UNIT-V - Stability Analysis

Syllabus:

* Importance of stability analysis in power system planning and operation.
* Classification of power system stability
* Angle and voltage stability
* Single machine Infinite Bus (SMIB) S/m.
* Developement of swing equation.
* Equal area criterion.
* Determination of critical clearing angle and time
* Solution of swing equation by
  - Modified Euler Method +
  - Runge-Kutta fourth order Method

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Introduction:

Stability studies:

The stability of a power system is defined as the ability of power system to return to stable operation when it is subjected to a disturbance (it may be small or large disturbance.

(or)

The stability of a physical system is referred as its capability to return to the original position on the occurance of a disturbance.

Steady state stability:

The steady state stability is defined as the ability of a power system to remain or return to stable operation when it is subjected to a small disturbance.

(or)

A power system has steady state stability if after a small slow disturbance it can regain and maintain synchronous speed.

Eg: Normal load Variations or changes.

Steady state stability limit:

It is the maximum possible flow of power without loss of stability when a small disturbance occurs.

Transient stability

The transient stability is defined as the ability of a power system to return to stable operation when it is subjected to a large disturbance.

(or)

A power system has transient stability if after a large disturbance it can regain and maintain synchronous speed.

Eg: Faults and switching.

Transient stability limit:

It refers to the maximum possible flow of power without loss of stability when a large disturbance occurs.

Rotor Angle Stability

It is concered with the ability of interconnected syn m/c of a power system to remain in synchronism under normal operating conditions and after being subjected to disturbance.

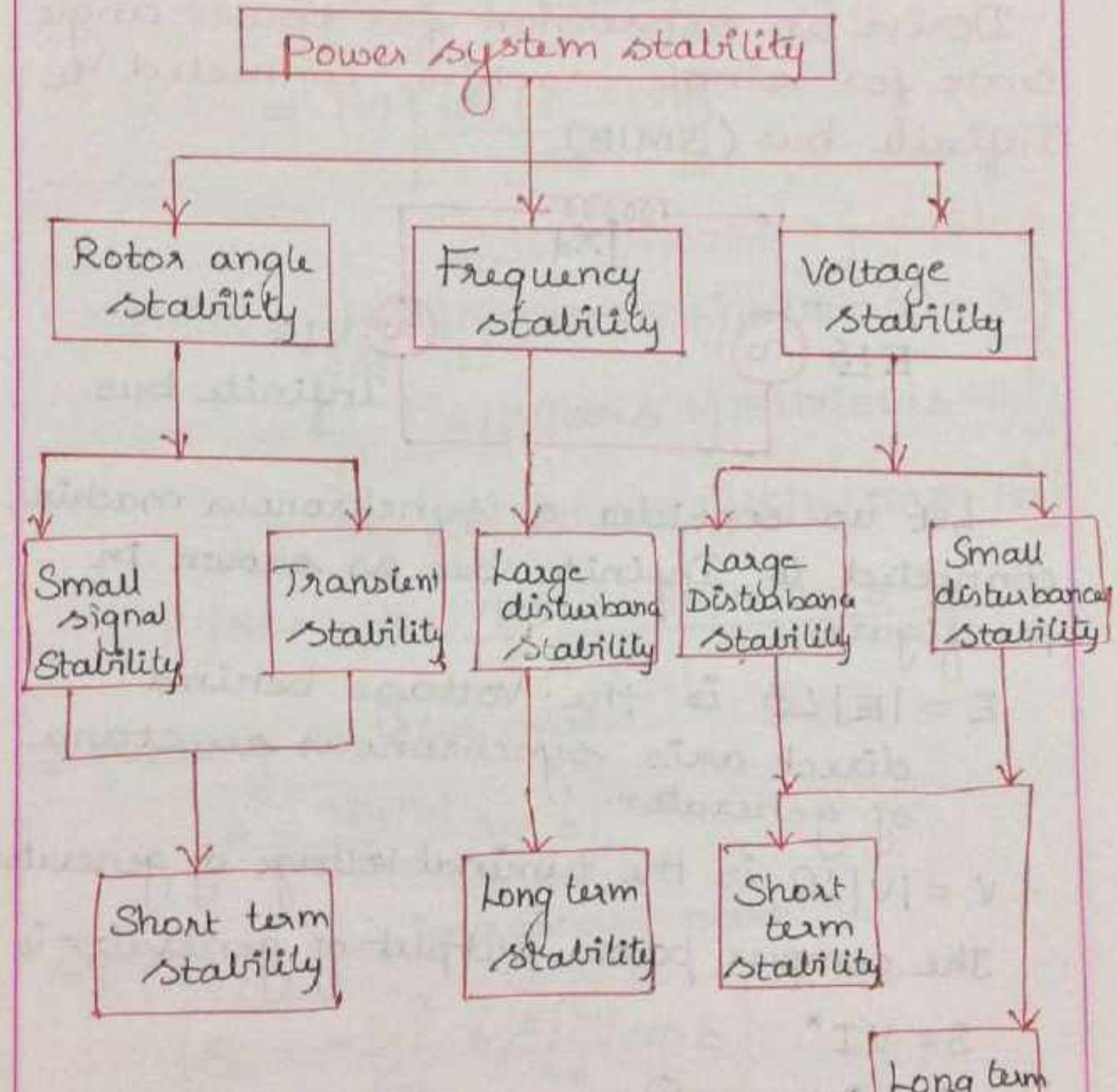
Voltage Stability:

