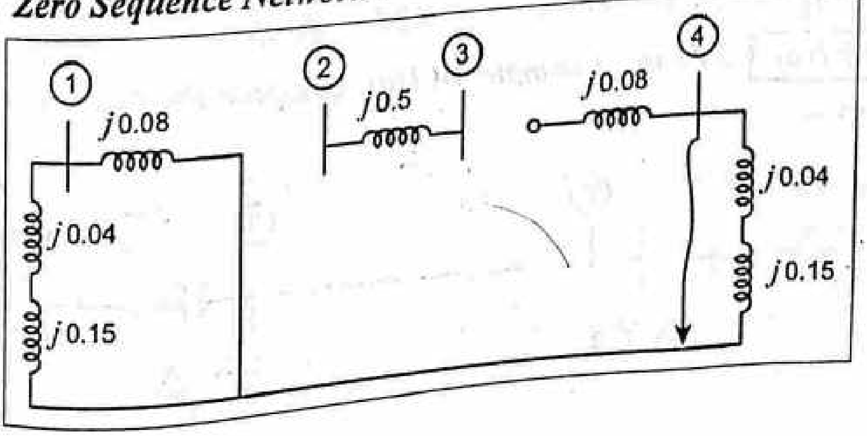
$$I_f = 3\,I_a^{+} = 3 \times -j2.094 = -j6.28 \text{ p.u.}$$

$$I_f = 6.28 \times 2886.75 = 18128.8 \text{ Amp}$$

Example 10.1(f) **For the Example 10.1(a), compute the fault current for the Fig., when fault at bus 4.**



Solution : ***Zero Sequence Network :***



$$Z^0_{44} = j0.19$$

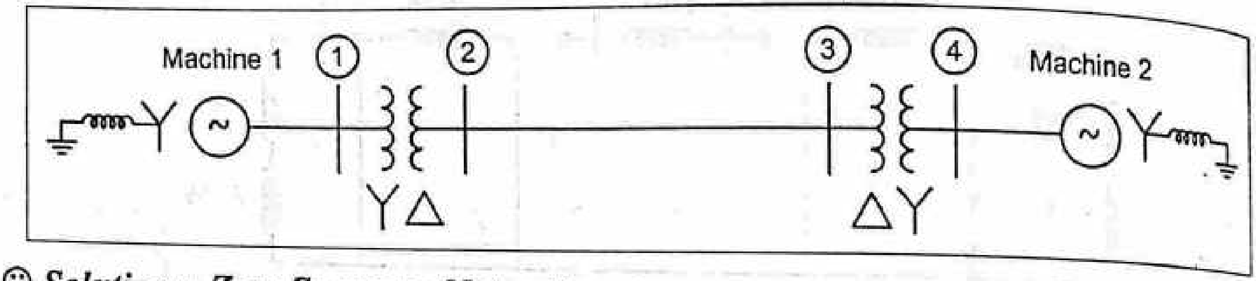
$$I^+_a = I^-_a = I^0_a = \frac{V^0}{Z^+_{44} + Z^-_{44} + Z^0_{44}} = \frac{1\angle 0°}{j0.1437 + j0.1437 + j0.19}$$

$$= -j2.094 \text{ p.u.}$$

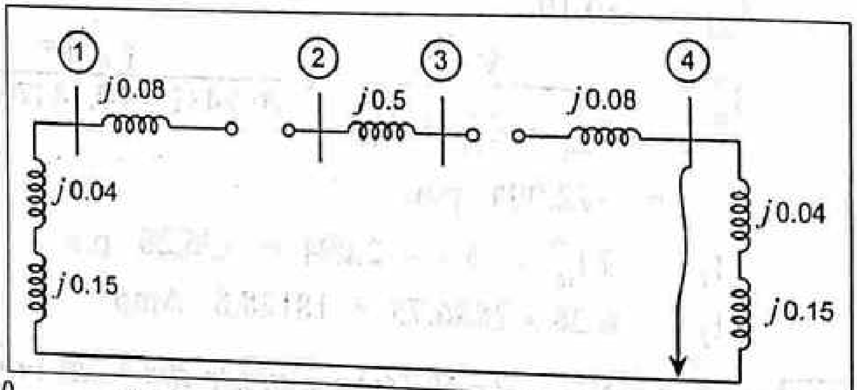
$$I_f = 3 \times I^+_a = 3 \times -j2.094 = -j6.28 \text{ p.u.}$$

$$I_f = 6.28 \times 2886.75 = 18128.8 \text{ Amp}$$

Example 10.1(g) ***For the Example 10.1(a), compute the fault current for the Fig., when fault at bus 4.***



☺ ***Solution : Zero Sequence Network :***



$$Z^0_{44} = j0.19$$

$$\text{Prefault voltage} = E_a = V^0 = 1\angle 0°$$

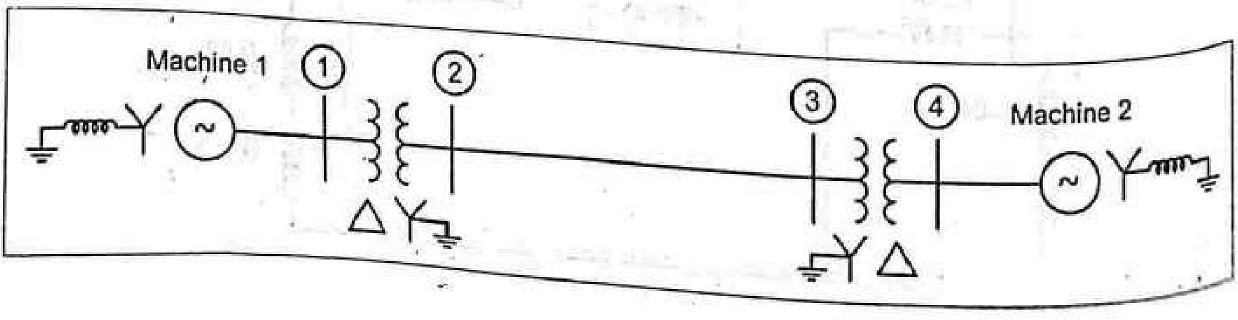
$$I^+_a = I^-_a = I^0_a = \frac{V^0}{Z^+_{44} + Z^-_{44} + Z^0_{44}} = \frac{1\angle 0°}{j0.1437 + j0.1437 + j0.19}$$

$$= -j2.094 \text{ p.u.}$$

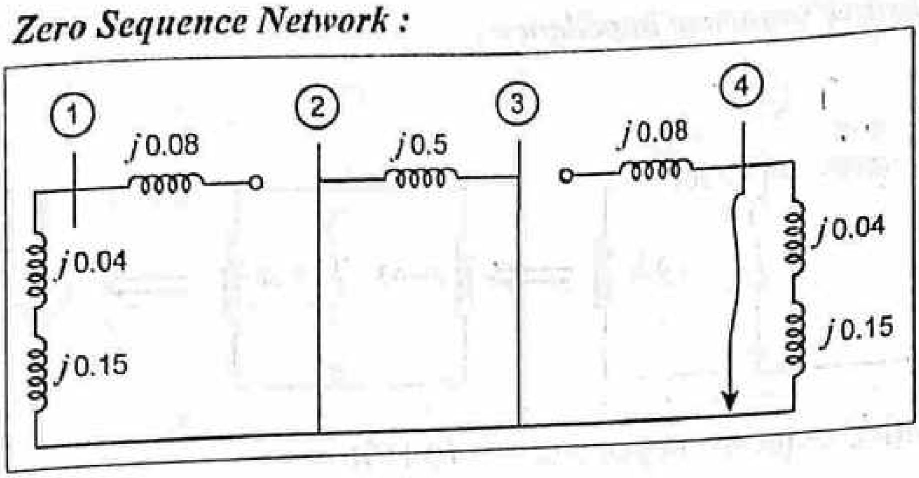
$$I_f = 3 \times I^+_a = 3 \times -j2.094 = -j6.28 \text{ p.u.}$$

$$I_f = 6.28 \times 2886.75 = 18128.8 \text{ Amp}$$

Example 10.1(h) ***For the Example 10.1(a), compute the fault current for the Fig., when fault at bus 4.***



☺ *Solution :* ***Zero Sequence Network :***



$$Z_{44}^{0} = j0.19$$

$$I_a^{+} = I_a^{-} = I_a^{0} = \frac{V^0}{Z_{44}^{+} + Z_{44}^{-} + Z_{44}^{0}}$$

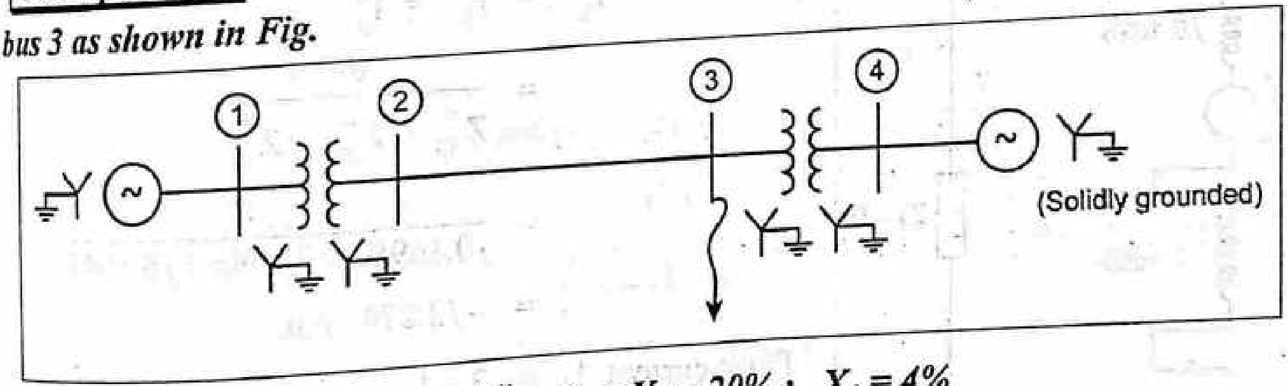
$$= \frac{1\angle 0°}{j0.1437 + j0.1437 + j0.19}$$

$$= -j2.094 \text{ p.u.}$$

$$I_f = 3 \times I_a^{+} = 3 \times -j2.094 = -j6.28 \text{ p.u.}$$

$$I_f = 6.28 \times 2886.75 = 18128.8 \text{ Amp}$$

Example 10.2 ***Determine the fault current and fault MVA for the L-G fault occurs at bus 3 as shown in Fig.***

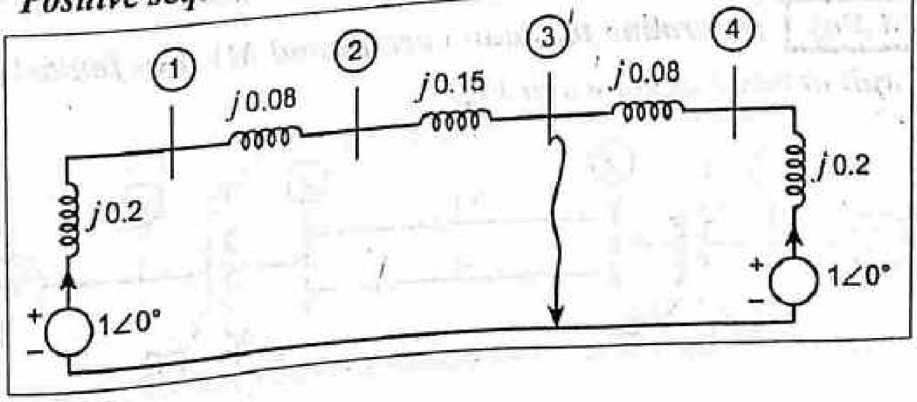


G_1, G_2: 100 MVA ; 20 KV ; $X_d'' = X_1 = X_2 = 20\%$; $X_0 = 4\%$

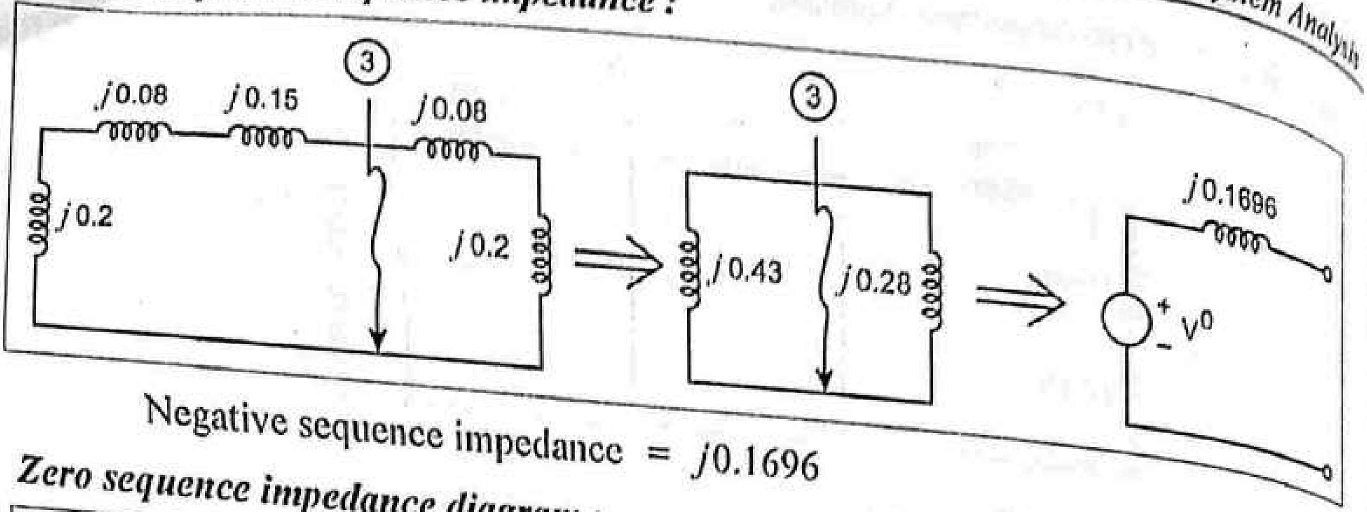
T_1, T_2: 100 MVA ; 20 / 345 KV (Y⏚ Y⏚) ; $X = 8\%$

Transmission line : $X_1 = X_2 = 15\%$; $X_0 = 50\%$

☺ *Solution :* ***Positive sequence impedance diagram :***

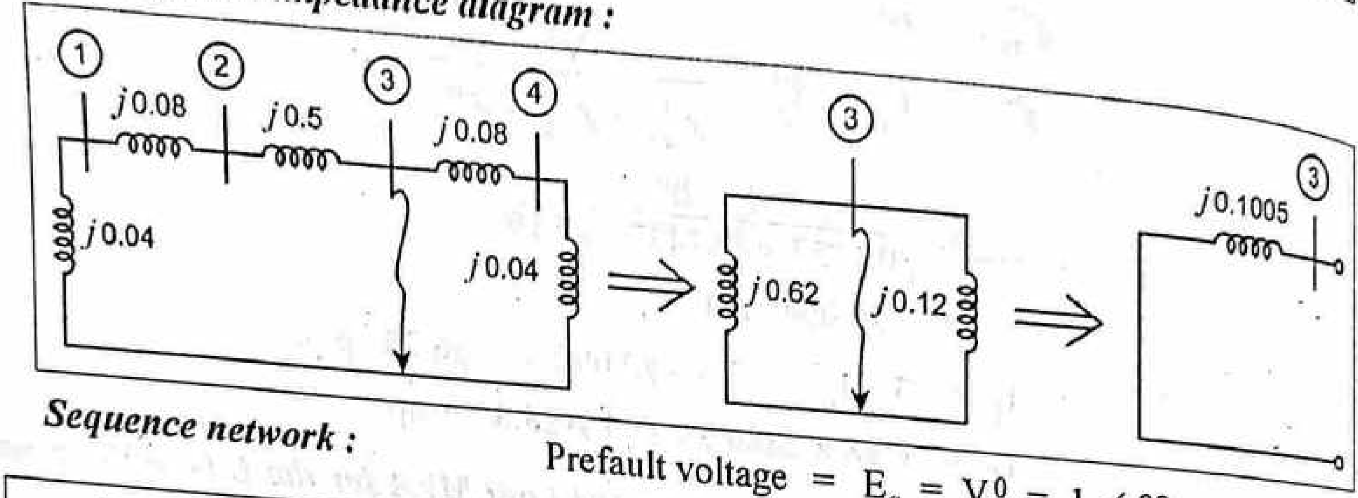


Determine positive sequence impedance :

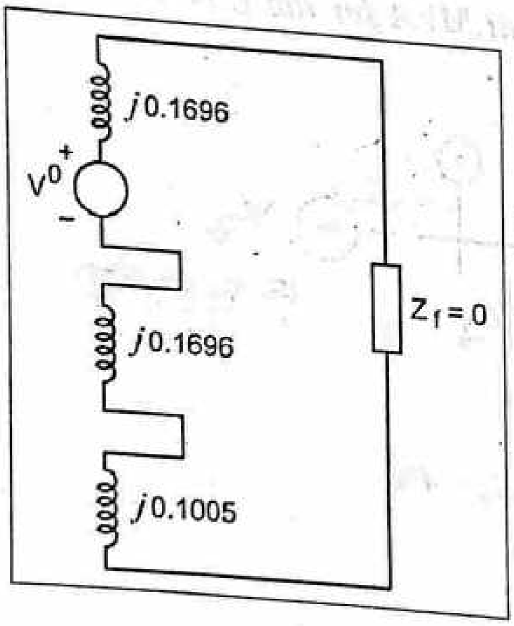


Negative sequence impedance = $j0.1696$

Zero sequence impedance diagram :



Sequence network :



Prefault voltage = $E_a = V^0 = 1\angle 0°$

$$I_a^+ = I_a^- = I_a^0$$

$$= \frac{V^0}{Z_{33}^+ + Z_{33}^- + Z_{33}^0}$$

$$= \frac{1\angle 0°}{j0.1696 + j0.1696 + j0.1005}$$

$$= -j2.274 \text{ p.u.}$$

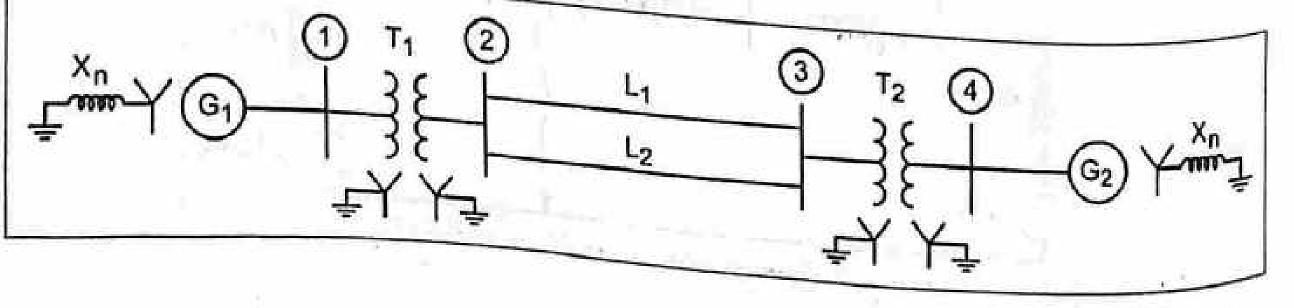
Fault current $I_f = 3 \times I_a^+$

$$= 3 \times (-j2.274) = -j.6.822 \text{ p.u.}$$

Fault MVA = $I_{f\ \text{p.u.}} \times$ MVA

$$= 6.822 \times 100 = 682.2 \text{ MVA}$$

Example 10.3(a) ***Determine the fault current and MVA at faulted bus for a line to ground (solid) fault at bus 4 as shown in Fig.***

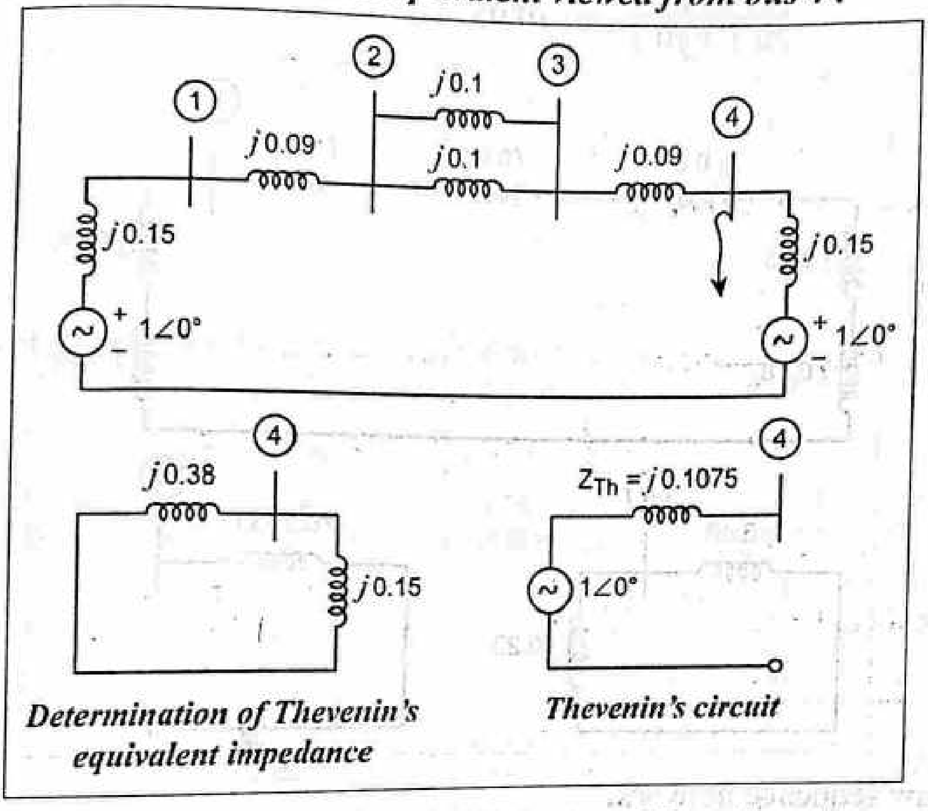


G_1, G_2: 100 MVA, 11 KV, $X^+ = X^- = 15\%$; $X^0 = 5\%$, $X_n = 6\%$
T_1, T_2: 100 MVA, 11 KV/220 KV, $X_{leak} = 9\%$
L_1, L_2: $X^+ = X^- = 10\%$; $X^0 = 10\%$ on a base of 100 MVA.

Consider a fault at phase 'a'.

☺ ***Solution :***

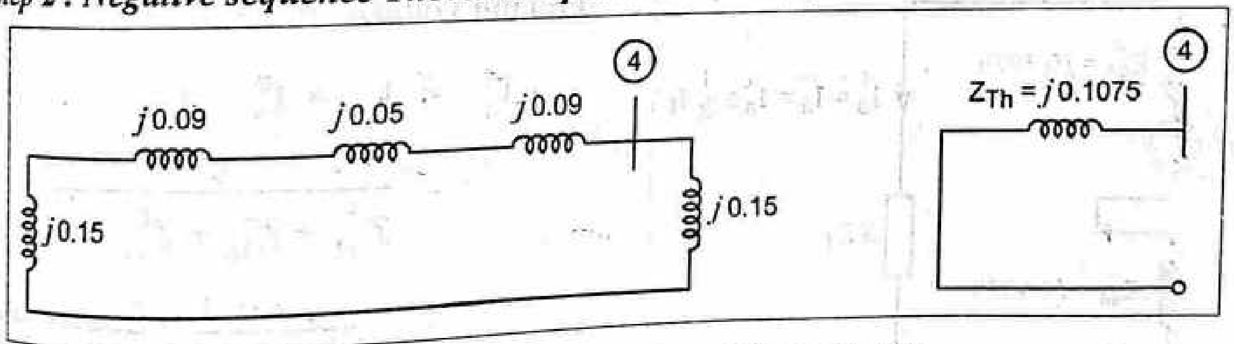
Step 1 : Positive sequence Thevenin equivalent viewed from bus 4 :



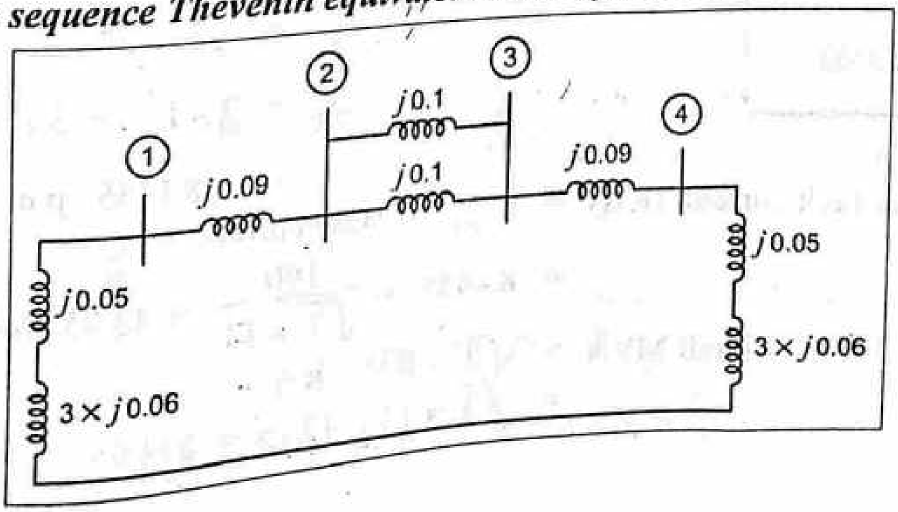
Determination of Thevenin's equivalent impedance

Thevenin's circuit

Step 2 : Negative sequence Thevenin equivalent viewed from bus 4 :



Step 3 : Zero sequence Thevenin equivalent viewed from bus 4 :

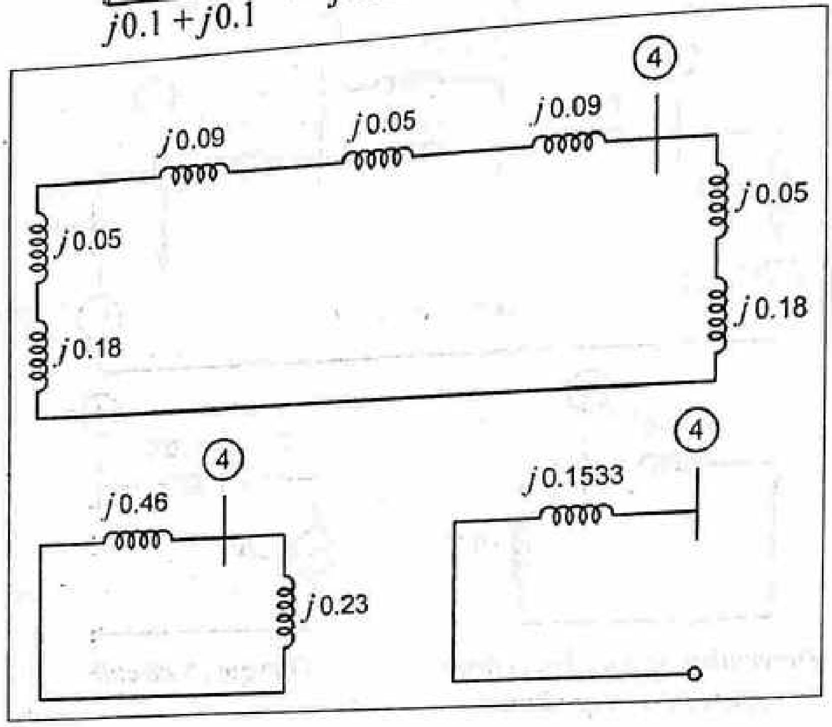


For transmission line $Z^{new}_{p.u.} = \frac{\text{Actual value}}{\text{Base value}} = \frac{j0.121}{\text{Base KV}^2} \times \text{Base MVA}$

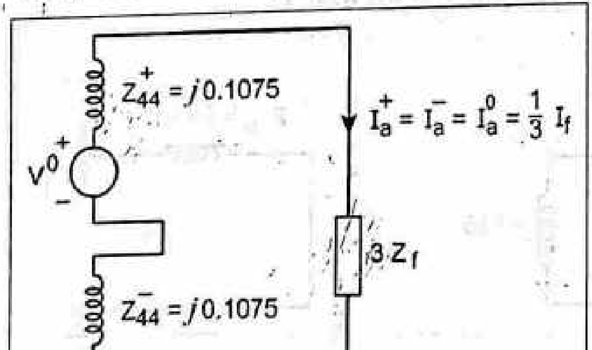
$$= \frac{j0.121}{11^2} \times 100 = j0.1$$

$j0.1$ and $j0.1$ are in parallel.

$$\frac{j0.1 \times j0.1}{j0.1 + j0.1} = j0.05$$



Step 4 : Draw sequence network.



Prefault voltage $E_a = V^0 = 1 \angle 0°$

$$I^{+}_{a} = I^{-}_{a} = I^{0}_{a}$$

$$= \frac{V^0}{Z^{+}_{44} + Z^{-}_{44} + Z^{0}_{44} + Z_f}$$

$$= \frac{1 \angle 0°}{j0.1075 + j0.1075 + j0.1533}$$

$$= -j2.7152$$

$$I_f = 3 \times I^{+}_{a} = 3 \times -j2.7152$$

$$= -j8.1455 \text{ p.u.}$$

$$\text{Actual fault current (KA)} = I_{f\,p.u.} \times \text{Base current}$$

$$= 8.1455 \times \frac{100}{\sqrt{3} \times 11} = 42.75 \text{ KA}$$

$$\text{Fault MVA} = \sqrt{3} \times \text{KV} \times \text{KA}$$

$$= \sqrt{3} \times 11 \times 42.75 = 814.55$$

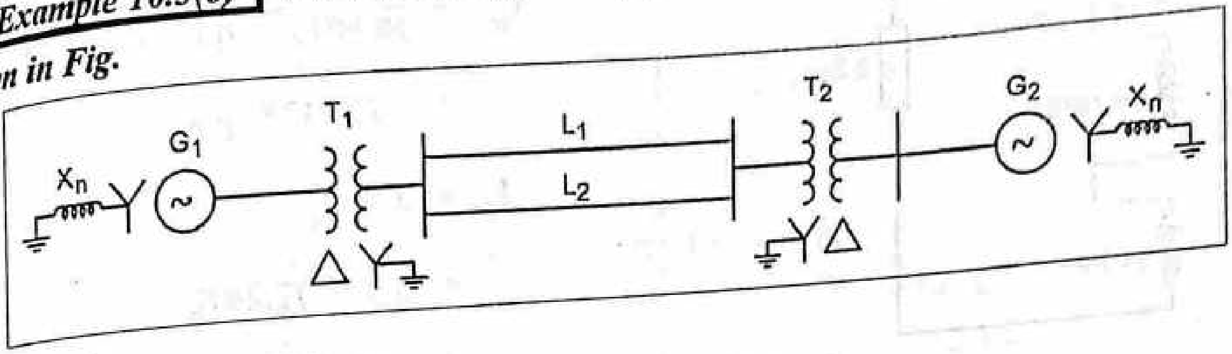
Current in phase domain :

$$I_a = I_f = 42.75 \text{ KA}$$

$$I_b = I_c = 0$$

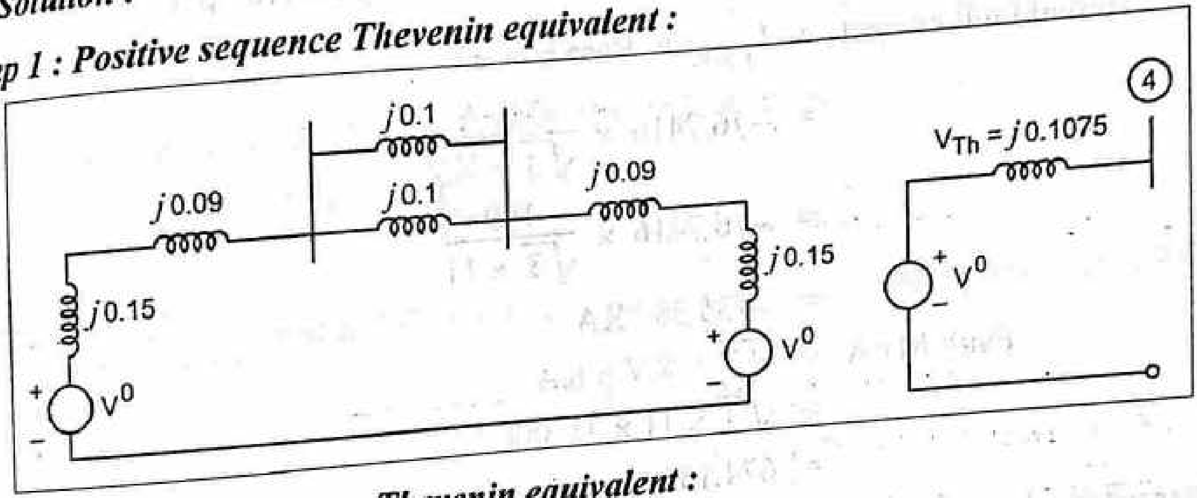
$$I_n = I_a + I_b + I_c = 42.75 \text{ KA}$$

Example 10.3(b) ***For the Example 10.3(a), compute the fault current, fault MVA as shown in Fig.***

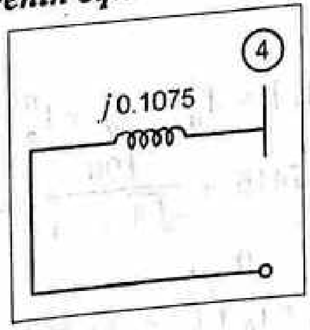


Solution :

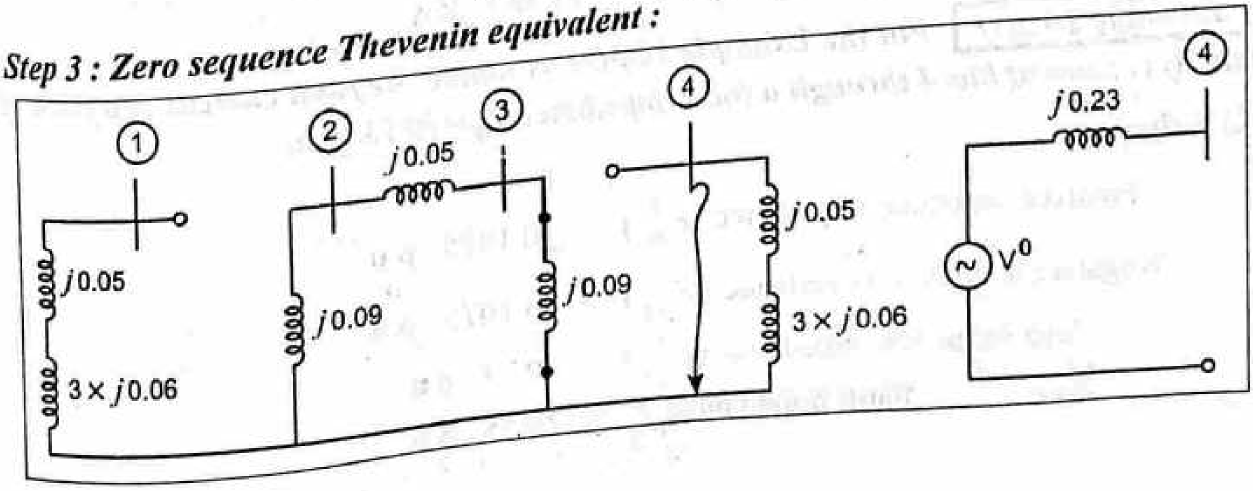
Step 1 : Positive sequence Thevenin equivalent :



Step 2 : Negative sequence Thevenin equivalent :

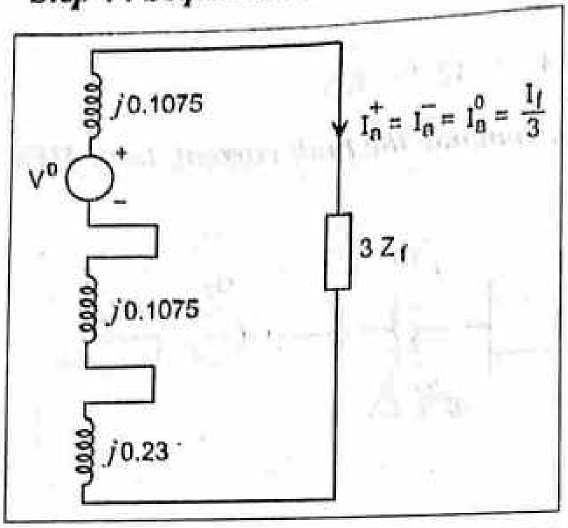


Step 3 : Zero sequence Thevenin equivalent :



Step 4 : Sequence network :

$$\text{Prefault voltage} = E_a = V^0 = 1\angle 0°$$



$$I_a^+ = I_a^- = I_a^0 = \frac{I_f}{3}$$

$$I_a^+ = \frac{V^0}{j0.1075 + j0.1075 + j0.23}$$

$$= -j2.2472 \quad \text{p.u.}$$

$$I_f = 3 \times I_a^+$$

$$= 3 \times -j2.2472$$

$$= -j6.7416 \quad \text{p.u.}$$

$$\text{Actual fault current} = I_{f\text{ p.u.}} \times \text{Base current}$$

$$= -j6.7416 \times \frac{\text{MVA}}{\sqrt{3} \times \text{KV}}$$

$$= -j6.7416 \times \frac{100}{\sqrt{3} \times 11}$$

$$= -j35.38 \quad \text{KA}$$

$$\text{Fault MVA} = \sqrt{3} \times \text{KV} \times \text{KA}$$

$$= \sqrt{3} \times 11 \times 35.384$$

$$= 674.156$$

Current in phase domain :

$$|I_a| = |I_f| = I_a^+ + I_a^- + I_a^0 = 6.7416 \quad \text{p.u.}$$

$$= 6.7416 \times \frac{100}{\sqrt{3} \times 11} = 35.384 \quad \text{KA}$$

$$I_b = I_c = 0$$

$$I_n = I_a + I_b + I_c = 35.384 \quad \text{KA}$$

Example 10.3(c) ***For the Example 10.3(b), compute the fault current and fault MVA for the L-G fault at bus 4 through a fault impedance $Z_f = j0.15$ p.u.***

☺ ***Solution :***

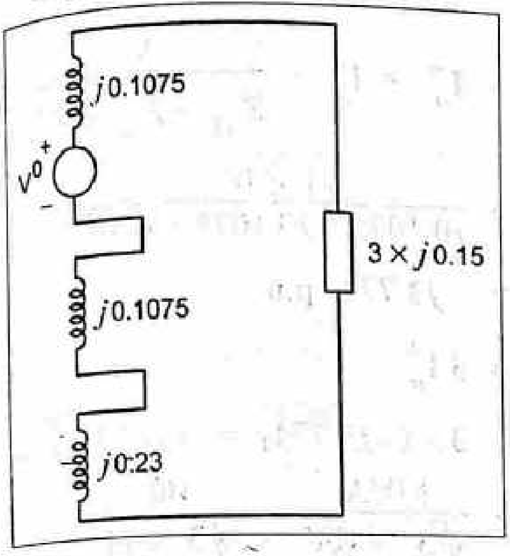
$$\text{Positive sequence impedance } (Z_{44}^+) = j0.1075 \quad \text{p.u.}$$

$$\text{Negative sequence impedance } (Z_{44}^-) = j0.1075 \quad \text{p.u.}$$

$$\text{Zero sequence impedance } (Z_{44}^0) = j0.23 \quad \text{p.u.}$$

$$\text{Fault impedance } Z_f = j0.15 \quad \text{p.u.}$$

Sequence network is



$$\text{Prefault voltage} = E_a = V^0 = 1\angle 0°$$

$$I_a^+ = I_a^- = I_a^0$$

$$= \frac{V^0}{Z_{44}^+ + Z_{44}^- + Z_{44}^0 + 3\times Z_f}$$

$$= \frac{1\angle 0°}{j0.1075 + j0.1075 + 3\times j0.15}$$

$$= -j1.5038 \text{ p.u.}$$

$$\text{Fault current } I_f = 3\, I_a^+$$

$$I_f = 3\times -j1.5038 = -j4.511 \text{ p.u.}$$

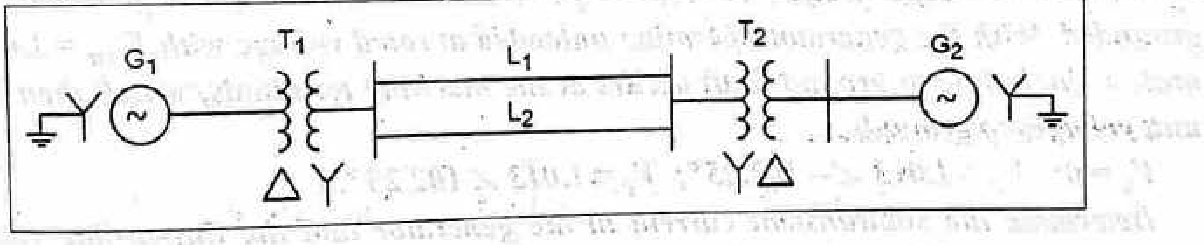
$$\text{Base current} = \frac{\text{MVA}}{\sqrt{3}\times \text{KV}} = \frac{100}{\sqrt{3}\times 11} = 5.248 \text{ KA}$$

$$\text{Fault current in amp} = 4.511\times 5.248 = 23.677 \text{ KA}$$

$$\text{Fault MVA} = \sqrt{3}\times \text{KA}\times \text{KV}$$

$$= \sqrt{3}\times 23.677\times 11 = 451.1$$

Example 10.3(d) ***For the Example 10.3(a), compute the fault current, fault MVA as shown in Fig.***

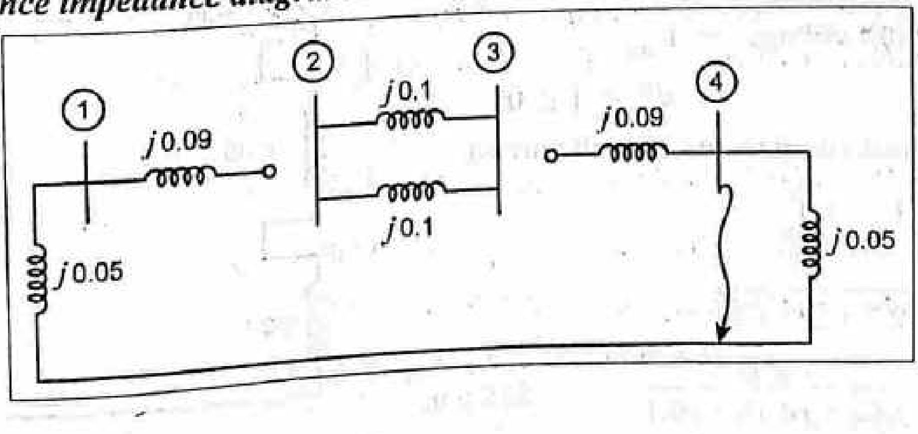


☺ ***Solution :***

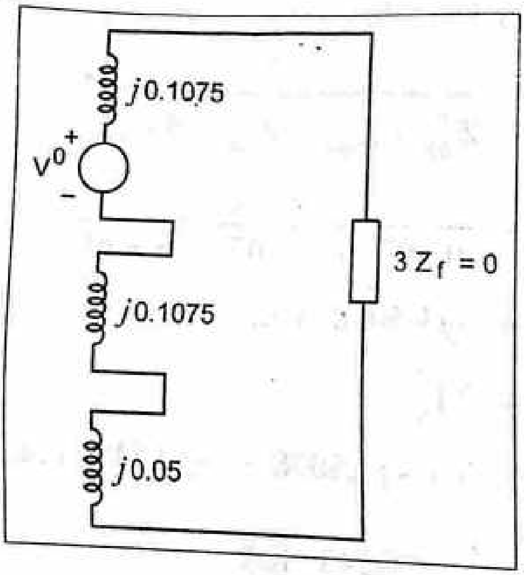
Positive sequence impedance = $j0.1075$

Negative sequence impedance = $j0.1075$

Zero sequence impedance diagram :



Sequence Network :



$$\text{Prefault voltage} = E_a = V^0 = 1\angle 0°$$

$$I_a^+ = I_a^- = I_a^0 = \frac{V^0}{Z_{44}^+ + Z_{44}^- + Z_{44}^0}$$

$$= \frac{1\angle 0°}{j0.1075 + j0.1075 + j0.05}$$

$$= -j3.774 \text{ p.u.}$$

$$\text{Fault current } I_f = 3\, I_a^+$$

$$= 3 \times (-j3.774) = -j11.32 \text{ p.u.}$$

$$\text{Base current} = \frac{\text{MVA}}{\sqrt{3} \times \text{KV}} = \frac{100}{\sqrt{3} \times 11}$$

$$= 5.2486 \text{ KA}$$

$$\text{Fault current in Amp} = 11.32 \times 5.2486 = 59.414 \text{ KA}$$

$$\text{Fault MVA} = \sqrt{3} \times \text{KV} \times \text{KA}$$

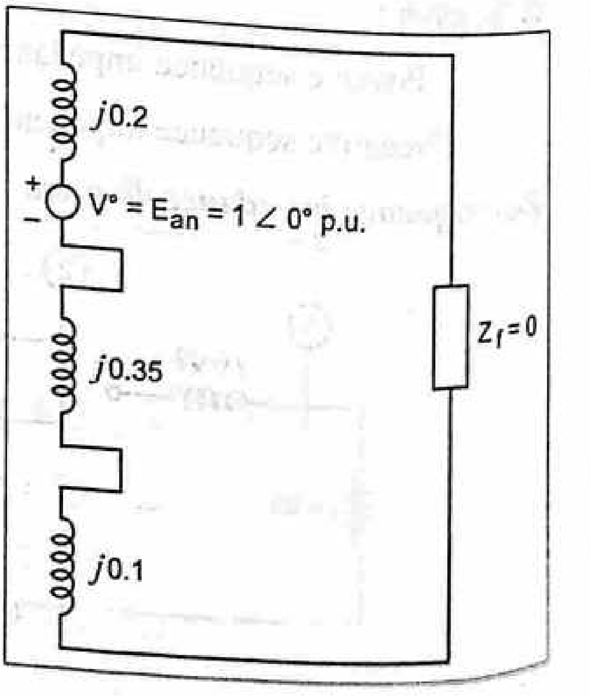
$$= \sqrt{3} \times 11 \times 59.414 = 1132$$

Example 10.4 *A salient pole generator without dampers is rated 20 MVA, 13.6 kV and has direct axis sub-transient reactance of 0.2 per unit. The negative and zero sequence reactances are, respectively, 0.35 and 0.1 per unit. The neutral of the generator is solidly grounded. With the generator operating unloaded at rated voltage with* $E_{an} = 1.0\angle 0°$ *per unit, a single line-to-ground fault occurs at the machine terminals, which then have per-unit voltages to ground,*

$$V_a = 0; \quad V_b = 1.013\angle -102.25°; \quad V_c = 1.013\angle 102.25°.$$

Determine the subtransient current in the generator and the line-to-line voltages for subtransient conditions due to the fault.