Voltage stability is concerned with the ability of a power system to maintain steady voltages @ all buses in the system under normal operating conditions and after being subjected to a disturbance.

Frequency stability

Frequency stability is concerned with the ability of a power system to maintain steady frequency with in a nominal range following a severe system upset, resulting in a significant imbalance between generation and load.

Classification of power system stability



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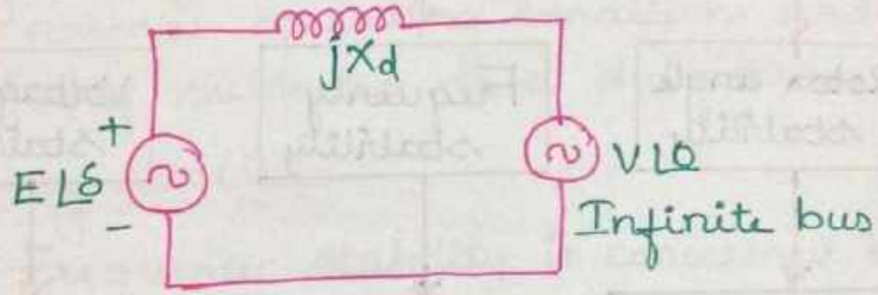
Power Angle curve:

Derive an expression for power angle curve for single machine connected to Infinite bus (SMIB)



Let us consider a synchronous machine connected to Infinite bus as shown in the figure.

$E = |E|\angle\delta$ is the voltage behind direct axis synchronous reactance of generator.

$V = |V|\angle 0$ is the terminal voltage of generator

The complex power output of generator is

$S = VI^*$

$S^* = V^* I \rightarrow ①$

w.r.T $S = P + jQ \Rightarrow S^* = P - jQ$

The current through the line is

$$I = \left[\frac{|E|\angle\delta - |V|\angle 0}{jX_d}\right]$$

① ⟹ $S^* = V^* I$

$$P - jQ = \left[|V|\angle 0\right]^* \left[\frac{|E|\angle\delta - |V|\angle 0}{jX_d}\right]$$

$$= |V| \left[\frac{|E|\angle\delta - |V|}{jX_d}\right]$$

$E\angle\delta = |E|\cos\delta + j|E|\sin\delta$

$$P - jQ = \frac{1}{jX_d}|V|\left[(|E|\cos\delta + j|E|\sin\delta) - |V|\right]$$