☺ **Solution:**



$$Z^+ = j0.2$$

$$Z^- = j0.35$$

$$Z^0 = j0.1$$

$$\text{Prefault voltage} = E_{an}$$

$$= V° = 1\angle 0°$$

Symmetrical components of fault current

$$I_a^+ = I_a^- = I_a^0$$

$$= \frac{V^0}{Z^+ + Z^- + Z^0}$$

$$= \frac{1\angle 0°}{j0.2 + j0.35 + j0.1} = -j1.538 \text{ p.u.}$$

$$\text{Fault current in p.u.} = 3\,I_a^+$$
$$= 3 \times -j1.538 = -j4.615 \text{ p.u.}$$
$$\text{Base current} = \frac{MVA_b \times 10^3}{\sqrt{3} \times KV_b}$$
$$= \frac{20 \times 10^3}{\sqrt{3} \times 13.6} = 849 \text{ Amp}$$
$$\text{Fault current in Amp} = -j4.615 \times 849 = 3918.14 \text{ Amp}$$

Subtransient current:

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_a^0 \\ I_a^+ \\ I_a^- \end{bmatrix}$$

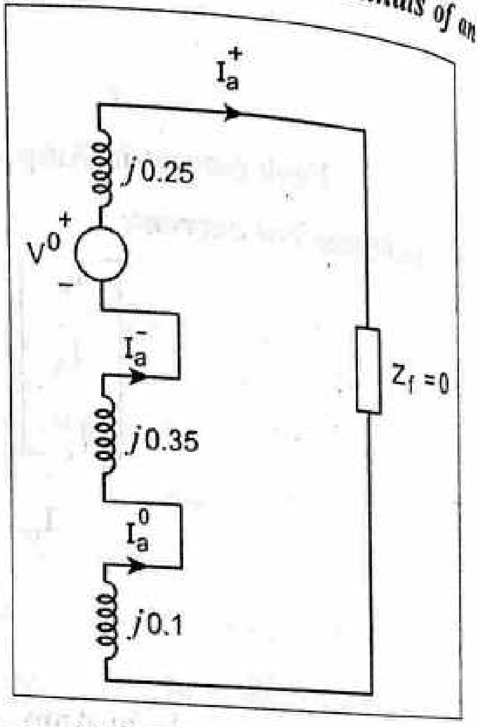
$$I_a = I_a^0 + I_a^+ + I_a^-$$
$$= -j1.538 + (-j1.538) + (-j1.538)$$
$$= -j4.615 \text{ p.u.}$$
$$I_a \text{ in Amp} = -j4.615 \times 849$$
$$= 3918.14 \text{ Amp}$$
$$I_b = I_a^0 + a^2 I_a^+ + a I_a^- = 0 \text{ p.u.}$$
$$I_c = I_a^0 + a I_a^+ + a^2 I_a^- = 0 \text{ p.u.}$$
$$I_b = I_c = 0 \text{ Amp}$$

Line-to-Line Voltage:

$$V_a = 0$$
$$V_b = 1.013 \angle -102.25°$$
$$V_c = 1.013 \angle 102.25°$$
$$V_{ab} = V_a - V_b$$
$$= 0 - [1.013 \angle -102.25°]$$
$$= 0.215 + j0.9899 \text{ p.u.}$$
$$V_{bc} = V_b - V_c$$
$$= 1.013 \angle -102.25° - 1.013 \angle 102.25° = -j1.979$$
$$V_{ac} = V_a - V_C = 0 - [1.013 \angle 102.25°]$$
$$= 0.215 - j0.989 \text{ p.u.}$$

Example 10.5 *A salient pole generator without dampers is rated 25 MVA, 13.2 KV and has a direct axis subtransient reactance of 0.25 p.u. The negative and zero sequence reactances are 0.35 and 0.1 p.u. respectively. The neutral of the generator is solidly grounded. Determine the subtransient current in the generator and line to line voltage for a subtransient conditions when a single line to ground fault occurs at the terminals of an unloaded generator.*

☺ ***Solution :***



$$Z^+ = j0.25 \text{ p.u.}$$
$$Z^- = j0.35 \text{ p.u.}$$
$$Z^0 = j0.1 \text{ p.u.}$$
$$\text{Prefault voltage} = E_a = V^0 = 1 \angle 0°$$

Symmetrical components of fault current

$$I_a^+ = I_a^- = I_a^0$$
$$= \frac{V^0}{Z^+ + Z^- + Z^0}$$
$$= \frac{1 \angle 0°}{j0.25 + j0.35 + j0.1}$$
$$= -j1.4286 \text{ p.u.}$$

$$\text{Fault current in p.u.} = 3\,I_a^+ = 3 \times -j1.4286 = -j4.2857 \text{ p.u.}$$

$$\text{Base current} = \frac{\text{MVA}_b \times 10^3}{\sqrt{3} \times \text{KV}_b} = \frac{25 \times 10^3}{\sqrt{3} \times 13.2} = 1093.466 \text{ Amp}$$

$$\text{Fault current in Amp} = -j4.2857 \times 1093.466 = 4686.28 \text{ Amp}$$

Subtransient current or phase current :

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_a^0 \\ I_a^+ \\ I_a^- \end{bmatrix}$$

$$I_a = I_a^0 + I_a^+ + I_a^- = -j1.4286 + (-j1.4286) + (-j1.4286)$$
$$= -j4.2858 \text{ p.u.}$$
$$I_a \text{ in Amp} = -j4.2858 \times 1093.466 = -j4686.376 \text{ Amp}$$

$$I_b = I_a^0 + a^2 I_a^+ + a\, I_a^-$$
$$= -j1.4286 + (-0.5 - j0.866) \times -j1.4286 + (-0.5 + j0.866)(-j1.4286) = 0 \text{ p.u.}$$
$$I_c = I_a^0 + a\, I_a^+ + a^2 I_a^- = 0 \text{ p.u.}$$
$$I_b = I_c = 0 \text{ Amp}$$

Symmetrical component of bus voltages for phase a :

$$V_a^0 = -Z^0 I_a^+ = -j0.1 \times -j1.4286 = -0.1429 \text{ p.u.}$$

$$V_a^+ = V^0 - Z^+ I_a^+ = 1\angle 0° - j0.25 \times (-j1.4286) = 0.6429 \text{ p.u.}$$

$$V_a^- = -Z^- I_a^+ = -j0.35 \times (-j1.4286) = -0.5 \text{ p.u.}$$

Subtransient phase voltages :

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_a^0 \\ V_a^+ \\ V_a^- \end{bmatrix}$$

$$V_a = V_a^0 + V_a^+ + V_a^- = -0.1429 + 0.6429 + (-0.5) = 0$$

$$V_b = V_a^0 + a^2 V_a^+ + a V_a^-$$

$$= -0.1429 + (-0.5 - j0.866) \times 0.6429 + (-0.5 + j0.866) \times -0.5$$

$$= -0.2144 - j0.9898$$

$$V_c = V_a^0 + a V_a^+ + a^2 V_a^- = -0.2144 + j0.9898$$

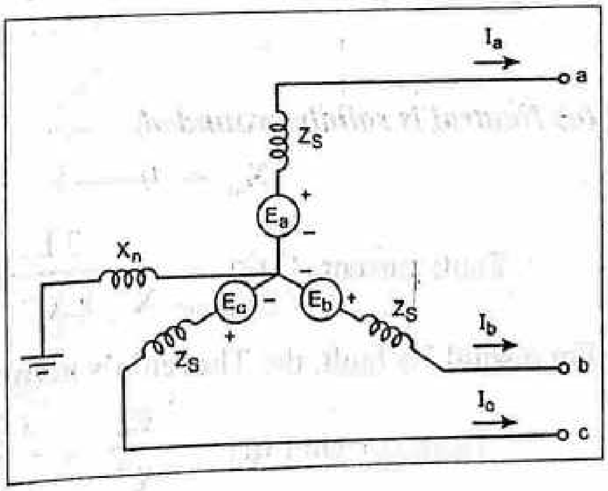
Line-to-Line voltage :

$$V_{ab} = V_a - V_b = 0 - [-0.2144 - j0.9898] = 0.2144 + j0.9898 \text{ p.u.}$$

$$V_{bc} = V_b - V_c = -0.2144 - j0.9898 - [-0.2144 + j0.9898] = -j1.9796 \text{ p.u.}$$

$$V_{ac} = V_a - V_c = 0 - [-0.2144 + j0.9898] = -0.2144 - j0.9898 \text{ p.u.}$$

Example 10.6 ***Fig. shows a synchro-ous generator whose neutral is grounded rough a reactance X_n. The generator has lanced emfs and sequence reactances X^+, X^- and X^0 such that $X^+ = X^- \geq X^0$.***



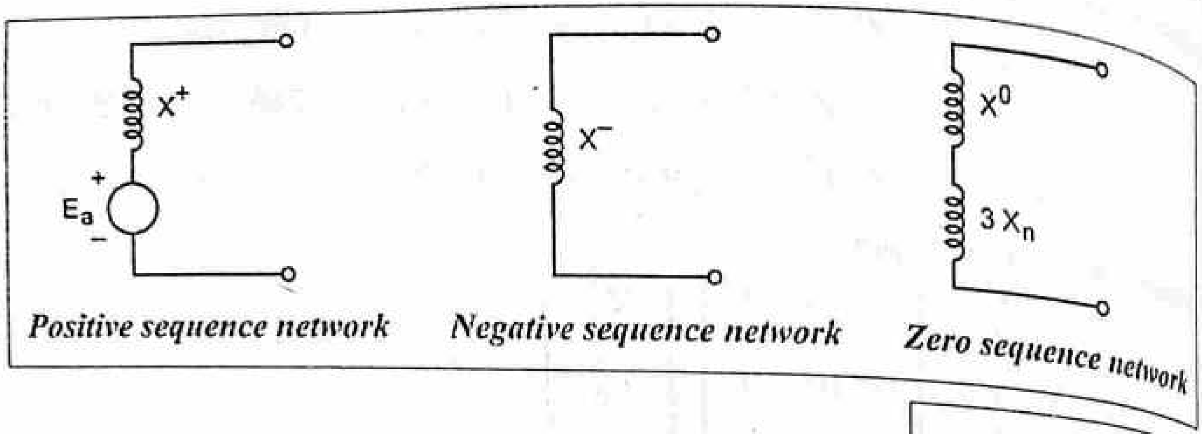
(a) Draw the sequence networks of the generator as seen from the terminals.

(b) Derive expression for fault current for a solid line-to-ground fault on phase a.

(c) Show that, if the neutral is grounded solidly, the LG fault current would be more than the 3ϕ fault current.

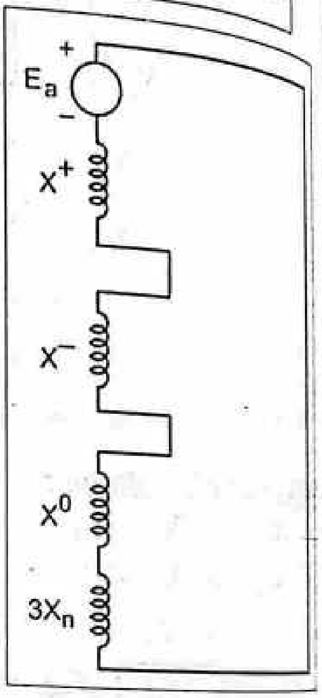
(d) Write expression for neutral grounding reactance, such that LG fault current is less than the 3ϕ fault current.

☺ ***Solution : (a) Sequence network of a generator :***



Positive sequence network *Negative sequence network* *Zero sequence network*

(b) Sequence network for a solid L-G fault :



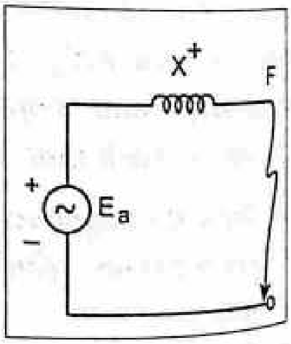
The sequence network is as shown.

$$\text{Fault current,} \quad I_f = 3 I_a^+$$

$$\therefore \quad I_a^+ = \frac{E_a}{X^+ + X^- + X^0 + 3X_n}$$

$$\therefore \quad I_f = \frac{3E_a}{X^+ + X^- + X^0 + 3X_n}$$

(c) Neutral is solidly grounded,



$$X_n = 0$$

$$\therefore \text{Fault current (L-G)} = \frac{3E_a}{X^+ + X^- + X^0}$$

For a solid 3ϕ fault, the Thevenin's network is as shown.

$$\text{Fault current } (3\phi) = \frac{E_a}{X^+} = \frac{3E_a}{3X^+} \qquad [\because 3X^+ > X^+ + X^- + X^0]$$

Fault current (3ϕ fault) < Fault current (L-G fault)

When $X_n = 0$, *i.e.*, generator neutral is solidly grounded, then line to ground fault is mor severe than a 3ϕ fault. Because

$$X^0 \le X^+, \;\; X^+ = X^- \;\text{ for generator}$$
$$X^0 \ge X^+, \;\; X^+ = X^- \;\text{ for line}$$

When a fault on a line is away from generator, then 3ϕ fault is more severe than L-G fau

(d) Generator neutral is grounded through reactance :

$$I_{f\,(L\text{-}G)} = \frac{3\,E_a}{X^+ + X^- + X^0 + 3\,X_n}$$

$$I_{f\,(3\phi)} = \frac{E_a}{X^+} = \frac{3\,E_a}{3\,X^+}$$

$$I_{f\,(L\text{-}G)} < I_{f\,(3\phi)}$$

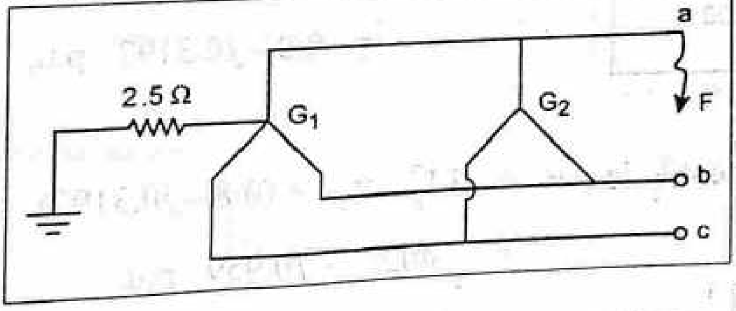
$$\frac{3\,E_a}{X^+ + X^- + X^0 + 3\,X_n} < \frac{3\,E_a}{3\,X^+} \qquad [\because 3\,X^+ < X^+ + X^- + X^0 + 3\,X_n]$$

$$2\,X^+ + X^0 + 3\,X_n > 3\,X^+$$

$$3\,X_n > 3\,X^+ - 2\,X^+ - X^0$$

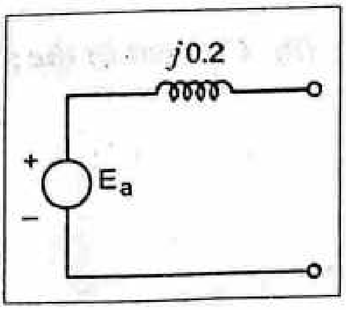
$$X_n > \frac{X^+ - X^0}{3}$$

Example 10.7 ***Two 13.2 KV, 25 MVA, three phase, star connected generators operate in parallel as shown in Fig., the positive, negative and zero sequence reactances are j0.4, j0.30 and j0.08 p.u. The star point of one of the generators is isolated and that of the other is earthed through a 2.5 ohm resistor. A single line to ground fault occurs at the terminals of one of the generators. Estimate (i) the fault current, (ii) current in the grounding resistor and (iii) the voltage across the grounding resistor.***



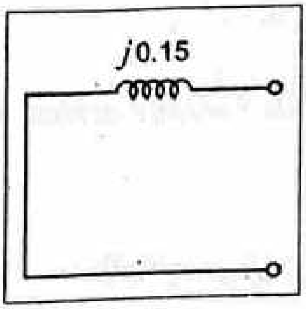
☺ ***Solution :***

Positive sequence network : (Generator G_1 & G_2 are in parallel)



$$Z^+ = \frac{j0.4 \times j0.4}{j0.4 + j0.4} = \frac{j0.4}{2} = j0.2$$

Negative sequence network :



$$Z^- = \frac{j0.3 \times j0.3}{j0.3 + j0.3} = j0.15$$

Zero sequence network :

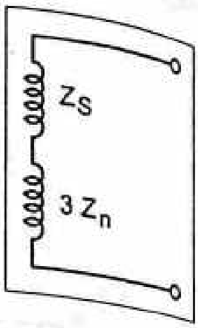
$$Z^0 = Z_S + 3\,Z_n$$

$$Z_{n\ \text{p.u.}} = \frac{2.5 \times \text{MVA}_b}{\text{KV}_b^2}$$

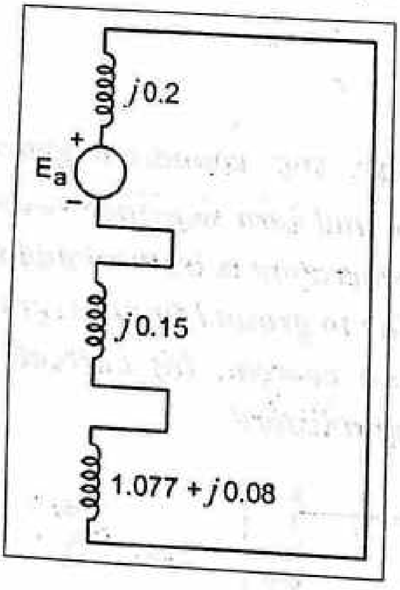
$$= \frac{2.5 \times 25}{13.2^2} = 0.359 \ \text{ p.u.}$$

$$Z^0 = j0.08 + 3 \times 0.359$$

$$= j0.08 + 1.077$$



Sequence network for L-G fault :



$$I_a^+ = I_a^- = I_a^0$$

$$= \frac{E_a}{Z^+ + Z^- + Z^0}$$

$$= \frac{1 \angle 0^\circ}{j0.2 + j0.15 + 1.077 + j0.08}$$

$$= 0.8 - j0.3197 \ \text{ p.u.}$$

(i) Fault current I_f in p.u. $= 3\,I_a^+ = 3 \times (0.8 - j0.3197)$

$$= 2.4025 - j0.959 \ \text{ p.u.}$$

(ii) Current in the grounding resistor I_r :

$$I_f = 2.4025 - j0.959 \ \text{ p.u.}$$

$$|I_f| = 2.5869 \ \text{ p.u.}$$

$$\text{Base current} = \frac{\text{MVA}_b}{\sqrt{3} \times \text{KV}_b} = \frac{25 \times 10^3}{\sqrt{3} \times 13.2} = 1093.466 \ \text{A}$$

$$|I_r| \text{ in Amp} = 2.5869 \times 1093.466 = 2828.69 \ \text{A}$$

(iii) Voltage across grounding resistor :

$$= I_r \text{ in Amp} \times 2.5\,\Omega$$

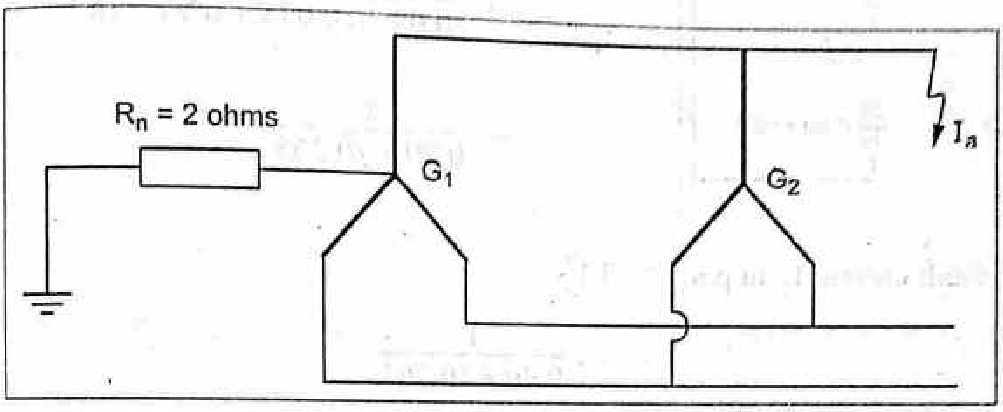
$$= 2828.69 \times 2.5 = 7.07 \ \text{ KV}$$

Example 10.8 ***Two 11 kV, 20 MVA, three phase, star connected generators operate in parallel as shown in Fig.; the positive, negative and zero sequence reactance's of each being, respectively, jo.18, j0.15, j0.10 pu. The star point of one of the generators is isolated and that of the other is earthed through a 2.0 ohms resistor. A single line to ground fault occurs at the terminals of one of the generators.***

Estimate ***(i) the fault current,***

(ii) current in grounding resistor, and

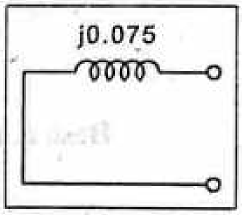
(iii) the voltage across grounding resistor.