$$= \frac{1}{jX_d}\left[|E||V|\cos\delta + j|E||V|\sin\delta - |V|^2\right]$$

$$= \frac{-j}{X_d}\left[|E||V|\cos\delta + j|E||V|\sin\delta - |V|^2\right]$$

$$P - jQ = \frac{-j|E||V|\cos\delta + |E||V|\sin\delta + j|V|^2}{X_d}$$

Equating the real part,

$$\boxed{P = \frac{|E||V|}{X_d}\sin\delta}$$

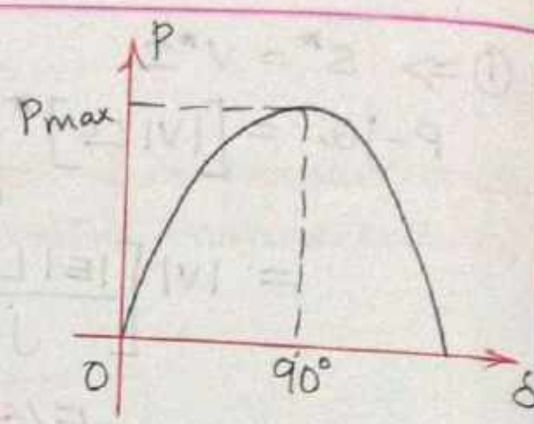
Equating the imaginary part

$$\boxed{Q = -\frac{|V|^2}{X_d} + \frac{|E||V|}{X_d}\cos\delta}$$

where, $P = \frac{|E||V|}{X_d}\sin\delta$ is the power angle equation or curve, when $\delta = 90^\circ$ the power output is maximum

$$P_{max} = \frac{|E||V|}{X_d} \sin 90^\circ$$

$$P_{max} = \frac{|E||V|}{X_d}$$



$$\boxed{\therefore P = P_{max} \sin \delta}$$

Methods of Improving steady state stability

The steady state stability limit is

$$P_{max} = \frac{|E||V|}{X_d}$$

We found that, P_{max} can be increased by

(i) Increasing either or both $|E|$ and $|V|$

(ii) Reducing X_d

(i) Methods for increasing system voltage.

Higher Excitation Voltage and fast response excitation system substantially increase the steady state stability limit. Rising the line voltage also increases the transmission system capabilities.

(ii) Methods for reducing the reactance between stations.

1) Use of double circuit line.

Even if one of the line fails, it can be switched off and hence the continuity of supply can be maintained. But the provision of additional line cannot be justified by stability consideration alone.

2) Use of bundled conductors

Bundling of conductors reduces the line reactance.

3) Series compensation of line reactance.

Inductive reactance of a line can be reduced by connecting static capacitors in series with line.

4) Use of synchronous phase Modifiers

Synchronous phase Modifiers may be installed in intermediate substations to increase the power limit.

5) Use of Machines with low impedance.

It improves the transmission capacity.

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Swing Equation

Derive an expression for swing equation.

A generator receives Mechanical torque input T_i via a shaft from the prime mover and develops a electro-magnetic torque T_e which opposes the Mechanical torque T_i.

The algebric difference between T_i and T_e is the net torque T_a which causes acceleration of rotor.

$$\boxed{T_a = T_i - T_e}$$

When the Machine is operating in steady state, T_a is zero.

When there is imbalance between T_i and T_e there is acceleration or retardation depending on whether T_a is +ve or −ve.

$T_a > 0$ indicates acceleration.

$T_a < 0$ indicates retardation.

From Law of Mechanics

$$\boxed{J \frac{d^2\theta}{dt^2} = T_a = T_i - T_e} \quad \text{—— (1)}$$

where

$J \rightarrow$ Moment of inertia in $kg\text{-}m^2$

$\frac{d^2\theta}{dt^2} \rightarrow$ angular acceleration in elec.deg/sec^2

The angular position θ of the rotor continuously varies with time. It is convenient to measure angular position with respect to synchronously rotating reference axis with respect to stationary axis.

Let, $\delta = \theta - \omega_s t \rightarrow (2)$

$\omega_s \rightarrow$ Synchronous speed in elec.deg/sec

$\delta \rightarrow$ angular displacement of rotor from the synchronously rotating reference axis.

Differentiating eqn (2) w.r.t 't'

$$\frac{d\delta}{dt} = \frac{d\theta}{dt} - \omega_s$$

again differentiating the above eqn w.r.t 't'

$$\frac{d^2\delta}{dt^2} = \frac{d^2\theta}{dt^2} \rightarrow (3)$$

Substituting eqn (3) in (1), we get.

$$J\frac{d^2\delta}{dt^2} = T_a = T_i - T_e \rightarrow (4)$$

Multiplying eqn (4) by 'ω'

$$J\omega\frac{d^2\delta}{dt^2} = T_a\omega = T_i\omega - T_e\omega \rightarrow (5)$$

$J\omega = M \rightarrow$ Angular Momentum

$T\omega = P \rightarrow$ Power

$\therefore$ The equation (5) becomes

$$\boxed{M\frac{d^2\delta}{dt^2} = P_a = P_i - P_e} \rightarrow (6)$$

Swing Equation.