☺ ***Solution:***

Positive sequence network : (Generator G_1 & G_2 are in parallel)

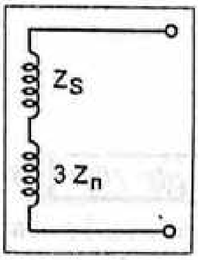
$$Z^+ = \frac{j0.18 \times j0.18}{j0.18 + j0.18} = j0.09$$



Negative sequence network :

$$Z^- = \frac{j0.15 \times j0.15}{j0.15 + j0.15} = j0.075$$

Zero sequence network :



$$Z^0 = Z_S + 3\ Z_n$$

$$Z_{n\ \text{p.u.}} = \frac{2 \times \text{MVA}_b}{\text{KV}_b^2}$$

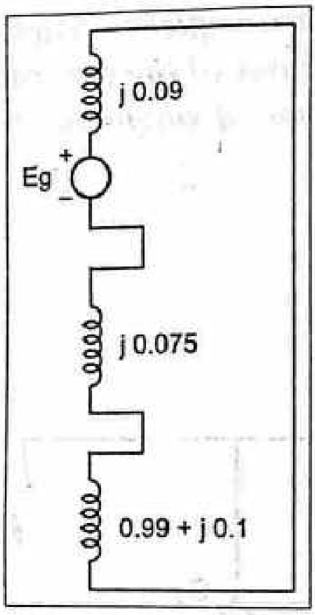
$$= 2 \times \frac{20}{11^2} = 0.33 \ \text{p.u.}$$

$$Z^0 = Z_S + 3\ Z_n$$

$$= j0.1 + 3 \times j0.33$$

$$= 0.99 + j0.1$$

Sequence network for L-G fault :



$$I_a^+ = I_a^- = I_a^0$$

$$= \frac{E_a}{Z^+ + Z^- + Z^0}$$

$$= \frac{1\angle 0°}{j0.09 + j0.075 + 0.99 + j0.1}$$

$$= \frac{1}{0.99 + j0.265}$$

(i) Fault current I_f in p.u. $= 3\,I_a^+$

$$= 3 \times \frac{1}{0.99 + j0.265}$$

$$= 2.827 - j0.756 \text{ p.u.}$$

(ii) Current in the grounding resistor I_r:

$$I_f = 2.827 - j0.756 \text{ p.u.}$$

$$|I_f| = 2.926 \text{ p.u.}$$

$$\text{Base current} = \frac{MVA_b}{\sqrt{3} \times KV_b} = \frac{20 \times 10^3}{\sqrt{3} \times 11} = 10497 \text{ A}$$

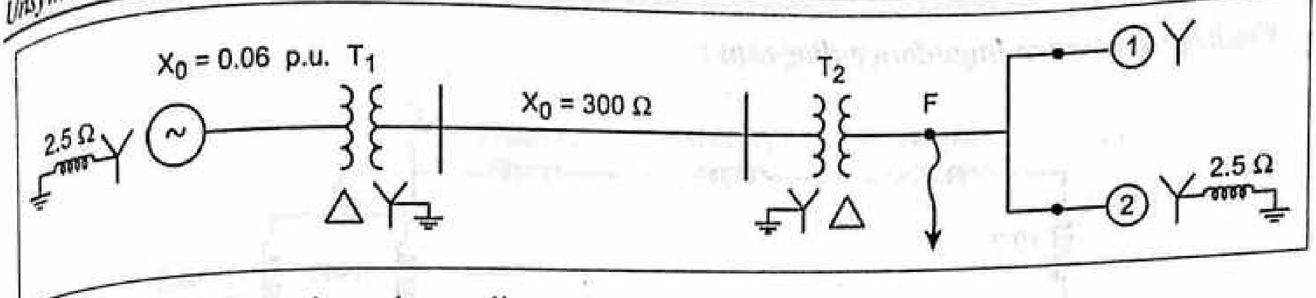
$$|I_r| \text{ in Amp} = 2.926 \times 10497 = 3.07 \text{ kA}$$

(iii) Voltage across grounding resistor:

$$= |I_r| \text{ in Amp} \times 2\,\Omega$$

$$= 3.07 \times 2 = 6.14 \text{ kV}$$

Example 10.9 *A 25 MVA and 11.0 KV generator has a subtransient reactance of 20% is connected to two motors through transmission line for zero sequence as shown in Fig. The motors are loaded to draw 15 and 7.5 MW at 10.8 KV, 0.8 power factor leading with 25% subtransient reactance. The 3ϕ transformers are rated 30 MVA, 10.8/121 KV, with leakage reactance of 10%, series reactance of transmission line is 80 Ω. If prefault current is neglected, calculate the fault current and subtransient current in all parts of the system when L-G fault occurs at fault F.*



☺ ***Solution :*** Draw impedance diagram.

$$\text{Base MVA} = 25$$

$$\text{Base KV} = 11$$

Generator :

$$Z_{p.u.} = Z_{p.u.\ old} \times \left[\frac{KV_{b\ given}}{KV_{b\ new}}\right]^2 \times \left[\frac{MVA_{b\ new}}{MVA_{b\ given}}\right]$$

$$= j0.2 \times \left[\frac{11}{11}\right]^2 \times \left[\frac{25}{25}\right]$$

$$= j0.2 \text{ p.u.}$$

Transformer 1, T_1 (sec) :

$$KV_b = 11 \times \frac{121}{10.8} = 123.24 \text{ KV}$$

$$Z_{p.u.} = j0.1 \times \left[\frac{121}{123.24}\right]^2 \times \left[\frac{25}{30}\right]$$

$$= 0.0803 \text{ p.u.}$$

Transmission line :

$$KV_b = 123.24 \text{ KV}$$

$$Z_{p.u.} = \frac{j100 \times MVA_b}{KV_b^2}$$

$$= \frac{j80}{123.24^2} \times 25 = j0.1317 \text{ p.u.}$$

Transformer 2, T_2 (prim):

$$KV_b = 123.24$$

$$Z_{p.u.} = j0.1 \times \left(\frac{121}{123.24}\right)^2 \times \frac{25}{30} = 0.0803 \text{ p.u.}$$

Motor :

$$KV_{b\ new} = 123.24 \times \frac{10.8}{121} = 11 \text{ KV}$$

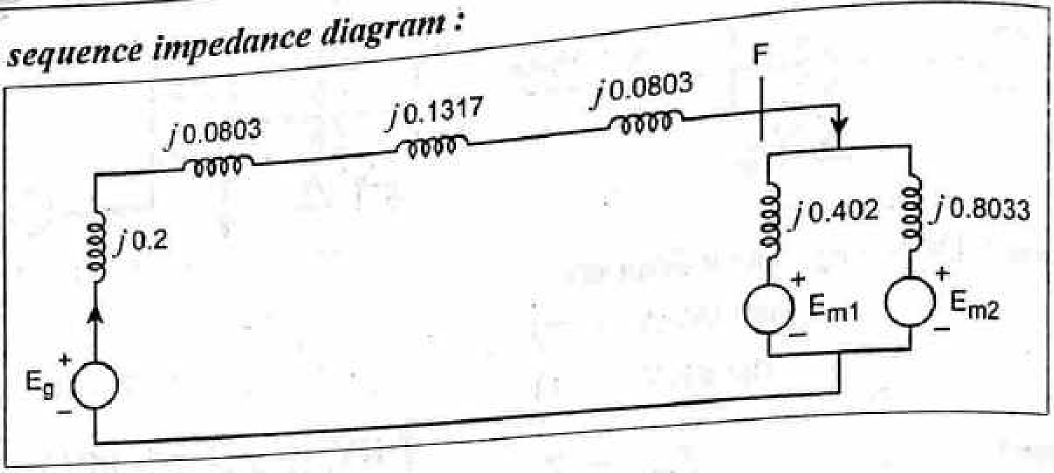
$$\text{Motor 1, } Z_{p.u.} = j0.25 \times \left[\frac{10.8}{11}\right]^2 \left[\frac{25}{15}\right]$$

$$= j0.402 \text{ p.u.}$$

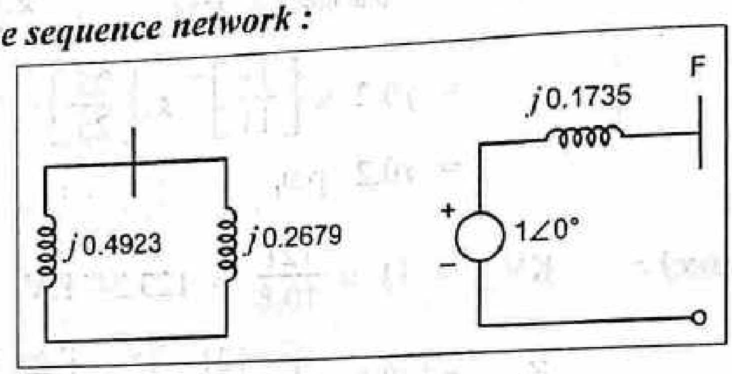
$$\text{Motor 2, } Z_{p.u.} = j0.25 \times \left[\frac{10.8}{11}\right]^2 \times \left[\frac{25}{7.5}\right]$$

$$= j0.8033 \text{ p.u.}$$

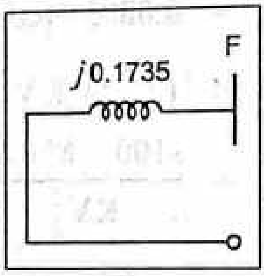
Positive sequence impedance diagram :



Thevenin's positive sequence network :



Thevenin's negative sequence network :



Zero sequence network :

Zero sequence impedance of transformer = Positive sequence reactance of transformer

$$= j0.0803 \text{ p.u.}$$

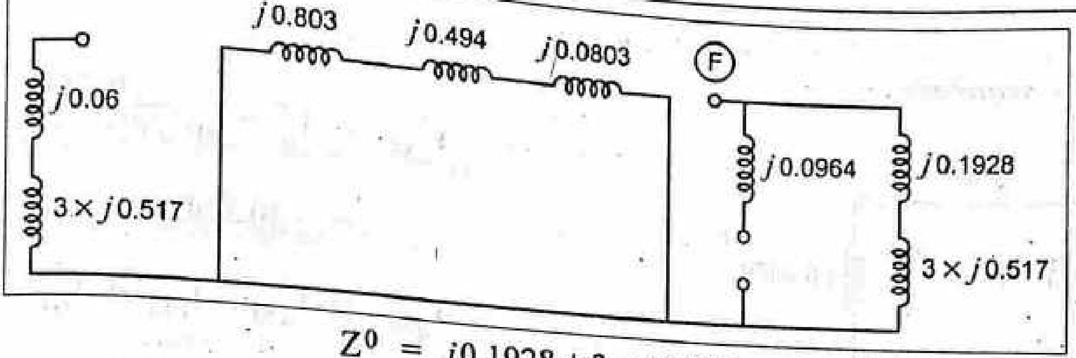
$$\text{Generator zero sequence impedance} = j0.06 \times \left[\frac{11}{11}\right]^2 \times \left[\frac{25}{25}\right] = j0.06 \text{ p.u.}$$

$$\text{Zero sequence impedance of line} = \frac{300 \times 25}{123.24^2} = j0.494 \text{ p.u.}$$

$$\text{Zero sequence impedance of motor 1} = j0.06 \times \left(\frac{10.8}{11}\right)^2 \times \left(\frac{25}{15}\right) = j0.0964 \text{ p.u.}$$

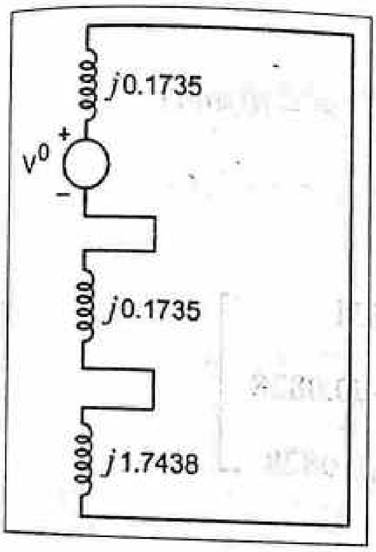
$$\text{Zero sequence impedance of motor 2} = j0.06 \times \left(\frac{10.8}{11}\right)^2 \left(\frac{25}{7.5}\right) = j0.1928 \text{ p.u.}$$

$$\text{Impedance of current limiting reactors } (jX_n) = \frac{j2.5 \times 25}{11^2} = j0.517 \text{ p.u.}$$



$$Z^0 = j0.1928 + 3 \times j0.517 = j1.7438 \text{ p.u.}$$

Sequence network :



$$\text{Prefault voltage } E_a = V^0 = \frac{\text{Actual value}}{\text{Base value}}$$

$$= \frac{10.8}{11} = 0.982 \text{ p.u.}$$

$$I_a^+ = I_a^- = I_a^0$$

$$= \frac{V^0}{Z^+ + Z^- + Z^0}$$

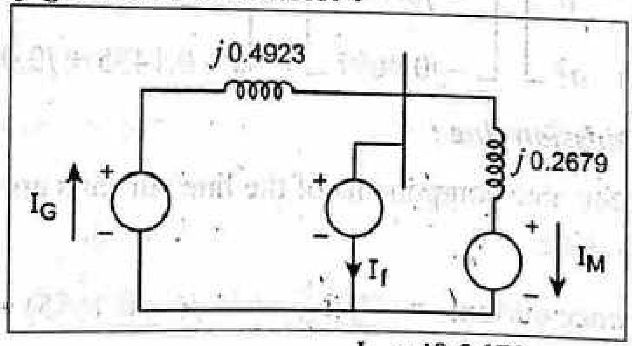
$$= \frac{0.982}{j0.1735 + j0.1735 + j1.7438}$$

$$= -j0.4697 \text{ p.u.}$$

$$\text{Fault current } I_f = 3 \times I_a^+ = 3 \times (-j0.4697)$$

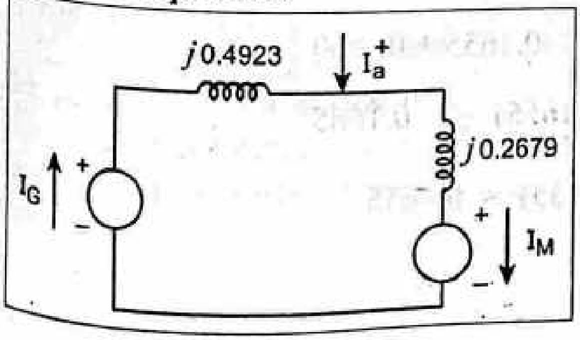
$$= -j1.409 \text{ p.u.}$$

Current contributed by generator and motor :



$$\text{Current contributed by generator} = \frac{I_f \times j0.2679}{j0.4923 + j0.2697}$$

Positive sequence :

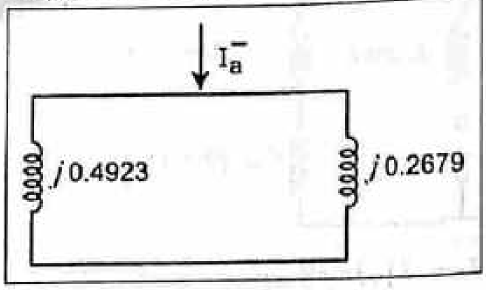


$$\left.\begin{array}{r}\text{Positive sequence}\\ \text{fault current}\end{array}\right\} I_{aG}^+ = \frac{I_a^+ \times j0.2679}{j0.4923 + j0.2679}$$

$$I_{aG}^+ = -j0.4697 \times 0.3524$$

$$= -j0.1655 \text{ p.u.}$$

Negative sequence :

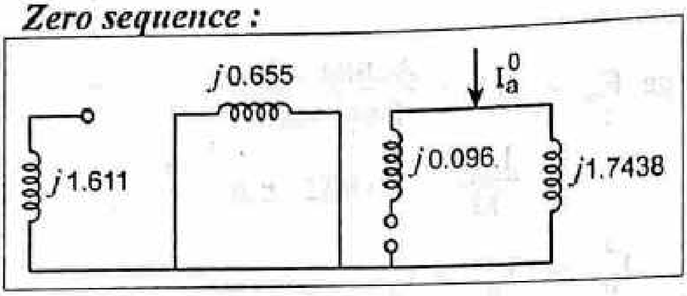


$$I_{aM}^{+} = I_a^{+} \times \frac{j0.4923}{j0.4923 + j0.2679}$$

$$= -j0.304 \text{ p.u.}$$

$$I_{aM}^{-} = I_{aM}^{+}, \quad I_{aG}^{-} = I_{aG}^{+}$$

Zero sequence :



$$I_{aG}^{0} = 0$$

$$I_{aM}^{0} = I_a^{0} = -j0.4697$$

Phase Fault current from the generator to the fault :

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} -j0.1655 \\ -j0.1655 \\ 0 \end{bmatrix} = \begin{bmatrix} -j0.331 \\ -0.1433 - j0.0828 \\ 0.1433 - j0.0828 \end{bmatrix}$$

Phase fault current from motor side to the fault :

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} -j0.304 \\ -j0.304 \\ -j0.4697 \end{bmatrix} = \begin{bmatrix} -j1.078 \\ 0.1435 + j0.0828 \\ -0.1435 + j0.0828 \end{bmatrix}$$

Current through transmission line :

Positive and negative sequence components of the line currents are shifted – 90° and + 90° respectively from generator side.

$$\text{Positive sequence current} = -j\,I_{aG}^{+} = -j(-j0.1655) = -0.1655 \text{ p.u.}$$

$$\text{Negative sequence current} = j \cdot I_{aG}^{+} = j(-j0.1655) = 0.1655 \text{ p.u.}$$

$$\text{Zero sequence current} = 0$$

$$\text{Current through line 'a'} = -0.1655 + 0.1655 + 0 = 0$$

$$I_{b\,\text{line}}^{+} = -j(-j0.1655) = -0.1655$$

$$I_{b\,\text{line}}^{-} = j(-j0.1655) = 0.1655$$

$$I_{b\,\text{line}}^{0} = 0$$

$$I_{b\,\text{line}} = -1655 + 0.1655 + 0 = 0$$

$$I^{+}_{c\,\text{line}} = I^{-}_{c\,\text{line}} = I^{0}_{c\,\text{line}} = 0$$

Voltage behind subtransient reactance :

$$\text{Prefault voltage in p.u.} = \frac{10.8}{11} = 0.982 \ \text{p.u.}$$

$$\text{Motor 1 current in p.u. } (I''_{M1}) = \frac{MVA_{(p.u.)}}{KV_{(p.u.)}} = \frac{15 \angle 36.86^\circ}{25 \times 0.8 \times 0.982}$$

$$= 0.764 \angle 36.86^\circ \ \text{p.u.}$$

$$\text{Motor 2 current in p.u. } (I''_{M2}) = \frac{7.5}{25 \times 0.8 \times 0.982} \angle 36.86^\circ = 0.382 \angle 36.86^\circ$$

$$\text{Total current drawn by the motors} = I''_{M1} + I''_{M2}$$

$$I''_{g} = 0.917 + j0.687 \ \text{p.u.} = 1.146 \angle 36.86^\circ$$

$$E''_{M1} = V^{0} - j0.402 \times I''_{M1} = 0.982 - j0.402 \times 0.764 \angle 36.86^\circ$$

$$= 1.166 - j0.2456 \ \text{p.u.}$$

$$E''_{M2} = V^{0} - j0.8033 \times I''_{M2} = 0.982 - j0.8033 \times 0.382 \angle 36.86^\circ$$

$$= 1.166 - j0.2456 \ \text{p.u.}$$

$$E''_{g} = V^{0} + j0.4923 \times I''_{g} = 0.982 + j0.4923 \times 1.146 \angle 36.86^\circ$$

$$= 0.644 + j0.4514 \ \text{p.u.}$$

Actual value of positive sequence fault current from generator and motor.

Generator :

$$I_{Gf} = I''_{g} + I^{+}_{aG} = 0.917 + j0.687 + (-j0.1655)$$

$$= 0.917 + j0.5215 \ \text{p.u.}$$

$$I_{Mf} = -I''_{g} + I^{+}_{aM} = -0.917 - j0.687 + (-j0.304)$$

$$= -0.917 - j0.991$$

Example 10.10 ***A 30 MVA, 11 kV generator has $Z_1 = Z_2 = j0.2$ p.u. $Z_0 = j0.05$ p.u. A line to ground fault occurs on the generator terminals. Find the fault current and line to line voltages during fault conditions. Assume that the generator neutral is solidly grounded and that the generator is operating at no-load and at rated voltage at the occurrence of fault.***

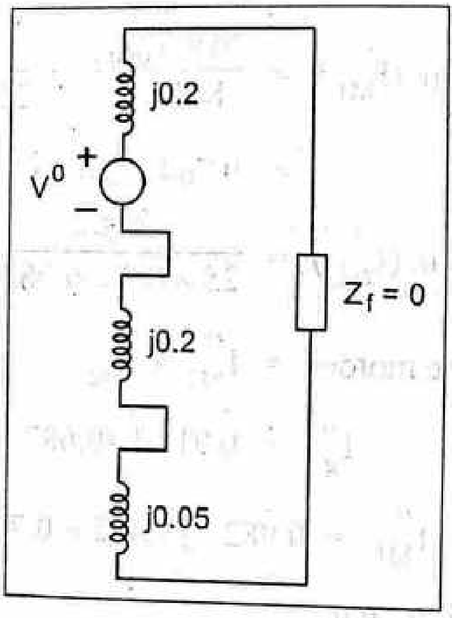
☺ ***Solution:***

$$Z^+ = j0.2 \quad \text{p.u}$$

$$Z^- = j0.2 \quad \text{p.u}$$

$$Z^0 = j0.05 \quad \text{p.u}$$

$$\text{Prefault voltage, } V^0 = 1 \angle 0°$$



Symmetrical components of fault current

$$I_a^+ = I_a^- = I_a^0 = \frac{V^0}{Z^+ + Z^- + Z^0}$$

$$= \frac{1 \angle 0°}{j0.2 + j0.2 + j0.05}$$

$$= -j2.222 \quad \text{p.u}$$

$$\text{Fault current in p.u} = 3 \ I_a^+$$

$$= 3 \times -j2.222$$

$$= -j6.666 \quad \text{p.u}$$

$$\text{Base current} = \frac{\text{MVA}_b \times 10^3}{\sqrt{3} \times \text{KV}_b} = \frac{30 \times 10^3}{\sqrt{3} \times 11}$$

$$= 1574.6 \quad \text{Amp}$$

$$\text{Fault current in Amp} = -j6.666 \times 1574.6$$

$$= 10496.3 \quad \text{Amp}$$

Line to Line Voltages during the fault:

$$\begin{aligned} V_a^0 &= -Z^0\, I_a^+ \\ &= -j0.05 \times -j2.222 = -0.1111 \\ V_a^+ &= V^0 - Z^+ I_a^+ \\ &= 1\angle 0^\circ - j0.2 \times -j2.222 \\ &= 0.5555 \\ V_a^- &= -Z^-\, I_a^+ \\ &= -j0.2 \times -j2.222 \\ &= -0.4444 \end{aligned}$$

Subtransient phase voltages

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_a^0 \\ V_a^+ \\ V_a^- \end{bmatrix}$$

$$\begin{aligned} V_a &= V_a^0 + V_a^+ + V_a^- \\ &= -0.1111 + 0.5555 - 0.4444 = 0 \\ V_b &= V_a^0 + a^2\, V_a^+ + a\, V_a^- \\ &= -0.1111 + 1\angle -240^\circ \times 0.5555 + 1\angle -120^\circ \times -0.4444 \\ &= -0.29 - j.0267 \\ V_c &= V_a^0 + a\, V_a^+ + a^2\, V_a^- \\ &= -0.29 + j0.267 \end{aligned}$$

Line to Line Voltages

$$\begin{aligned} V_{ab} = V_a - V_b &= 0 - [-0.29 - j0.267] \\ &= 0.29 + j0.267 \ \text{p.u} \\ V_{bc} = V_b - V_c &= -0.29 - j0.267 - [-0.29 + j0.267] \\ &= -j0.534 \ \text{p.u} \\ V_{ac} = V_a - V_c &= 0 - [-0.29 + j0.267] \\ &= 0.29 - j0.267 \end{aligned}$$

10.4. LINE TO LINE FAULT

Consider a three phase generator with a fault through an impedance Z_f between phases b and c shown in Fig.10.5. Assume the generator is unloaded (no load), the conditions at the fault bus K are expressed by the following relations.

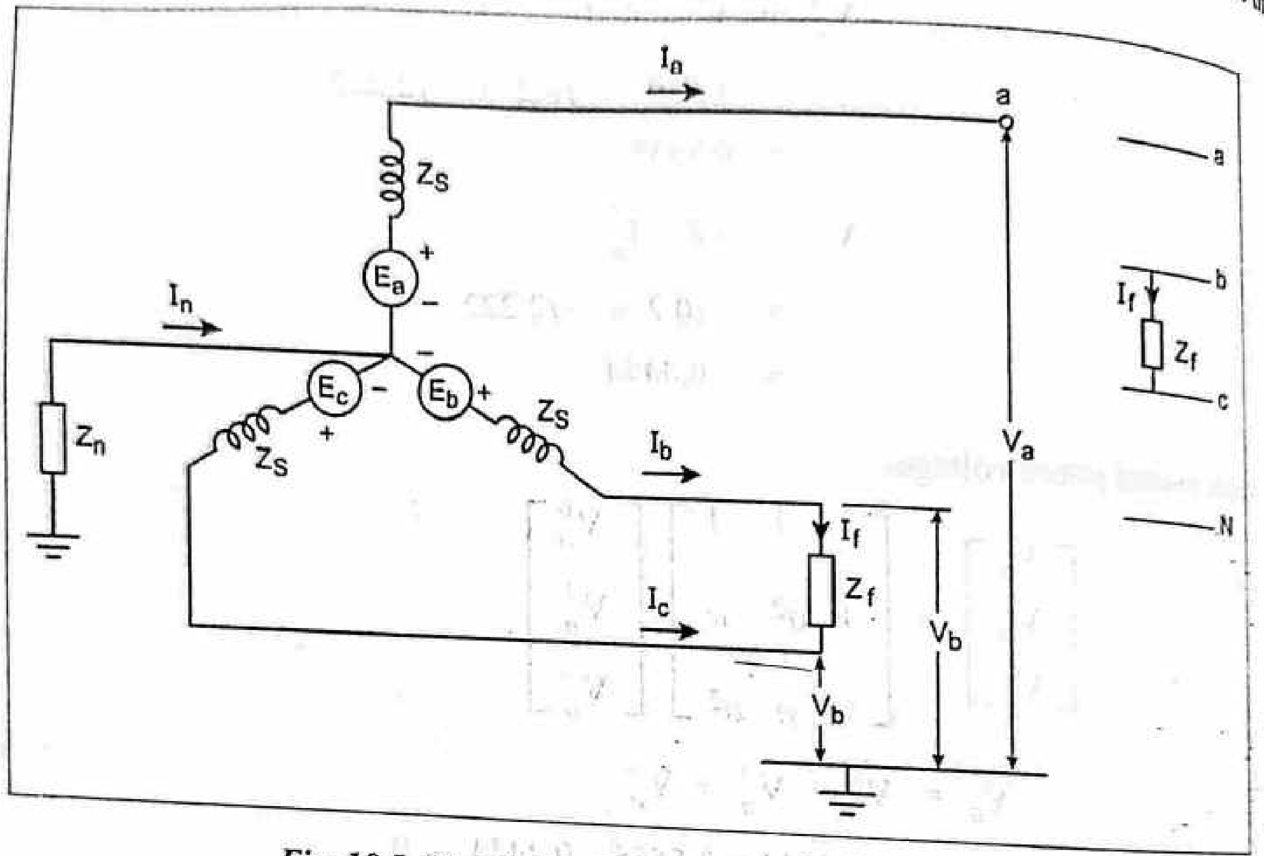


Fig. 10.5. Line to line fault between phases b and c

$$\left.\begin{aligned} I_b &= -I_c \\ I_a &= 0 \text{ (unloaded generator)} \\ V_b - V_c &= Z_f I_b \Rightarrow V_c = V_b - Z_f I_b \end{aligned}\right\} \quad \ldots (10.20)$$

Substitute for $I_b = -I_c$, $I_a = 0$, the symmetrical components of currents are :

$$\begin{bmatrix} I_a^0 \\ I_a^+ \\ I_a^- \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

$$\begin{bmatrix} I_a^0 \\ I_a^+ \\ I_a^- \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} 0 \\ I_b \\ -I_b \end{bmatrix}$$

$$I_a^0 = \frac{1}{3}[0 + I_b - I_b] = 0$$

$$I_a^+ = \frac{1}{3}[a\, I_b - a^2\, I_b]$$

$$I_a^- = \frac{1}{3}[a^2 I_b - a I_b]$$

$$\therefore \boxed{I_a^+ = -I_a^- \text{ and } I_a^0 = 0} \qquad \dots (10.21)$$

From sequence networks of the generator, the symmetrical voltages are given by

$$\begin{bmatrix} V_a^0 \\ V_a^+ \\ V_a^- \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z_{KK}^0 & 0 & 0 \\ 0 & Z_{KK}^+ & 0 \\ 0 & 0 & Z_{KK}^- \end{bmatrix} \begin{bmatrix} I_a^0 \\ I_a^+ \\ -I_a^+ \end{bmatrix}$$

$$\left.\begin{aligned} V_a^0 &= -Z_{KK}^0\, I_a^0 = -Z_{KK}^0 \times 0 = 0 \\ V_a^+ &= E_a - Z_{KK}^+\, I_a^+ \\ V_a^- &= -Z_{KK}^-\, I_a^- = Z_{KK}^-\, I_a^+ \end{aligned}\right\} \qquad \dots (10.22)$$

The phase currents are given by

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_a^0 \\ I_a^+ \\ I_a^- \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ I_a^+ \\ -I_a^+ \end{bmatrix}$$

$$\left.\begin{aligned} I_a = 0,\ I_b &= a^2 I_a^+ - a\, I_a^+ = I_a^+ (a^2 - a) \\ I_c &= a\, I_a^+ - a^2 I_a^+ = I_a^+ (a - a^2) = -I_b \end{aligned}\right\} \qquad \dots (10.23)$$

The voltages throughout the zero sequence network must be zero since there are no zero sequence sources and because $I_a^0 = 0$, current is not being injected into that network due to the fault. Hence LL fault calculation do not involve zero sequence network.

The phase voltages are

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ V_a^+ \\ V_a^- \end{bmatrix}$$

$$V_a^0 = 0$$

$$\left.\begin{aligned} V_a &= V_a^+ + V_a^- \\ V_b &= a^2 V_a^+ + a\, V_a^- \\ V_c &= a\, V_a^+ + a^2 V_a^- \end{aligned}\right\} \qquad \dots (10.24)$$

From the condition $V_b - V_c = Z_f I_b$

Substituting V_b and V_c from equation (10.24), we get,

$$a^2 V_a^+ + a V_a^- - a V_a^+ - a^2 V_a^- = Z_f I_b$$

$$V_a^+ (a^2 - a) - V_a^- [a^2 - a] = Z_f I_b$$

$$(a^2 - a) [V_a^+ - V_a^-] = Z_f I_b \qquad \dots (10.25)$$

Substitute the value of I_b from equation (10.23), we get,

$$(a^2 - a)(V_a^+ - V_a^-) = (a^2 - a) I_a^+ Z_f$$

$$V_a^+ - V_a^- = I_a^+ Z_f \qquad \dots (10.26)$$

Substitute V_a^+, V_a^- from equation (10.22), we get,

$$E_a - Z_{KK}^+ I_a^+ - [-Z_{KK}^- I_a^-] = I_a^+ Z_f$$

$$E_a - (Z_{KK}^+ + Z_{KK}^-) I_a^+ = I_a^+ Z_f \qquad [i.e.,\ I_a^+ = -I_a^-]$$

$$E_a = [Z_{KK}^+ + Z_{KK}^- + Z_f] I_a^+$$

$$I_a^+ = \frac{E_a}{Z_{KK}^+ + Z_{KK}^- + Z_f} \qquad \dots (10.27)$$

$$I_a^- = -I_a^+$$

$$I_a^0 = 0$$

Current in phase domain

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_a^0 \\ I_a^+ \\ I_a^- \end{bmatrix} = \begin{bmatrix} 0 + I_a^+ - I_a^+ \\ 0 + (a^2 - a) I_a^+ \\ 0 + (a - a^2) I_a^+ \end{bmatrix} = \begin{bmatrix} 0 \\ (a^2 - a) I_a^+ \\ -(a^2 - a) I_a^+ \end{bmatrix}$$

The fault current is $I_b = -I_c = (a^2 - a) I_a^+$

$$= (-0.5 - j0.866 + 0.5 - j0.866) I_a^+ = -j1.732 I_a^+$$

$$= -j\sqrt{3}\, I_a^+$$

Substituting I_a^+ from equation (10.27), we get

$$\boxed{I_f = I_b = \frac{-j\sqrt{3}\ E_a}{Z_{KK}^+ + Z_{KK}^- + Z_f}} \qquad \dots (10.28)$$

From equation (10.21) and (10.26), the positive sequence network is connected in parallel with the negative sequence network through the fault impedance Z_f as shown in Fig.10.6 and no connection for zero sequence network because $V_a^0 = 0$.

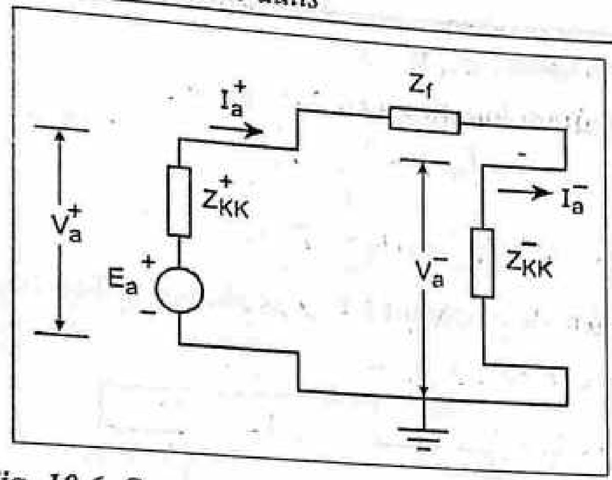


Fig. 10.6. Sequence network for LL fault with Z_f

From the network, we know

$$I_a^- = -I_a^+,\ I_a^0 = 0$$

$$I_a^+ = \frac{E_a}{Z_{KK}^+ + Z_{KK}^- + Z_f} \quad \dots (10.29)$$

$$V_a^+ = V_a^- + Z_f I_a^+$$

$$I_f = I_b = I_a^+ (a^2 - a) = -j\sqrt{3}\ I_a^+ \quad \dots (10.30)$$

Substituting for the symmetrical components of currents from equation (10.21) in equations (10.22) and (10.28), the symmetrical components of voltages and phase voltages at the fault point are obtained.

In many practical applications,

$$Z_{KK}^+ = Z_{KK}^-$$

10.4.1. DIRECT SHORT CIRCUIT OR BOLTED L-L FAULT

Fig.10.7 shows the direct short circuit or bolted LL fault.

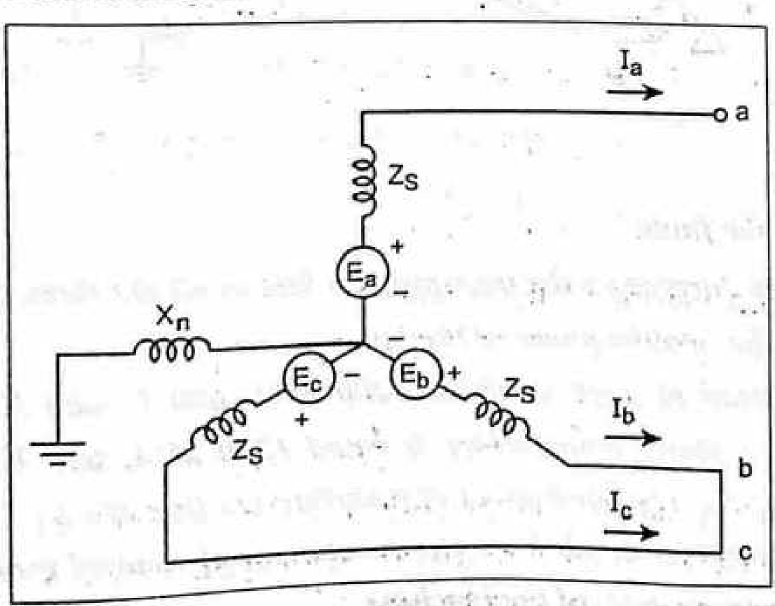


Fig. 10.7. Direct short circuit LL fault

Fault impedance, $Z_f = 0$

The conditions of the fault at bus K are

$$\left.\begin{aligned} I_a &= 0 \\ I_b &= -I_c \\ V_b &= V_c \end{aligned}\right\} \qquad \dots (10.31)$$

The sequence network for short circuit LL is as shown in Fig.10.8.

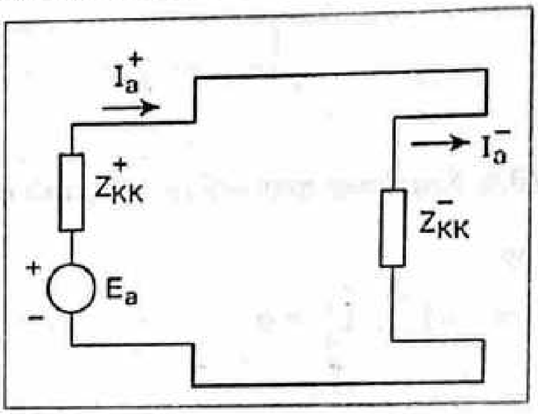
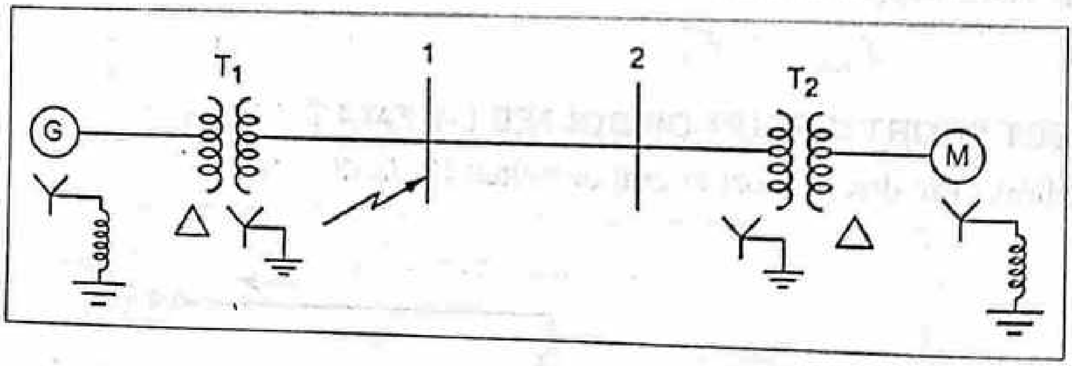


Fig. 10.8. Sequence network for short circuit LL fault

$$I_a^+ = -I_a^-$$

$$I_f = \frac{E_a\,(a^2-a)}{Z_{KK}^+ + Z_{KK}^-} = \frac{-j\sqrt{3}\;E_a}{Z_{KK}^+ + Z_{KK}^-} \qquad \dots (10.32)$$

Example 10.11 *A single line to ground fault occurs on the bus 1 of the system of Fig.*



Find:

(i) Current in the fault.

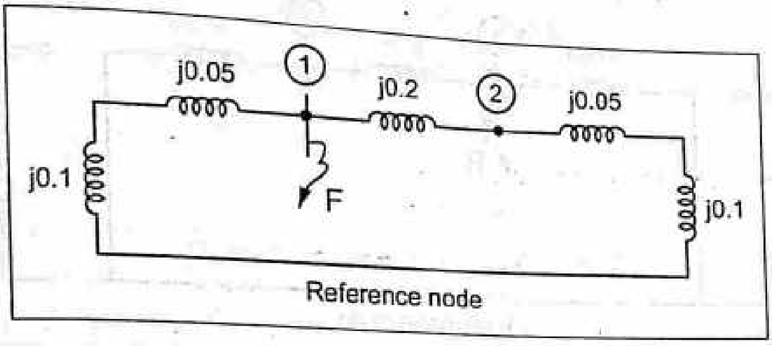
(ii) Short circuit current on the transmission line in all the three phases.

(iii) Voltage of the healthy phase of the bus 1.

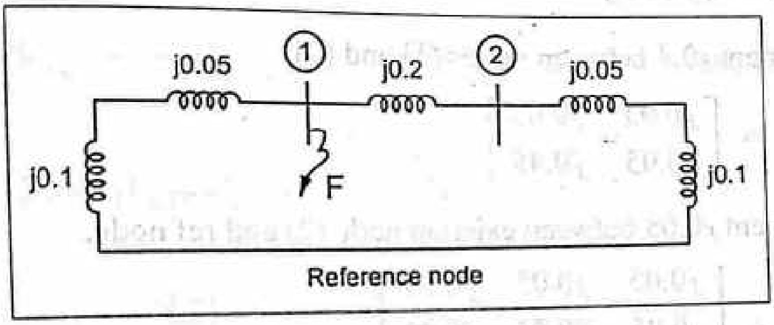
Given values: Rating of each machine 1200 kVA, 600 V with $X_1 = X_2 = 10\%$ and $X_0 = 5\%$. Each three phase transformer is rated 1200 kVA, 600 V/3300 V (Δ/Y) with leakage reactance of 5%. The reactances of transmission line are $X_1 = X_2 = 20\%$ and $X_0 = 40\%$ on the base of 1200 kVA, 3300 V. The reactances of neutral grounding reactors are 5% on the kVA and voltage base of the machine.

☺ Solution:

Positive sequence network:



Negative sequence network:



Formulate Z_{bus}:

$$Z_{\text{bus}}^{\text{new}} = \begin{array}{cc} & (1) \\ (1) & [j0.15] \end{array}$$

Adding an element $j0.2$ between nodes (1) and (2),

$$Z_{\text{bus}}^{\text{new}} = \begin{array}{ccc} & (1) & (2) \\ (1) & j0.15 & j0.15 \\ (2) & j0.15 & j0.35 \end{array}$$

Adding an element between existing node and ref.

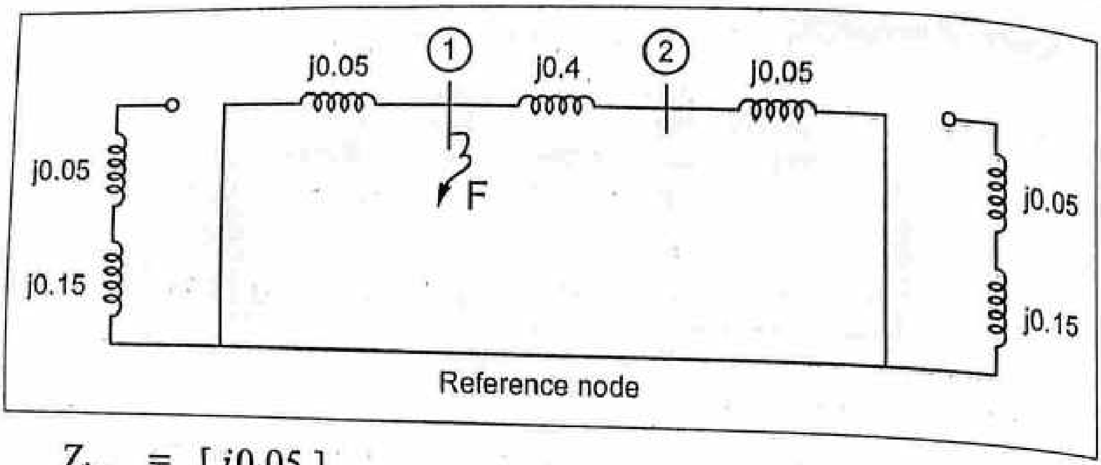
$$Z_{\text{bus}}^{\text{new}} = \begin{array}{cccc} & (1) & (2) & (3) \\ (1) & j0.15 & j0.15 & j0.15 \\ (2) & j0.15 & j0.35 & j0.35 \\ (3) & j0.15 & j0.35 & j0.5 \end{array}$$

Apply Kron reduction,

$$Z_{\text{bus}}^{+} = Z_{\text{bus}}^{-} = \begin{bmatrix} j0.105 & j0.045 \\ j0.045 & j0.105 \end{bmatrix}$$

$$Z_{11}^{+} = Z_{11}^{-} = j0.105$$

Zero sequence network:



$$Z_{bus} = [j0.05]$$

Adding an element $j0.4$ between nodes (1) and (2),

$$Z_{bus} = \begin{bmatrix} j0.05 & j0.05 \\ j0.05 & j0.45 \end{bmatrix}$$

Adding an element $j0.05$ between existing node (2) and ref node,

$$Z_{bus} = \begin{bmatrix} j0.05 & j0.05 & j0.05 \\ j0.05 & j0.45 & j0.45 \\ j0.05 & j0.45 & j0.5 \end{bmatrix}$$

Apply Kron reduction,

$$Z^{o}_{bus} = \begin{bmatrix} j0.045 & j0.005 \\ j0.005 & j0.045 \end{bmatrix}$$

$$Z^{o}_{11} = j0.045$$

Current in the fault $I_f = 3\, I^{+}_{a}$

$$I^{+}_{a} = \frac{1\angle 0°}{Z^{+}_{11} + Z^{-}_{11} + Z^{0}_{11} + 3\, Z_f}$$

$$= \frac{1\angle 0°}{j0.105 + j0.105 + j0.045 + 0} = -j3.92 \text{ p.u}$$

Current in the fault $I_f = 3 \times (-j3.92) = -j11.7$ p.u

(ii) Short circuit current on transmission lines

Positive sequence post fault bus voltages,

$$V^{+}_{f_1} = V^{+}_{0} - Z^{+}_{11}\, I^{+}_{f}$$

$$= 1.0 - j0.105 \times (-j3.92) = 0.5884$$

$$V_{f_2}^{+} = V_0^{+} - Z_{12}^{+} I_f^{+}$$

$$= 1.0 - j0.045 \times (-j3.92) = 0.8236$$

Negative sequence post fault bus voltages

$$V_{f_1}^{-} = -Z_{11}^{-} I_f^{-} = -j0.105 \times (-j3.92) = -0.4116$$

$$V_{f_2}^{-} = -Z_{12}^{-} I_f^{-} = -j0.045 \times (-j3.92) = -0.1764$$

Zero sequence post fault bus voltages

$$V_{f_1}^{0} = -Z_{11}^{0} I_f^{0} = -j0.045 \times (-j3.92) = -0.1764$$

$$V_{f_2}^{0} = -Z_{12}^{0} I_f^{0} = -j0.005 \times (-j3.92) = -0.0196$$

Positive sequence current $I_{12}^{+} = \dfrac{V_{f_1}^{+} - V_{f_2}^{+}}{Z_{12(\text{line})}^{+}}$

$$= \frac{0.5884 - 0.8236}{j0.2} = j1.176 \text{ p.u}$$

$$I_{12}^{-} = \frac{V_{f_1}^{-} - V_{f_2}^{-}}{Z_{12(\text{line})}^{-}} = \frac{-0.4116 - (-0.1764)}{j0.2} = j1.176 \text{ p.u}$$

$$I_{12}^{0} = \frac{V_{f_1}^{0} - V_{f_2}^{0}}{Z_{12(\text{line})}^{0}} = \frac{-0.1764 - (-0.0196)}{j0.4} = j0.392 \text{ p.u}$$