W.K.T Kinetic Energy (KE)

$$KE = \frac{1}{2} J\omega^2$$

$$= \frac{1}{2} \underline{J\omega}\, \omega = \frac{1}{2} M\omega$$

The Inertia constant 'H' is defined as

$$H = \frac{\text{Stored Energy in Mega-joules}}{\text{Machine rating in MVA (G)}}$$

$$H = \frac{\frac{1}{2}M\omega}{G}$$

$$GH = \frac{1}{2} M\omega \Rightarrow M = \frac{2GH}{\omega}$$

$$= \frac{\cancel{2}GH}{\cancel{2}\pi f}$$

$$\boxed{M = \frac{GH}{\pi f}}$$

Substitute $M = \frac{GH}{\pi f}$ in equation (6), we get

$$\boxed{\frac{GH}{\pi f}\frac{d^2\delta}{dt^2} = P_a = P_i - P_e}$$

Sub $\pi = 180^\circ$, we get

$$\boxed{\frac{GH}{180^\circ f}\frac{d^2\delta}{dt^2} = P_a = P_i - P_e} \rightarrow \text{Swing Equation.}$$

Assumptions Made in swing equation

1. The input remains constant during the entire period of analysis. This assumption is quite valid for syn. motors because the speed of the motor doesnot significantly change unless the motor loses synchronism.
2. The resistances of lines and machines are neglected.
3. The effect of damper winding is neglected
4. The line shunt capacitance is also neglected.
5. The effect of saliency is neglected in synchronous machines. The machine is represented as constant voltage behind transient reactance.

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Constants Used in stability Analysis:

Inertia constant M.

From the swing equation

$$\boxed{M\frac{d^2\delta}{dt^2} = P_a}$$

If power is in watts, δ in radians & 't' in sec then M is in Watts/radian/sec^2 (or) Watt sec^2/rad

Since 1 Joule = 1 Watt sec,

$\therefore$ M can have its unit as Joule sec/rad

If power is in MW, then M has MJ sec/rad.

Kinetic energy (N)

$$KE = \frac{1}{2}J\omega^2$$

$$= \frac{1}{2}\underline{J\omega}\,\omega \qquad M = J\omega$$

$$\boxed{KE = \frac{1}{2}M\omega_s = N}$$

$\omega_s = 2\pi n_s$; Mech rad/sec

$= 2\pi f$; elect rad/sec

$= 360f$; elect deg/sec.

Inertia constant (H)

$$\boxed{H = \frac{K.E}{G}}\ \text{MJ/MVA}$$

$H = \frac{N}{G}$ $\quad$ $G \rightarrow$ MVA rating

$N = GH$

Relationship between M & H

We have

$$K.E = N = \frac{1}{2} M \omega_s$$

$$M = \frac{2N}{\omega_s} = \frac{2N}{360f} = \frac{N}{180f}$$

Sub $N = GH$, we get

$$\boxed{M = \frac{GH}{180f}} \text{ MJ sec/elect degree}$$

If the angle is expressed in radians

$$\boxed{M = \frac{GH}{\pi f}} \text{ MJ sec/elect radian}$$

Problems:

1) A 50 HZ, 4 pole turbo alternator rated 100 MVA, 11 KV has an inertia constant of 8 MJ/MVA. Find

1) the energy stored in the rotor @ synchronous speed.

2) the rotor acceleration if the Mechanical i/p is suddenly raised to 80 MW for an electric load of 50 MW (Neglect Mechanical and electrical losses)

Solution:

$H = 8$ MJ/MVA ; $G = 100$ MVA

(i) Kinetic Energy / K.E $= GH$

$= 100 \times 8 = 800$ MJ

(ii) Swing equation is

$$\boxed{M\frac{d^2\delta}{dt^2} = P_a = P_i - P_e}$$

Here $P_i = 80\ MW$

$P_e = 50\ MW$

$$\boxed{P_a = P_i - P_e = 30\ MW}$$

$$M = \frac{GH}{180f}$$

$$= \frac{800}{180 \times 50} = 0.0889\ MJ\ sec$$

∴ Accelearation

$$\alpha = \frac{d^2\delta}{dt^2} = \frac{P_a}{M}$$

$$= \frac{30}{0.0889}$$

$$\boxed{\frac{d^2\delta}{dt^2} = 337.5\ \text{Elect. deg/sec}}$$

3) case

If the accelearation calculated in part (b) is maintained for 10 cycles, find the change in torque angle and rotor speed in revolution per minute @ the end of this period.