(iii) *Voltage of healthy phase of the bus 1:*

$$V_a = 0$$

$$V_b = a^2 V_1^{+} + a V_1^{-} + V_1^{0}$$

$$= 1\angle 240° \times 0.5884 + 1\angle 120° \times (-0.4116) + (-0.1764)$$

$$= -0.2646 - j0.866$$

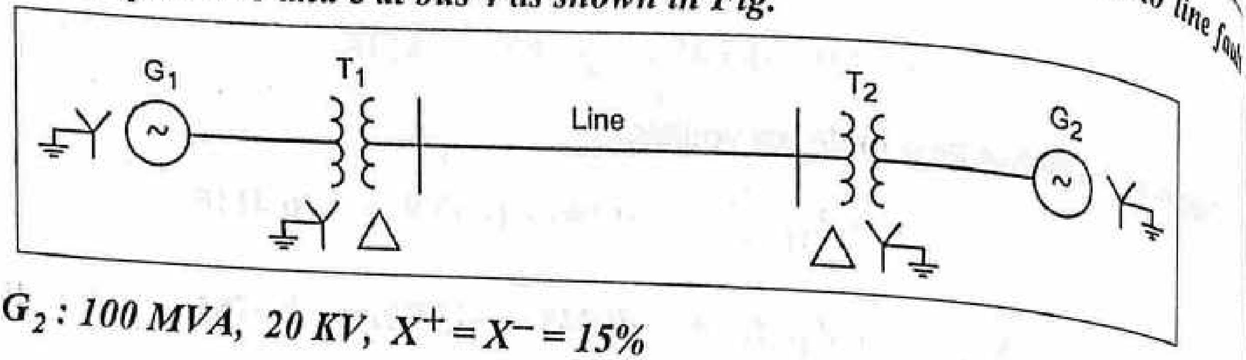
$$= 0.9056 \angle -107°$$

$$V_c = a V_1^{+} + a^2 V_1^{-} + V_1^{0}$$

$$= 1\angle 120° \times 0.5884 + 1\angle 240° \times (-0.4116) + (-0.1764)$$

$$= 0.9056 \angle 107°$$

Example 10.12 ***Determine the fault current and fault MVA for a line to line fau occurs between phases b and c at bus 4 as shown in Fig.***

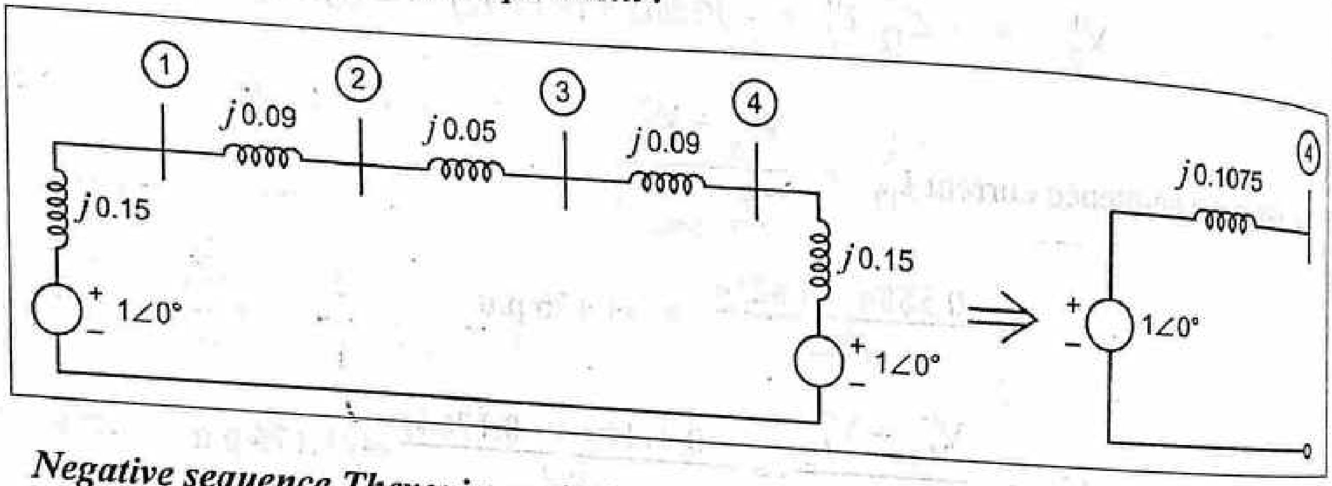


G_1, G_2 : 100 MVA, 20 KV, $X^+ = X^- = 15\%$

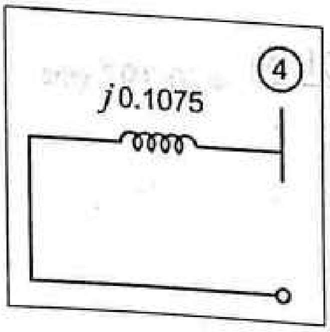
T_1, T_2 : 100 MVA, 20/345 KV, $X_{leak} = 9\%$; Line : $X^+ = X^- = 5\%$

☺ Solution :

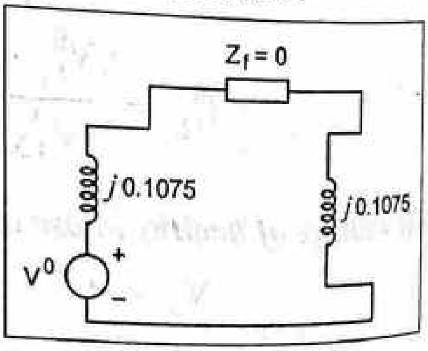
Positive sequence Thevenin equivalent :



Negative sequence Thevenin equivalent :



Sequence network :



$$\text{Prefault voltage} = E_a = V^0 = 1\angle 0°$$

$$I_a^+ = -I_a^- = \frac{1\angle 0°}{j0.1075 + j0.1075} = -j4.651 \text{ p.u.}$$

$$|I_a^+| = |I_a^-| = 4.651 \times \frac{100}{\sqrt{3} \times 20} = 13.426 \text{ KA}$$

$$I_a^+ = -j13.426, \; I_a^- = j13.426 \text{ KA}$$

$$I_a^0 = 0$$

Current in phase domain :

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_a^0 \\ I_a^+ \\ I_a^- \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ -j13.426 \\ j13.426 \end{bmatrix}$$

$$I_a = 0$$

$$I_b = 1 \times 0 + a^2 (-j13.426) + a\,(j13.426) = -23.254 \text{ KA}$$

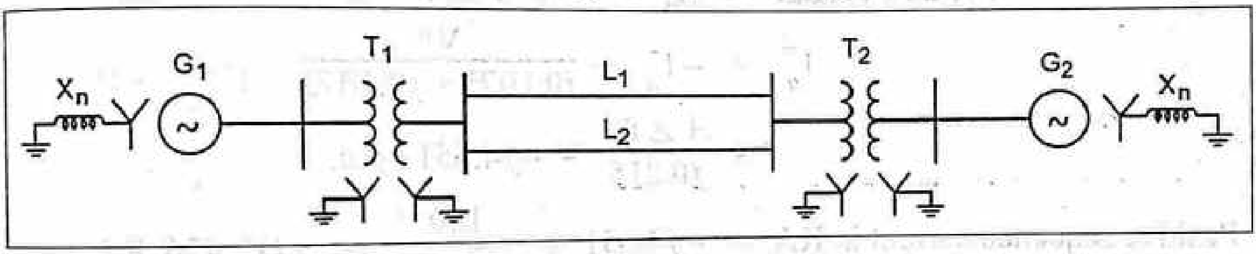
$$I_c = -I_b = 23.254 \text{ KA}$$

$$I_n = I_a + I_b + I_c = 0$$

$$\text{Fault MVA} = \sqrt{3} \times I_f\,(\text{KA}) \times \text{KV} = \sqrt{3} \times 23.254 \times 20$$

$$= 805.542$$

Example 10.13 ***Determine the fault current and fault MVA for a line to line fault occurs between phases 'b' and 'c' at bus (4) as shown in Fig.***



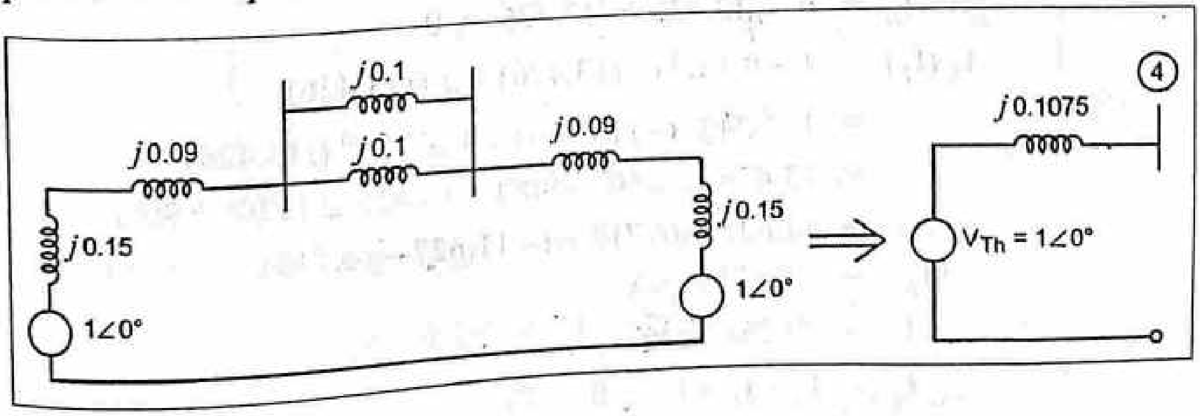
G_1, G_2: 100 MVA, 20 KV, $X^+ = X^- = 15\%$, $X^0 = 4\%$, $X_n = 6\%$

T_1, T_2: 100 MVA, 20/345 KV, $X_{leak} = 9\%$

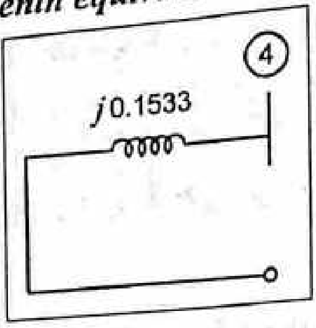
L_1, L_2: $X^+ = X^- = 10\%$, $X^0 = 40\%$ on a base of 100 MVA

☺ ***Solution :***

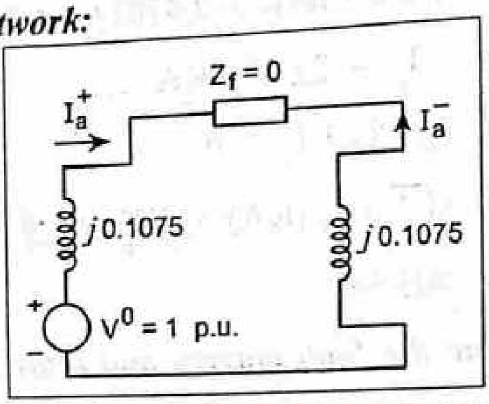
Step 1 : Positive sequence Thevenin equivalent :



Step 2 : Negative sequence Thevenin equivalent :



Step 3 : Draw sequence network:



$$\text{Prefault voltage} = E_a = V^0 = 1 \angle 0^\circ$$

$$I_a^+ = -I_a^- = \frac{V^0}{j0.1075 + j0.1075}$$

$$= \frac{1 \angle 0^\circ}{j0.215} = -j4.651 \quad \text{p.u.}$$

$$\text{Positive sequence current in KA} = -j4.651 \times \frac{100}{\sqrt{3} \times 20} = -j13.426 \text{ KA}$$

$$|I_a^+| = |I_a^-| = 13.426 \text{ KA}, \; I_a^0 = 0$$

Current in phase domain :

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ -j13.426 \\ j13.426 \end{bmatrix}$$

$$I_a = 0 - j13.426 + j13.426 = 0$$

$$I_b (I_f) = 1 \times 0 + a^2 (-j13.426) + a (j13.426)$$

$$= 1 \angle 240^\circ (-j13.426) + 1 \angle 120^\circ (j13.426)$$

$$= 13.426 \angle (240^\circ - 90^\circ) + 13.426 \angle (120^\circ + 90^\circ)$$

$$= 11.627 + j6.713 + (-11.627 - j6.713)$$

$$|I_b| = -23.254 \text{ KA}$$

$$I_c = 23.254 \text{ KA}$$

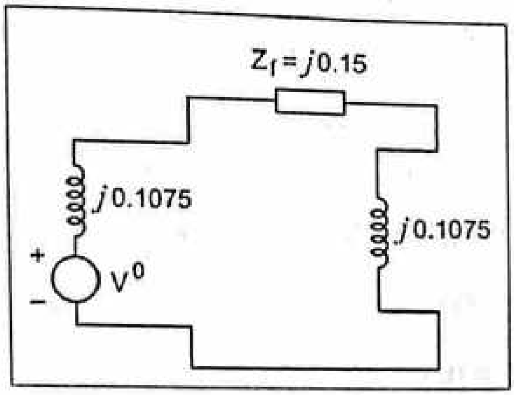
$$I_n = I_a + I_b + I_c = 0$$

$$\text{Fault MVA} = \sqrt{3} \times I_f \times \text{KV} = \sqrt{3} \times 23.254 \times 20 = 805.542$$

Example 10.14 *Determine the fault current, fault MVA when a L-L fault occurs between phases b and c at bus (4) and the fault impedance is j0.15 p.u.*

Positive sequence reactance = j0.1075 p.u.
Negative sequence reactance = j0.1075 p.u.
Zero sequence reactance = j0.1533 p.u.

☺ ***Solution : Sequence network :***



$$\text{Prefault voltage} = E_a = V^0 = 1\angle 0°$$

Symmetrical components of currents

$$I_a^0 = 0$$

$$I_a^+ = -I_a^- = \frac{V^0}{Z_{44}^+ + Z_{44}^- + Z_f}$$

$$= \frac{1\angle 0°}{j0.1075 + j0.1075 + j0.15} = -j2.7397 \text{ p.u.}$$

Fault current at phase a, b, c is

$$\begin{bmatrix} I_{af} \\ I_{bf} \\ I_{cf} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_a^0 \\ I_a^+ \\ I_a^- \end{bmatrix}$$

$$\begin{bmatrix} I_{af} \\ I_{bf} \\ I_{cf} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ -j2.7397 \\ j2.7397 \end{bmatrix} = \begin{bmatrix} 0 \\ -4.745 \\ 4.745 \end{bmatrix} \text{ p.u.}$$

$$\text{Fault current } I_f = |I_{bf}| = 4.745 \text{ p.u.} \times \text{Base current}$$

$$= 4.745 \times \frac{100}{\sqrt{3} \times 20} = 13.698 \text{ KA}$$

$$\text{Fault MVA} = I_f \times \sqrt{3} \times \text{KV} = 13.698 \times \sqrt{3} \times 20$$

$$= 474.5$$

Example 10.15 *A 30 MVA, 11 KV, 3 phase synchronous generator has a direct sub. transient reactance of 0.25 p.u. The negative and zero sequence reactances are 0.35 and 0.1 p.u. respectively. The neutral of the generator is solidly grounded. Find the sub. transient currents and the line-to-line voltages at the fault under sub-transient conditions when a line-to-line fault occurs at the terminals of the generator. Assume that the generator is unloaded and operating at rated terminal voltage when the fault occurs.*

☺ ***Solution :***

Base MVA = 30

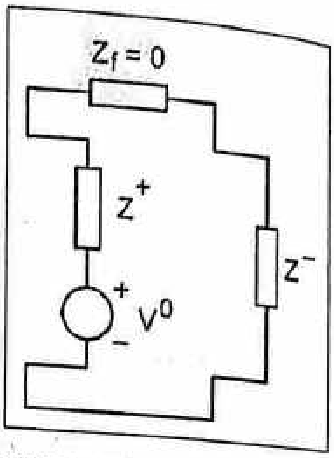
Base KV = 11

$Z^+ = j0.25$ p.u.

$Z^- = j0.35$ p.u.

$Z^0 = j0.1$ p.u.

Prefault voltage $= E_a = V^0 = 1\angle 0°$



Sequence network for L-L fault

$$\text{Positive sequence current } I_a^+ = \frac{V^0}{Z_{KK}^+ + Z_{KK}^- + Z_f} = \frac{1\angle 0°}{j0.25 + j0.35 + 0}$$

$$= -j1.667 \text{ p.u.}$$

$$\text{Negative sequence current } I_a^- = -I_a^+ = j1.667 \text{ p.u.}$$

$$\text{Zero sequence current } I_a^0 = 0$$

Subtransient current or Phase currents :

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_a^0 \\ I_a^+ \\ I_a^- \end{bmatrix}$$

$$I_a = I_a^0 + I_a^+ + I_a^-$$

$$= 0 - j1.667 + j1.667 = 0$$

$$I_b = I_a^0 + a^2 I_a^+ + a I_a^-$$

$$= 0 + (-0.5 - j0.866) \times (-j1.667) + (-0.5 + j0.866)(j1.667)$$

$$= -2.887 + j0 \text{ p.u.}$$

$$= -2.887 \text{ p.u.}$$

$$I_c = -I_b = 2.887 \text{ p.u.}$$

$$\text{Base current} = \frac{\text{MVA}_b}{\sqrt{3} \times \text{KV}_b} = \frac{30 \times 10^3}{\sqrt{3} \times 11} = 1574.59 \text{ Amp}$$

$$\therefore \quad I_b = -2.887 \times 1574.59 = -4545.85 \text{ Amp}$$

$$I_c = 2.887 \times 1574.59 = 4545.85 \text{ Amp}$$

Symmetrical component of voltages :

$$V_a^0 = -Z_{KK}^0 \; I_a^0 = 0$$

$$V_a^+ = E_a - Z_{KK}^+ \; I_a^+ = 1 \angle 0° - j0.25 \times (-j1.667)$$

$$= 1 - 0.41675 = 0.58325 \text{ p.u.}$$

$$V_a^- = -Z_{KK}^- \; I_a^- = Z_{KK}^- \; I_a^+$$

$$= j0.35 \times -j1.667 = 0.58345 \text{ p.u.}$$

Phase voltages :

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_a^0 \\ V_a^+ \\ V_a^- \end{bmatrix}$$

$$V_a = V_a^0 + V_a^+ + V_a^- = V_a^+ + V_a^- = 0.58325 + 0.58345$$

$$= 1.1667 \text{ p.u.}$$

$$V_b = V_a^0 + a^2 V_a^+ + a \, V_a^-$$

$$= 0 + (-0.5 - j0.866) \times 0.58325 + (-0.5 + j0.866) \times 0.58345$$

$$= -0.58335 \text{ p.u.}$$

$$V_c = V_a^0 + a \, V_a^+ + a^2 V_a^-$$

$$= 0 + (-0.5 + j0.866) \times 0.58325 + (-0.5 - j0.866) \times 0.58345$$

$$= -0.58335 \text{ p.u.}$$

$$\boxed{V_b = V_c}$$

Sub-transient line-to-line voltages :

$$V_{ab} = V_a - V_b$$

$$= 1.1667 - (-0.58335) = 1.75005 \text{ p.u.}$$

$$V_{bc} = V_b - V_c$$

$$= -0.58335 - (-0.58335) = 0 \text{ p.u.}$$

$$V_{ac} = V_a - V_c$$

$$= 1.1667 - (-0.58335) = 1.75005 \text{ p.u.}$$

$$\text{Base voltage, } KV_b = \frac{11}{\sqrt{3}} = 6.3508 \text{ KV}$$

$$V_{ab} = 1.75005 \times 6.3508 = 11.1143 \text{ KV}$$

$$V_{ac} = 1.75005 \times 6.3508 = 11.1143 \text{ KV}$$

$$V_{bc} = 0 \text{ p.u.} = 0 \text{ KV}$$

10.5. DOUBLE LINE-TO-GROUND FAULT

Fig.10.9 shows a three phase generator with a fault on phases b and c through an impedance Z_f to ground. Assuming the generator is initially on no-load, the conditions at the fault K are expressed by the following relations.

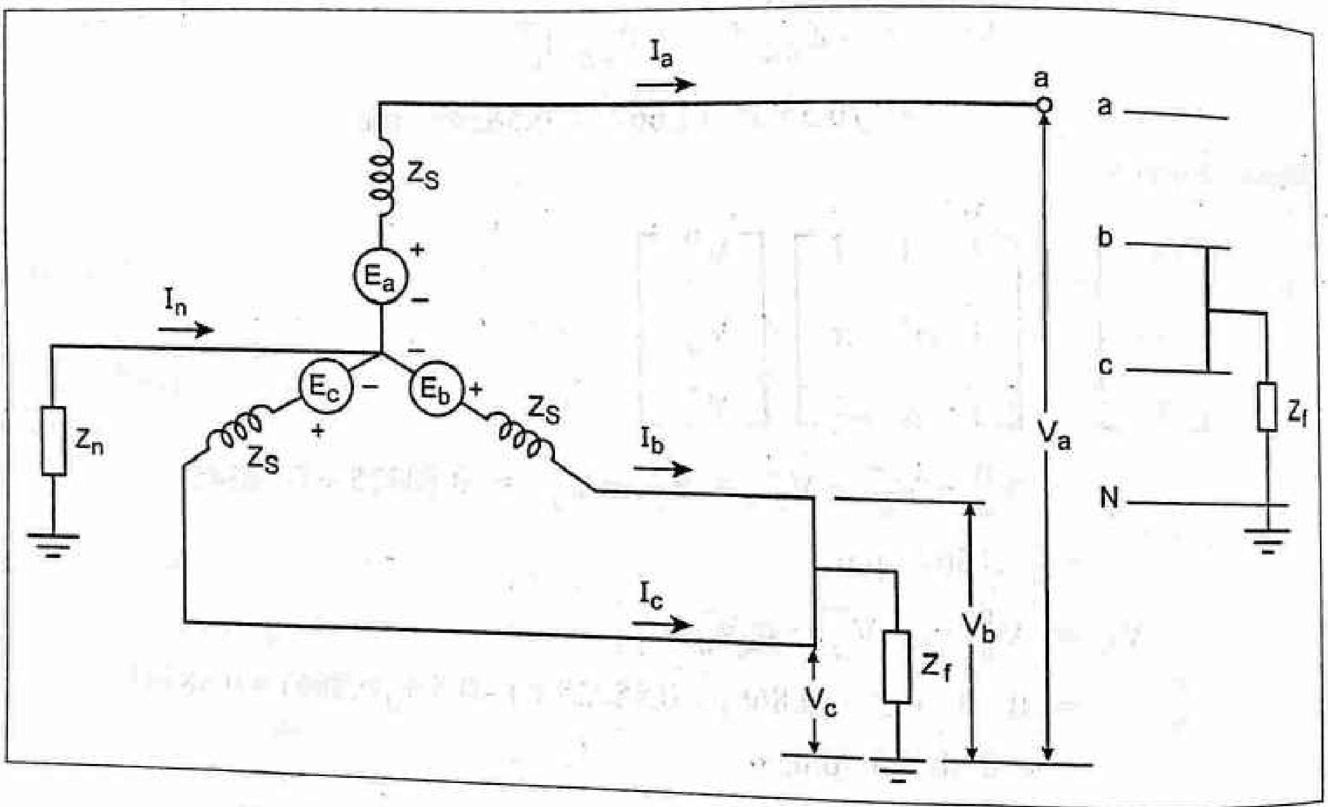


Fig. 10.9. Double line to ground fault between phases b and c

$$\left.\begin{aligned} I_a &= 0 \\ I_b + I_c &= I_f \\ V_b &= V_c = Z_f I_f = Z_f(I_b + I_c) \end{aligned}\right\} \quad \dots (10.33)$$

The symmetrical components of voltages are

$$\begin{bmatrix} V_a^0 \\ V_a^+ \\ V_a^- \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \quad \dots (10.34)$$