$1\ \text{cycle} = \frac{1}{50} = 0.02\ sec$ $\left[\text{freq} = 50\ Hz \quad \text{Time} = \frac{1}{\text{freq}}\right]$

$10\ \text{cycles} = 0.02 \times 10 = 0.2\ sec$

Let $\frac{d^2\delta}{dt^2} = \alpha$

$$\frac{d\delta}{dt} = \int \alpha\, dt$$
$$= \alpha t$$
$$\delta = \int \alpha t\, dt$$
$$\delta = \frac{1}{2}\alpha t^2$$

$\therefore$ change in $\delta = \frac{1}{2}\alpha t^2$

$$= \frac{1}{2} \times 337.5 \times 0.2^2$$
$$= 6.75 \text{ elect. degree}$$

Since it is a four pole machine

1 Revolution corresponds to $4 \times 180^\circ$

$$= 720^\circ \text{ elect. deg}$$

$$\alpha = 337.5 \times \frac{60}{720}$$
$$= 28.125 \text{ rpm/sec}$$

Syn. speed $N_s = \frac{120 f}{P} = \frac{120 \times 50}{4}$

$$N_s = 1500 \text{ rpm.}$$

Rotor speed @ the end of 10 cycles $\Big\}$ = Syn. speed + change in speed

$$= 1500 + (28.125 \times 0.2)$$
$$= 1500 + 5.625$$
$$= 1505.625 \text{ rpm}$$

— x —

(2) The moment of inertia of a 4 pole, 100 MVA, 11 kV, 3ϕ, 0.8 pf, 50 Hz turbo alternator is 10000 kg-m². Calculate H and M.

Solution :-

$$J = 10000 \text{ kg-m}^2 \qquad P = 4 \text{ pole}$$
$$G = 100 \text{ MVA} \qquad f = 50 \text{ Hz}$$

$$N_s = \frac{120f}{P} = \frac{120 \times 50}{4} = 1500 \text{ rpm}$$

$$n_s = \frac{N_s}{60} = \frac{1500}{60} = 25 \text{ rps}$$

$$\omega_s = 2\pi n_s = 2\pi \times 25 = 50\pi$$

$$K.E = N = \frac{1}{2} J \omega_s^2$$
$$= \frac{1}{2} \times 10000 \times (50\pi)^2$$

$$\boxed{K.E = 123.37 \text{ MJ}}$$

Inertia constant $H = \frac{K.E}{G}$

$$\boxed{H = \frac{123.37}{100} = 1.2337 \text{ MJ/MVA}}$$

$$M = \frac{GH}{180f}$$
$$= \frac{1.2337}{180 \times 50}$$

$$\boxed{M = 0.0137 \text{ MJ sec/elect.deg}}$$

(3) A 2 pole, 50 Hz, 11 kV turbo alternator has a rating of 100 MW, pf 0.85 lagging. The rotor has a moment of Inertia of 10000 kg m^2. Calculate H and M.

Soln: $J = 10000 \text{ kg-m}^2$ $\quad p = 2$ pole. $\quad f = 50$ Hz

$G = 100$ MW $\quad$ pf $= 0.85$ (Lag)