Substitute $V_b = V_c$ in equation (10.34), we get

$$\begin{bmatrix} V_a^0 \\ V_a^+ \\ V_a^- \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} V_a \\ V_b \\ V_b \end{bmatrix}$$

$$V_a^0 = \frac{1}{3}(V_a + V_b + V_b) = \frac{1}{3}(V_a + 2\,V_b)$$

$$V_a^+ = \frac{1}{3}(V_a + a\,V_b + a^2\,V_b)$$

$$= \frac{1}{3}[V_a + V_b(a + a^2)] \qquad [1 + a + a^2 = 0\,;\; a + a^2 = -1]$$

$$= \frac{1}{3}[V_a - V_b]$$

$$V_a^- = \frac{1}{3}[V_a + a^2\,V_b + a\,V_b]$$

$$= \frac{1}{3}[V_a + V_b(a^2 + a)] = \frac{1}{3}[V_a - V_b]$$

$$\boxed{V_a^+ = V_a^-} \qquad \ldots(10.35)$$

The phase currents are given by

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}\begin{bmatrix} I_a^0 \\ I_a^+ \\ I_a^- \end{bmatrix}$$

$$I_a = I_a^0 + I_a^+ + I_a^-$$

$$I_b = I_a^0 + a^2\,I_a^+ + a\,I_a^-$$

$$I_c = I_a^0 + a\,I_a^+ + a^2\,I_a^-$$

$$I_f = I_b + I_c = I_a^0 + a^2\,I_a^+ + a\,I_a^- + I_a^0 + a\,I_a^+ + a^2\,I_a^-$$

$$= 2\,I_a^0 + I_a^+\,(a^2 + a) + I_a^-\,(a + a^2)$$

$$= 2\,I_a^0 + I_a^+\,(-1) + I_a^-\,(-1)$$

$$= 2\,I_a^0 - I_a^+ - I_a^- = 2\,I_a^0 - (I_a^+ + I_a^-) \qquad \ldots(10.36)$$

From the condition, $\mathrm{I}_a = \mathrm{I}_a^0 + \mathrm{I}_a^- + \mathrm{I}_a^- = 0$

$$(\mathrm{I}_a^+ + \mathrm{I}_a^-) = -\mathrm{I}_a^0 \qquad \dots (10.37)$$

Substituting (10.37) in (10.36), we get

$$\mathrm{I}_b + \mathrm{I}_c = 2\,\mathrm{I}_a^0 + \mathrm{I}_a^0 = 3\,\mathrm{I}_a^0$$

From the condition, $\mathrm{V}_b = Z_f(\mathrm{I}_b + \mathrm{I}_c)$

$$\mathrm{V}_b = 3\,Z_f\,\mathrm{I}_a^0 \qquad \dots (10.38)$$

The phase voltages are given by

$$\begin{bmatrix} \mathrm{V}_a \\ \mathrm{V}_b \\ \mathrm{V}_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} \mathrm{V}_a^0 \\ \mathrm{V}_a^+ \\ \mathrm{V}_a^- \end{bmatrix}$$

$$\begin{aligned}
\mathrm{V}_a &= \mathrm{V}_a^0 + \mathrm{V}_a^+ + \mathrm{V}_a^- \\
\mathrm{V}_b &= \mathrm{V}_a^0 + a^2\,\mathrm{V}_a^+ + a\,\mathrm{V}_a^- \\
&= \mathrm{V}_a^0 + a^2\,\mathrm{V}_a^+ + a\,\mathrm{V}_a^+ && [\because \mathrm{V}_a^+ = \mathrm{V}_a^-] \\
&= \mathrm{V}_a^0 + \mathrm{V}_a^+\,(a^2 + a) \\
\mathrm{V}_b &= \mathrm{V}_a^0 - \mathrm{V}_a^+ && [\therefore \mathrm{V}_b = 3\,Z_f\mathrm{I}_a^0] \qquad \dots (10.39)
\end{aligned}$$

Equate (10.38) and (10.39),

$$\mathrm{V}_a^0 - \mathrm{V}_a^+ = 3\,Z_f\mathrm{I}_a^0 \qquad \dots (10.40)$$

The symmetrical component voltages are given by

$$\begin{bmatrix} \mathrm{V}_a^0 \\ \mathrm{V}_a^+ \\ \mathrm{V}_a^- \end{bmatrix} = \begin{bmatrix} 0 \\ \mathrm{E}_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z_{\mathrm{KK}}^0 & 0 & 0 \\ 0 & Z_{\mathrm{KK}}^+ & 0 \\ 0 & 0 & Z_{\mathrm{KK}}^- \end{bmatrix} \begin{bmatrix} \mathrm{I}_a^0 \\ \mathrm{I}_a^+ \\ \mathrm{I}_a^- \end{bmatrix}$$

$$\left.\begin{aligned}
\mathrm{V}_a^0 &= -Z_{\mathrm{KK}}^0\,\mathrm{I}_a^0 \\
\mathrm{V}_a^+ &= \mathrm{E}_a - Z_{\mathrm{KK}}^+\,\mathrm{I}_a^+ \\
\mathrm{V}_a^- &= -Z_{\mathrm{KK}}^-\,\mathrm{I}_a^-
\end{aligned}\right\} \qquad \dots (10.41)$$

Substitute V_a^0, V_a^+ in equation (10.40), we get

$$-Z_{KK}^0\ I_a^0 - [\,E_a - Z_{KK}^+\ I_a^+\,] = 3\,Z_f I_a^0$$

$$-[\,E_a - Z_{KK}^+\ I_a^+\,] = [\,Z_{KK}^0 + 3\,Z_f]\,I_a^0$$

$$I_a^0 = \frac{-[\,E_a - Z_{KK}^+\ I_a^+\,]}{Z_{KK}^0\ + 3\,Z_f} \qquad \ldots (10.42)$$

From equation (10.35), $\quad V_a^+ = V_a^-$

$$E_a - Z_{KK}^+\ I_a^+ = -Z_{KK}^-\ I_a^-$$

$$I_a^- = \frac{-(E_a - Z_{KK}^+\ I_a^+)}{Z_{KK}^-} \qquad \ldots (10.43)$$

From equation (10.37), $\quad -I_a^0 = I_a^+ + I_a^-$

$$I_a^+ = -[\,I_a^- + I_a^0\,] = -I_a^- - I_a^0$$

$$= \left[\frac{E_a - Z_{KK}^+\ I_a^+}{Z_{KK}^-}\right] + \left[\frac{E_a - Z_{KK}^+\ I_a^+}{Z_{KK}^0\ + 3\,Z_f}\right]$$

$$= I_a^+ \times \left[\,1 + \frac{Z_{KK}^+}{Z_{KK}^-} + \frac{Z_{KK}^+}{Z_{KK}^0\ + 3\,Z_f}\right]$$

$$= \frac{E_a}{Z_{KK}^-} + \frac{E_a}{Z_{KK}^0 + 3\,Z_f}$$

$$I_a^+[Z_{KK}^-\,(Z_{KK}^0 + 3\,Z_f) + Z_{KK}^+\,(Z_{KK}^0\ + 3\,Z_f) + Z_{KK}^+\ Z_{KK}^-\,] = E_a\,[\,Z_{KK}^0\ + 3\,Z_f + Z_{KK}^-\,]$$

$$I_a^+\,[\,Z_{KK}^+\,(Z_{KK}^0\ + 3\,Z_f + Z_{KK}^-) + Z_{KK}^-\ (Z_{KK}^0\ + 3\,Z_f) = E_a\,(Z_{KK}^0\ + 3\,Z_f + Z_{KK}^-)$$

$$I_a^+ = \frac{E_a\,(Z_{KK}^0 + 3\,Z_f + Z_{KK}^-)}{(Z_{KK}^0 + 3\,Z_f + Z_{KK}^-)\left[\,Z_{KK}^+ + \dfrac{Z_{KK}^-\ (Z_{KK}^0\ + 3\,Z_f)}{(Z_{KK}^0\ + 3\,Z_f + Z_{KK}^-)}\right]}$$

$$\boxed{I_a^+ = \frac{E_a}{Z_{KK}^+ + \dfrac{Z_{KK}^-\ (Z_{KK}^0\ + 3\,Z_f)}{Z_{KK}^0\ + 3\,Z_f + Z_{KK}^-}}} \qquad \ldots (10.44)$$

The fault current, $I_f = 3 I_a^0$

Substituting from equation (10.42), we get

$$I_f = -3 \times \left[\frac{E_a - Z_{KK}^+ \; I_a^+}{Z_{KK}^0 + 3 Z_f} \right]$$

Substituting I_a^+ from equation (10.44), we get

$$I_f = \frac{-3}{Z_{KK}^0 + 3 Z_f} \left[E_a - \frac{Z_{KK}^+ \; E_a}{Z_{KK}^+ + \dfrac{Z_{KK}^- \; (Z_{KK}^0 + 3 Z_f)}{Z_{KK}^0 + Z_{KK}^- + 3 Z_f}} \right]$$

$$I_f = \frac{-3}{Z_{KK}^0 + 3 Z_f} \left[\frac{E_a \times Z_{KK}^- \; (Z_{KK}^0 + 3 Z_f)}{Z_{KK}^+ \times Z_{KK}^0 + 3 Z_f \, Z_{KK}^+ + Z_{KK}^+ \, Z_{KK}^- + Z_{KK}^- \, Z_{KK}^0 + 3 Z_f \, Z_{KK}^-} \right] \quad \dots (10.45)$$

Sequence Network :

From equations (10.35), (10.37) and (10.40),

The positive, negative and zero sequence networks are connected in parallel as shown in Fig.10.10.

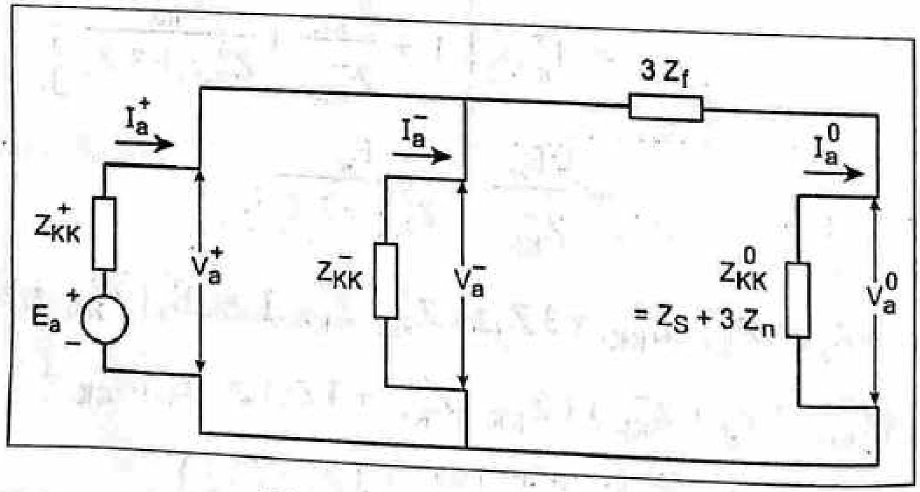


Fig. 10.10. Sequence network

From sequence network,

$$V_a^+ = V_a^- = V_a^0 + 3 Z_f \, I_a^0$$

$$I_a^+ + I_a^- = -I_a^0$$

Total Z consisting of Z_{KK}^+ in series with the parallel combination of Z_{KK}^- and $(Z_{KK}^0 + 3 Z_f)$

$$\left.\begin{aligned} I_a^+ &= \frac{E_a}{Z_{KK}^+ + \left[\dfrac{Z_{KK}^-\,(3\,Z_f + Z_{KK}^0)}{(Z_{KK}^- + 3\,Z_f + Z_{KK}^0)}\right]} \\ I_a^- &= -I_a^+ \times \frac{3\,Z_f + Z_{KK}^0}{Z_{KK}^- + 3\,Z_f + Z_{KK}^0} \\ I_a^0 &= -I_a^+ \times \frac{Z_{KK}^-}{Z_{KK}^- + 3\,Z_f + Z_{KK}^0} \end{aligned}\right\} \text{[From current division method]} \quad \ldots (10.46)$$

Substituting for the symmetrical components of currents, we can calculate sequence voltage changes at all buses of the system.

10.5.1. DIRECT SHORT CIRCUIT OR BOLTED LLG FAULT

Fig.10.11 shows the direct short circuit or bolted double line to ground fault.

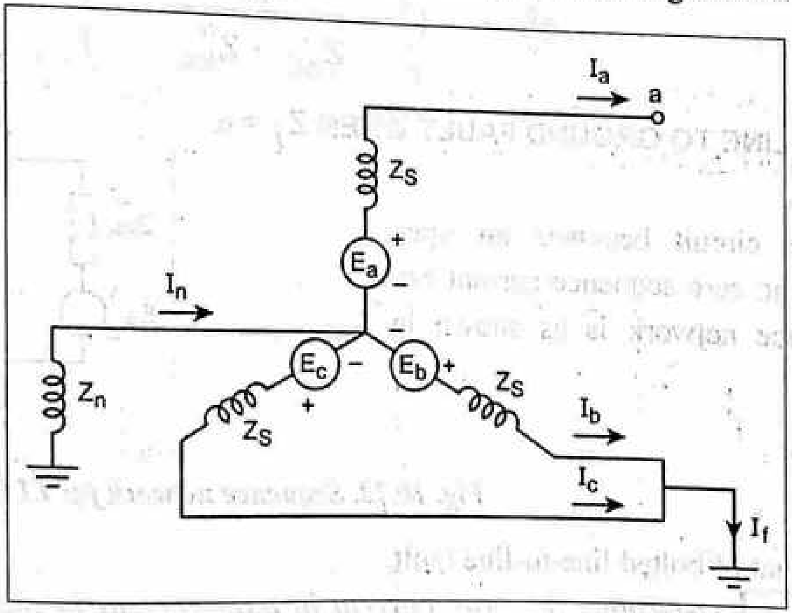


Fig. 10.11. Direct short circuit LLG fault

Fault impedance, $Z_f = 0$

The conditions of the fault at bus K are

$$\left.\begin{aligned} \therefore\ I_a &= 0,\ V_b = 0,\ V_c = 0 \\ I_f &= I_b + I_c \end{aligned}\right\} \quad \ldots (10.47)$$

The sequence network for short circuit LLG fault is as shown in Fig.10.12.

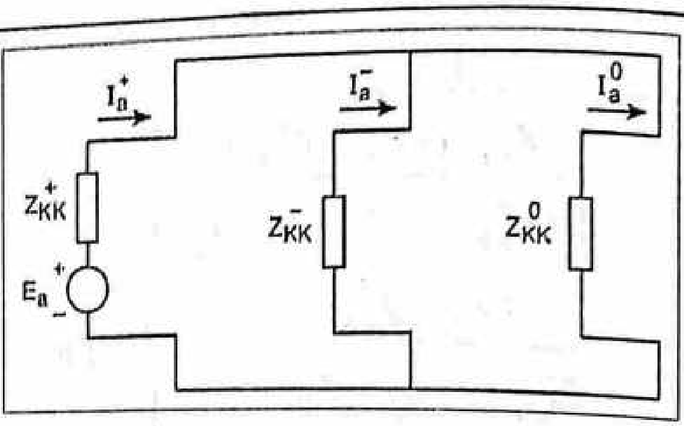


Fig. 10.12. Sequence network for short circuit LLG fault

$$\left.\begin{aligned} I_a^+ &= \frac{E_a}{Z_{KK}^+ + \left[\dfrac{Z_{KK}^- \times Z_{KK}^0}{Z_{KK}^- + Z_{KK}^0}\right]} \\ I_a^- &= -I_a^+ \times \frac{Z_{KK}^0}{Z_{KK}^- + Z_{KK}^0} \\ I_a^0 &= -I_a^+ \times \frac{Z_{KK}^-}{Z_{KK}^- + Z_{KK}^0} \end{aligned}\right\} \quad \dots (10.48)$$

10.5.2. DOUBLE LINE TO GROUND FAULT WHEN $Z_f = \alpha$

When $Z_f = \alpha$,

Zero sequence circuit becomes an open circuit. Therefore no zero sequence current can flow. The sequence network is as shown in Fig.10.13.

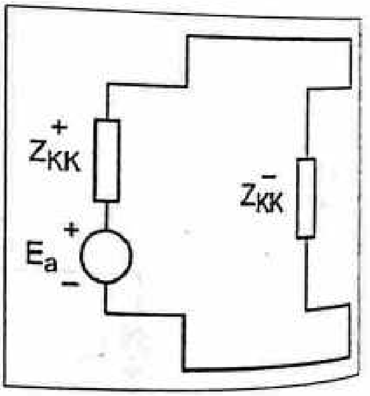
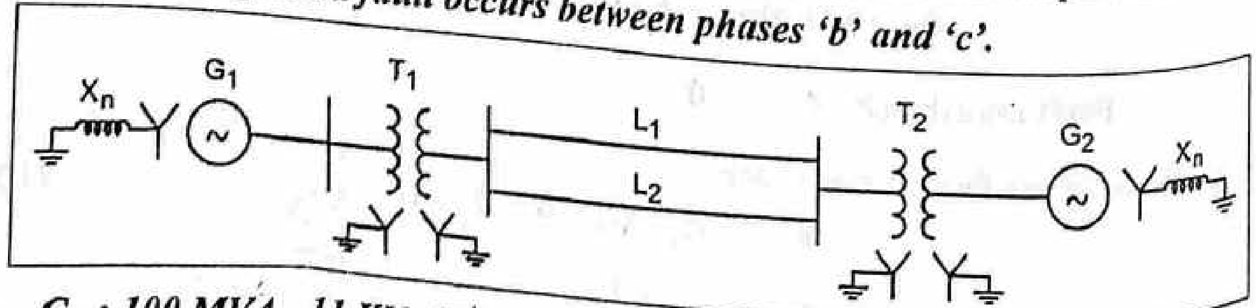


Fig. 10.13. Sequence network for LLG fault with $Z_f = \alpha$

It is similar to that of bolted line-to-line fault.

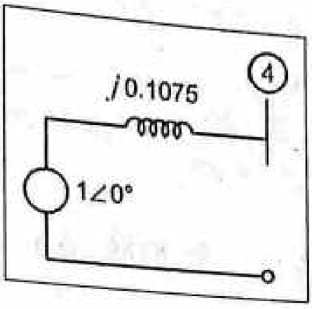
Example 10.16 *Determine the fault current in p.u., current in phase domain form for a double line to ground fault occurs between phases 'b' and 'c'.*



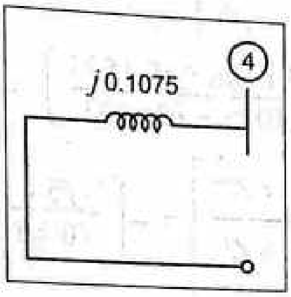
G_1, G_2 : 100 MVA, 11 KV, $X^+ = X^- = 15\%$, $X^0 = 5\%$, $X_n = 6\%$
T_1, T_2 : 100 MVA, 11/220 KV, $X_{leak} = 9\%$
L_1, L_2 : $X^+ = X^- = 10\%$, $X^0 = 10\%$ on a base of 100 MVA

Solution :

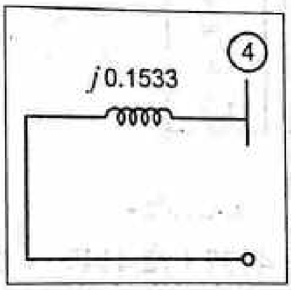
Step 1 : Positive sequence Thevenin's equivalent



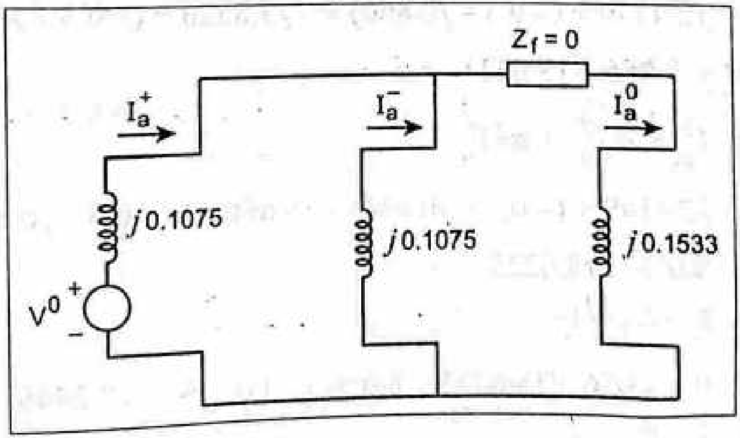
Step 2 : Negative sequence Thevenin equivalent



Step 3 : Zero sequence Thevenin equivalent



Step 4 : Sequence network



$$I_a^+ = \frac{V^0}{Z^+ + \dfrac{Z^-(Z^0+3\,Z_f)}{Z^-+Z^0+3\,Z_f}}$$

$$Z_f = 0\ ;\ I_a^+ = \frac{V^0}{Z^+ + \dfrac{Z^- Z^0}{Z^-+Z^0}} = \frac{1\angle 0°}{j0.1075 + \dfrac{j0.1075 \times j0.1533}{j0.1075 + j0.1533}}$$

$$= \frac{1\angle 0°}{j0.1707} = -j5.8586 \text{ p.u.}$$

$$I_a^- = -\left[\frac{I_a^+ \times Z^0}{Z^- + Z^0}\right]$$

$$= -\left[\frac{-j5.8586 \times j0.1533}{j0.1075 + j0.1533}\right] = j3.4437 \text{ p.u.}$$

$$I_a^0 = -\left[\frac{I_a^+ \times Z^-}{Z^- + Z^0}\right] = -\left[\frac{-j5.8586 \times j0.1075}{j0.1075 + j0.1533}\right] = j2.4149 \text{ p.u.}$$

Current in phase domain :

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_a^0 \\ I_a^+ \\ I_a^- \end{bmatrix}$$

$$I_a = I_a^+ + I_a^- + I_a^0$$

$$= -j5.8586 + j3.4437 + j2.4149 = 0$$

$$I_b = I_a^0 + a^2\, I_a^+ + a\, I_a^-$$

$$= j2.4149 + (-0.5 - j0.866) \times -j5.8586 + (-0.5 + j0.866) \times j3.4437$$

$$= -8.056 + j3.6223 \text{ p.u.}$$

$$I_c = I_a^0 + a\, I_a^+ + a^2\, I_a^-$$

$$= j2.4149 + (-0.5 + j0.866) \times j5.8586 + (-0.5 - j0.866) \times j3.4437$$

$$= 8.056 + j3.6223$$

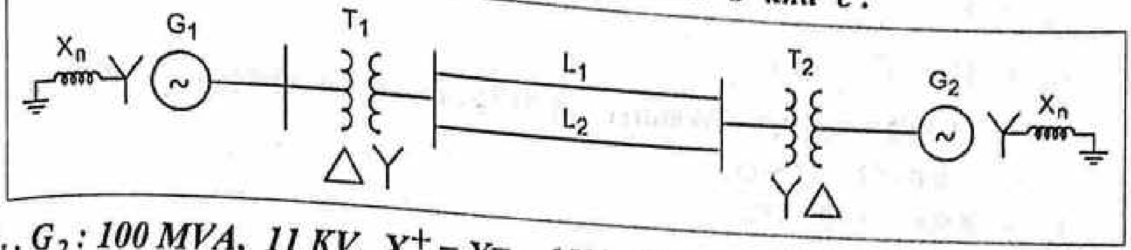
$$I_n = I_a + I_b + I_c$$

$$= 0 - 8.056 + j3.6223 + 8.056 + j3.6223 = j7.2446 \text{ p.u.}$$

$$\text{Fault current } I_f = 3 \times I_a^0$$

$$= 3 \times j2.4149 = j7.2447 \text{ p.u.}$$

Example 10.17 **Determine the fault current in p.u., current in phase domain form for a double line to ground fault occurs between phases 'b' and 'c'.**



G_1, G_2 **: 100 MVA, 11 KV,** $X^+ = X^- = 15\%, X^0 = 5\%, X_n = 6\%$

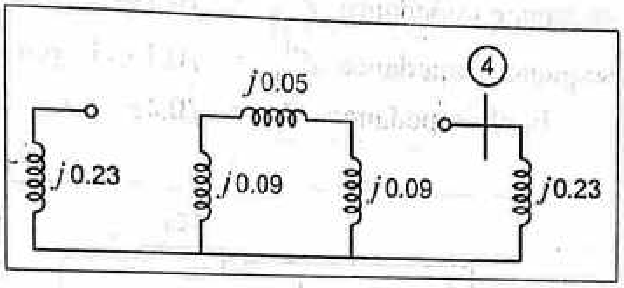
T_1, T_2 **: 100 MVA, 11/220 KV,** $X_{leak} = 9\%$

L_1, L_2 **:** $X^+ = X^- = 10\%, X^0 = 10\%$ **on a base of 100 MVA**

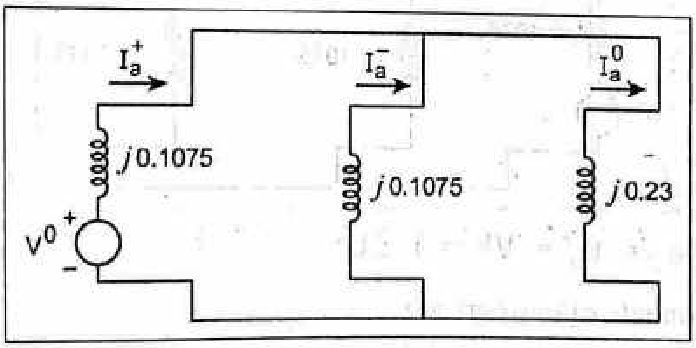
☺ **Solution :**

$$Z^+ = j0.1075, \quad Z^- = j0.1075$$

$$Z^0 = j0.23$$



Sequence network :



$$\text{Prefault voltage} = E_a = V^0 = 1\angle 0°$$

$$I_a^+ = \frac{V^0}{Z^+ + \dfrac{Z^- Z^0}{Z^- + Z^0}} = \frac{1\angle 0°}{j0.1075 + \dfrac{j0.1075 \times j0.23}{j0.1075 + j0.23}}$$

$$= -j5.5322 \text{ p.u.}$$

$$I_a^- = -\left[\frac{-j5.5322 \times j0.23}{j0.1075 + j0.23}\right] = j3.77 \text{ p.u.}$$

$$I_a^0 = -\left[\frac{-j5.5322 \times j0.1075}{j0.1075 + j0.23}\right] = j1.7621$$

$$\text{Fault current } I_f = 3 I_a^0 = 3 \times j1.7621 = j5.2863 \text{ p.u.}$$

Current in phase domain :

$$I_a = I_a^0 + I_a^+ + I_a^- = 0$$

$$I_b = I_a^0 + a^2 I_a^+ + a\, I_a^-$$

$$= j1.7621 + (-0.5 - j0.866)(-j5.5322) + (-0.5 + j0.866) \times j3.77$$

$$= -8.0557 + j2.6431$$

$$I_c = 8.0557 + j2.6431$$

$$I_n = 5.286 \text{ p.u.}$$

Example 10.18 ***For an Example 10.16, determine the fault current when LLG fault occurs between phases b and c. Fault impedance is j0.15 p.u.***

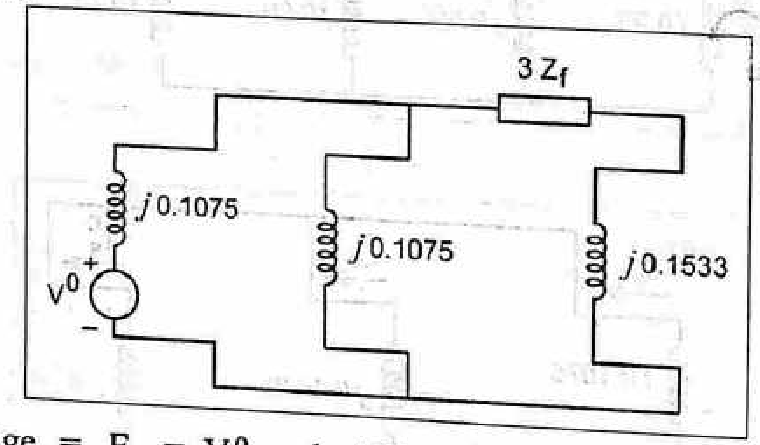
☺ ***Solution :*** Positive sequence impedance $Z_{44}^+ = j0.1075$ p.u.

Negative sequence impedance $Z_{44}^- = j0.1075$ p.u.

Zero sequence impedance $Z_{44}^0 = j0.1533$ p.u.

Fault impedance $Z_f = j0.15$ p.u.

Sequence network is



Prefault voltage $= E_a = V^0 = 1 \angle 0°$

Symmetrical components of currents are :

$$I_a^+ = \frac{V^0}{Z_{44}^+ + \dfrac{Z_{44}^- (Z_{44}^0 + 3Z_f)}{Z_{44}^0 + 3Z_f + Z_{44}^-}}$$

$$= \frac{1 \angle 0°}{j0.1075 + \dfrac{j0.1075\,(j0.1533 + 3 \times j0.15)}{j0.1533 + 3 \times j0.15 + j0.1075}} = -j5.0317 \text{ p.u.}$$

$$I_a^- = -I_a^+ \left[\frac{3Z_f + Z_{44}^0}{Z_{44}^- + 3Z_f + Z_{44}^0} \right]$$

$$= -(-j5.0317) \left[\frac{3 \times j0.15 + j0.1533}{j0.1075 + 3 \times j0.15 + j0.1533} \right] = j4.2707$$

$$I_a^0 = -I_a^+ \left[\frac{Z_{44}^-}{Z_{44}^- + 3\,Z_f + Z_{44}^0} \right]$$

$$= -(-j5.0317) \left[\frac{j0.1075}{j0.1075 + 3 \times j0.15 + j0.1533} \right]$$

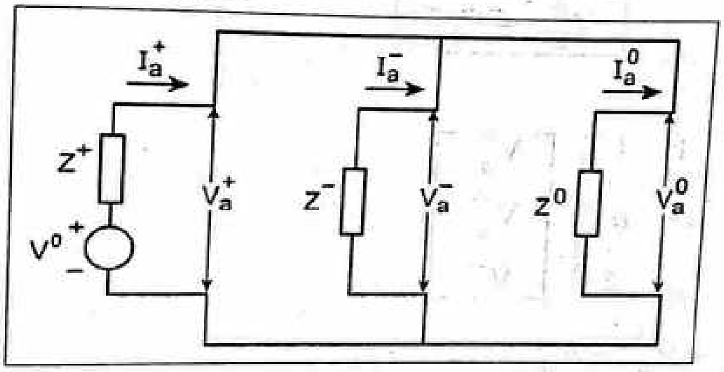
$$= j0.761 \quad \text{p.u.}$$

$$\text{Fault current } I_f = 3\,I_a^0$$

$$= 3 \times j0.761 = j2.2829 \quad \text{p.u.}$$

Example 10.19 ***A 25 MVA, 13.2 KV alternator with solidly grounded neutral has a subtransient reactance of 0.25 p.u. The negative and zero sequence reactances are 0.35 and 0.1 p.u. respectively. A double line to ground fault occurs at the terminals of the alternator, determine the fault current and line-to-line voltages.***

☺ ***Solution :*** Sequence network is



$$\text{Prefault voltage } = E_a = V^0 = 1 \angle 0°$$

$$\text{Positive sequence current } I_a^+ = \frac{V^0}{Z^+ + \left(\frac{Z^- \times Z^0}{Z^- + Z^0} \right)}$$

$$= \frac{1 \angle 0°}{j0.25 + \left(\frac{j0.35 \times j0.1}{j0.35 + j0.1} \right)} = -j3.0508 \quad \text{p.u.}$$

$$I_a^- = -I_a^+ \times \frac{Z^0}{Z^- + Z^0}$$

$$= -(-j3.0508) \times \frac{j0.1}{j0.35 + j0.1} = j0.678 \quad \text{p.u.}$$

$$I_a^0 = -I_a^+ \times \frac{Z^-}{Z^- + Z^0}$$

$$= -(-j3.0508) \times \frac{j0.35}{j0.35 + j0.1} = j2.373 \quad \text{p.u.}$$

$$\text{Fault current} = 3\,I_a^0 = 3 \times j2.373 = j7.119 \quad \text{p.u.}$$

$$\text{Base current} = \frac{\text{MVA}_b}{\sqrt{3} \times \text{KV}_b} = \frac{25 \times 10^3}{\sqrt{3} \times 132} = 1093.466 \quad \text{Amp}$$

$$I_f \text{ in Amp} = j7.119 \times 1093.466 = j7.784 \quad \text{Amp}$$

Symmetrical component of voltages :

$$V_a^0 = -Z^0 I_a^0$$

$$= -j0.1 \times j2.373 = 0.2373 \quad \text{p.u.}$$

$$V_a^+ = E_a - Z^+ I_a^+$$

$$= 1 \angle 0° - j0.25 \times -j3.0508 = 0.2373 \quad \text{p.u.}$$

$$V_a^- = -Z^- I_a^- = -j0.35 \times j0.678$$

$$= 0.2373 \quad \text{p.u.}$$

$$\therefore \boxed{V_a^+ = V_a^-}$$

Phase voltages :

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_a^0 \\ V_a^+ \\ V_a^- \end{bmatrix}$$

$$V_a = V_a^0 + V_a^+ + V_a^-$$

$$= 0.2373 + 0.2373 + 0.2373 = 0.7119$$

$$V_b = V_a^0 + a^2 V_a^+ + a\,V_a^-$$

$$= 0.2373 + (-0.5 - j0.866)\,0.2373 + (-0.5 + j0.866)\,0.2373 = 0$$

$$V_c = V_a^0 + a\,V_a^+ + a^2 V_a^- = 0$$

$$\therefore \boxed{V_b = V_c = 0}$$

Line-to-Line voltages :

$$V_{ab} = V_a - V_b = 0.7119 - 0 = 0.7119 \quad \text{p.u.}$$

$$V_{ac} = V_a - V_c = 0.7119 - 0 = 0.7119 \quad \text{p.u.}$$

$$V_{bc} = V_b - V_c = 0 - 0 = 0 \quad \text{p.u.}$$

Example 10.20 ***A 50 MVA, 11 kV, 3-ph alternator was subjected to different types of faults. The fault currents are 3-ph fault 1870 A, line to line fault 2590 A, single line to ground fault 4130 A. The alternator neutral is solidly grounded. Find the p.u values of the three sequence reactances of the alternator.***

☺ ***Solution:***

$$\text{MVA}_b = 50$$

$$\text{KV}_b = 11$$

$$I_f\ (3\phi \text{ fault}) = 1870 \text{ A}$$

$$I_f\ (\text{L}-\text{L fault}) = 2590 \text{ A}$$

$$I_f\ (\text{L}-\text{G fault}) = 4130 \text{ A}$$

$$Z_0,\ Z^+,\ Z^- = ?$$

$$\text{Base current} = \frac{\text{MVA}_b \times 10^3}{\sqrt{3} \times \text{KV}_b}$$

$$= \frac{50 \times 10^3}{\sqrt{3} \times 11} = 2624.3 \text{ A}$$

$$I_{f\,\text{p.u.}}\ (3\phi) = \frac{V^0}{Z^+}$$

$$= \frac{1870}{2624.3} = -j0.713$$

$$\Rightarrow Z^+ = \frac{1\angle 0°}{-j0.713} = j1.4 \text{ p.u}$$

$$I_{f\,\text{p.u.}}\ (\text{L.L}) = \frac{-j\sqrt{3}\ V^0}{Z^+ + Z^-} = \frac{2590}{2624.3} = -j0.99$$

$$Z^+ + Z^- = \frac{-j\sqrt{3} \times 1\angle 0°}{-j0.99} = +j1.75 \text{ p.u}$$

$$Z^- = j1.75 - Z^+$$

$$= j1.75 - j1.4 = j0.35$$

$$I_{f\,\text{p.u.}}\ (\text{L.G}) = \frac{3\,V^0}{Z^+ + Z^- + Z^0}$$

$$= \frac{4130}{2624.3} = -j1.57$$

$$Z^+ + Z^- + Z^0 = \frac{3 \times 1\angle 0°}{-j1.57} = j1.91$$

$$Z^0 = j1.91 - Z^+ - Z^-$$

$$= j1.91 - j1.4 - j0.35$$

$$= j0.16 \text{ p.u}$$

10.6. SHORT CIRCUIT ANALYSIS OF UNBALANCED LARGE-SCALE SYSTEMS

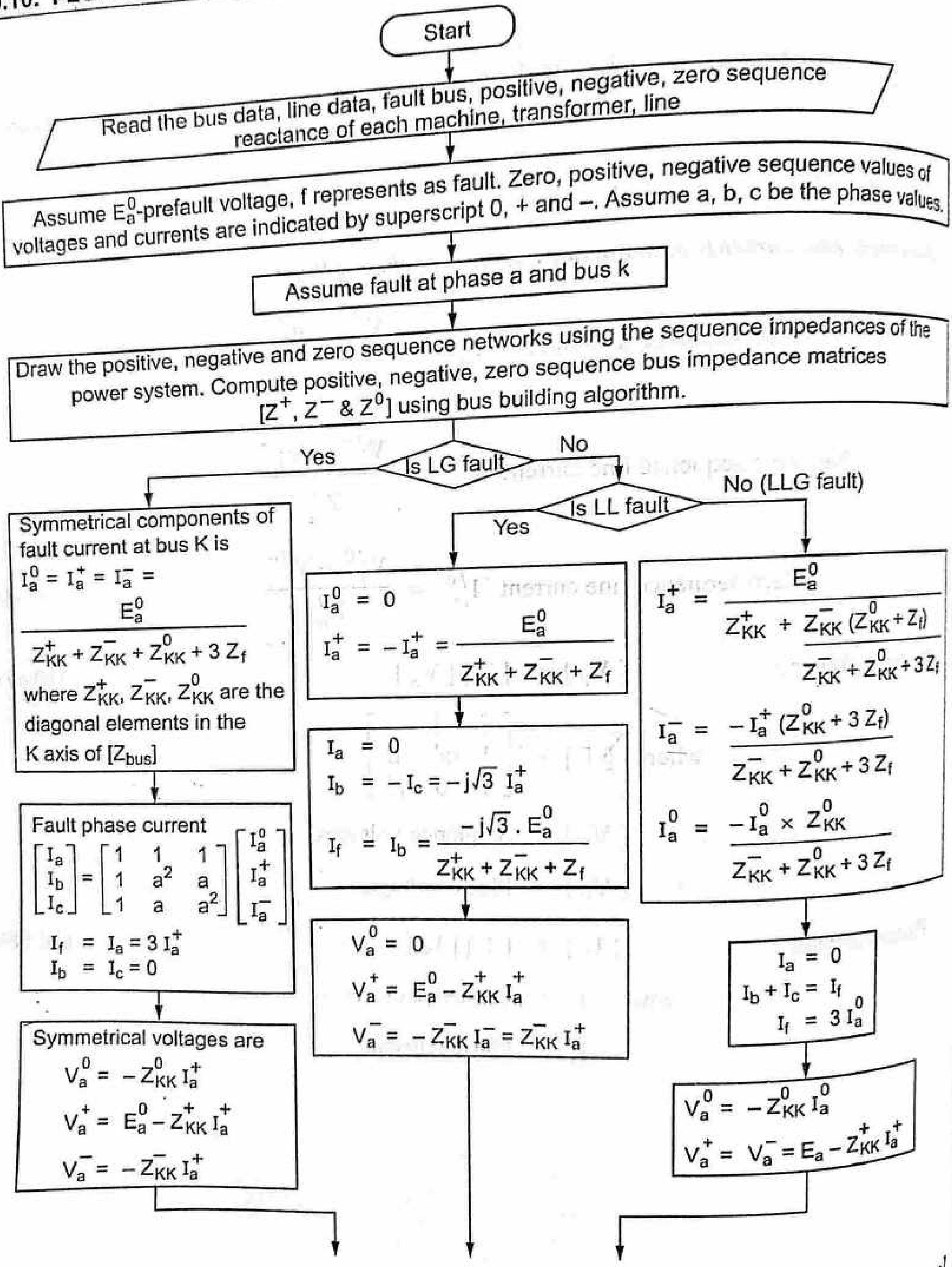
The method of fault analysis explained for symmetrical fault can be extended to unsymmetrical faults. The following symbols are used in unsymmetrical fault calculation.

- Superscript *f* represents post fault or fault values.
- Superscript +, – and 0 represents positive, negative and zero sequence voltages, currents and impedances.
- A number subscript following this positive (+), negative (–) and zero (0) represent the bus code.
- Phase values of voltages and currents are indicated collectively by subscript *p* and individually by the subscript *a*, *b* and *c*.

10.6.1. PROBLEM STATEMENT

- The unsymmetrical fault analysis can be done by using the following steps.
- Assemble the Thevenin's equivalent positive, negative and zero sequence networks separately using the sequence impedances of various power system components like generators, motors, transformers and transmission lines.
- Compute the positive, negative and zero sequence bus impedance matrices Z^+, Z^- and Z^0 using bus building algorithm or short circuit fault impedance matrix $Z_{S, bus}$.
- Select the type (L-L, L-G, L-L-G), location (bus number) and mathematical description of the fault.
- Determine the fault current at the fault bus using the sequence networks for a L-G, L-L and L-L-G faults.
- Determine the prefault sequence voltages and post fault sequence voltages.
- Compute the positive, negative and zero sequence line currents.

10.10. FLOW CHART FOR UNSYMMETRICAL FAULT ANALYSIS



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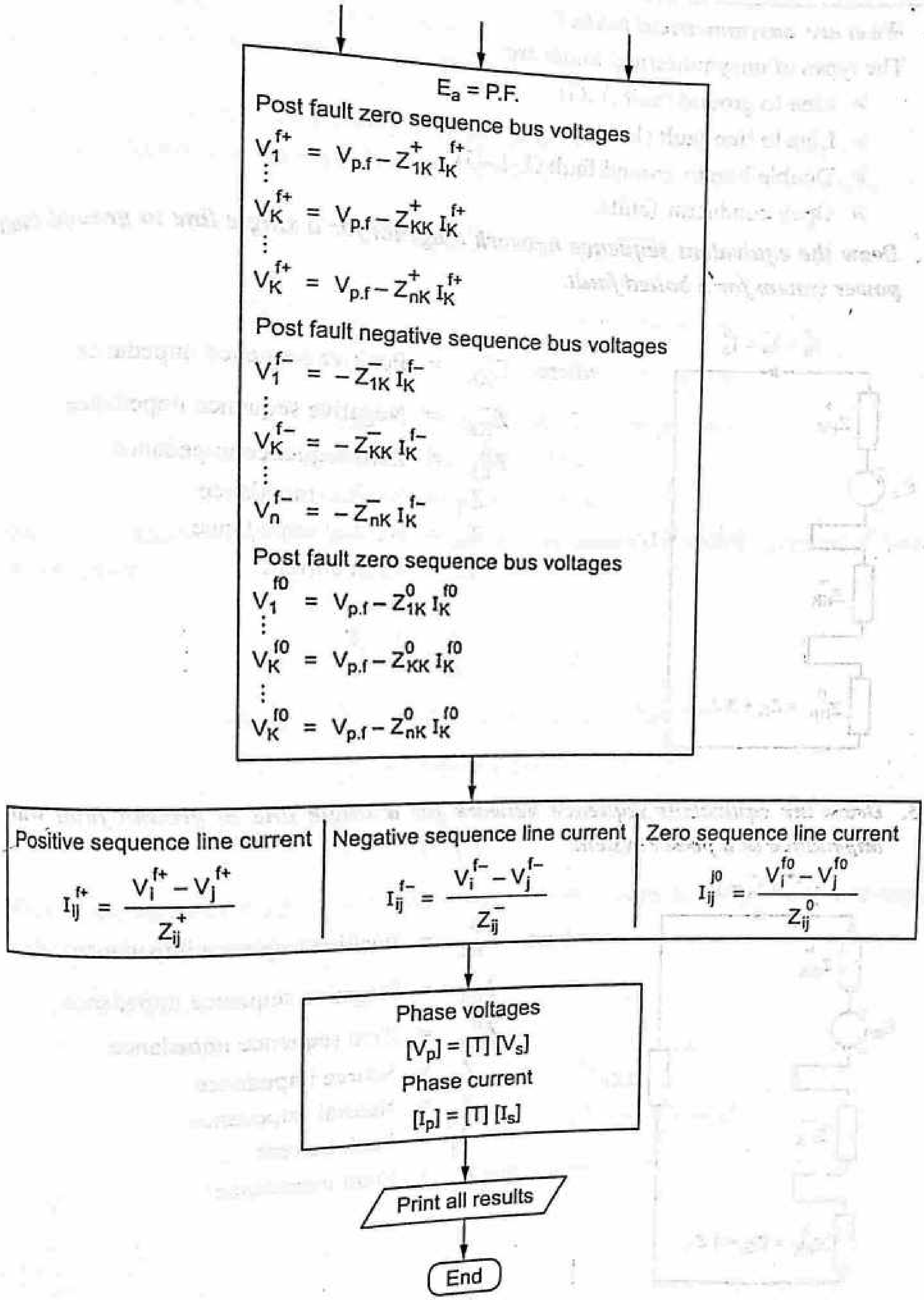
E_a = P.F.
Post fault zero sequence bus voltages
$V_1^{f+} = V_{p.f} - Z_{1K}^{+} I_K^{f+}$
$V_K^{f+} = V_{p.f} - Z_{KK}^{+} I_K^{f+}$
$V_K^{f+} = V_{p.f} - Z_{nK}^{+} I_K^{f+}$
Post fault negative sequence bus voltages
$V_1^{f-} = -Z_{1K}^{-} I_K^{f-}$
$V_K^{f-} = -Z_{KK}^{-} I_K^{f-}$
$V_n^{f-} = -Z_{nK}^{-} I_K^{f-}$
Post fault zero sequence bus voltages
$V_1^{f0} = V_{p.f} - Z_{1K}^{0} I_K^{f0}$
$V_K^{f0} = V_{p.f} - Z_{KK}^{0} I_K^{f0}$
$V_K^{f0} = V_{p.f} - Z_{nK}^{0} I_K^{f0}$
Positive sequence line current
$I_{ij}^{f+} = \frac{V_i^{f+} - V_j^{f+}}{Z_{ij}^{+}}$
Negative sequence line current
$I_{ij}^{f-} = \frac{V_i^{f-} - V_j^{f-}}{Z_{ij}^{-}}$
Zero sequence line current
$I_{ij}^{j0} = \frac{V_i^{f0} - V_j^{f0}}{Z_{ij}^{0}}$
Phase voltages
$[V_p] = [T] [V_s]$
Phase current
$[I_p] = [T] [I_s]$
Print all results
End