G in MVA $= \frac{100}{0.85} = 117.675$ MVA

$N_s = \frac{120 f}{P} = \frac{120 \times 50}{2} = 3000$ rpm

$n_s = \frac{N_s}{60} = \frac{3000}{60} = 50$ rps

$\omega_s = 2\pi n_s = 2\pi \times 50 = 100\pi$

$K.E = N = \frac{1}{2} J \omega_s^2$

$= \frac{1}{2} \times 10{,}000 \times (100\pi)^2$

$= 492.980$ MJ

Inertia constant

$H = \frac{K.E}{G}$

$= \frac{492.980}{117.675} = 4.189$

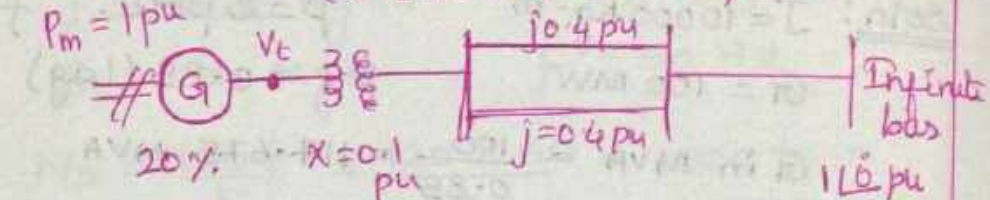
$\boxed{H = 4.189 \text{ MJ/MVA}}$

$M = \frac{GH}{180 f}$

$= \frac{117.675 \times 4.189}{180 \times 50}$

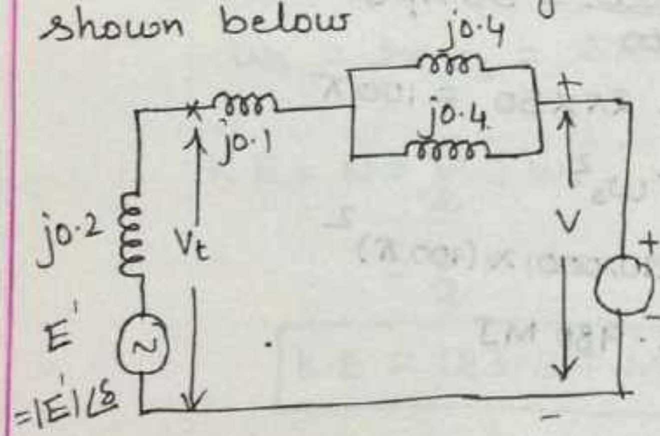
$\boxed{M = 0.0547 \text{ MJ sec/elect.deg}}$

4) The Generator shown in figure is delivering power to infinite bus. Take $|V_t| = 1.1$ pu. Find the maximum power that can be transferred when (i) system is healthy (2) one line is open.

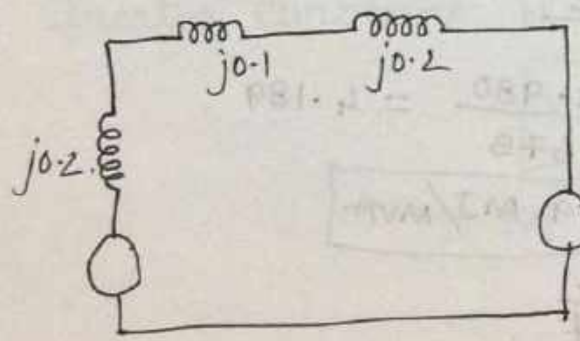


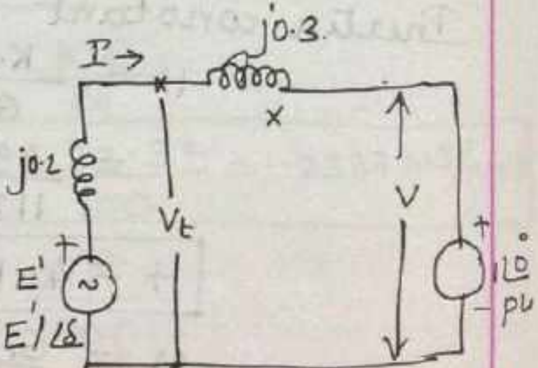
Solution:

The reactance diagram of the s/m is shown below



Here $V_t = |V_t|\angle\theta$
$= 1.1\angle\theta$
$V = 1\angle 0^\circ$ pu





Let the electrical power transferred @ steady state be equal to P_m

$\therefore P_e = P_m = 1$ pu

W.K.T

$$P_m = \frac{|E||V|}{X}\sin\theta$$

$$= \frac{|V_t||V|}{X}\sin\theta$$

$$1 = \frac{1.1 \times 1}{0.3}\sin\delta \Rightarrow \sin\theta = \left[\frac{1\times 0.3}{1.1\times 1}\right]$$

$$\theta = \sin^{-1}\left[\frac{1\times 0.3}{1.1\times 1}\right]$$

$$\theta = 15.8^\circ$$

$$\therefore V_t = |V_t|\angle\theta = 1.1\angle 15.8^\circ = 1.058 + j0.3$$

current $I = \frac{V_t - V}{X} = \frac{1.058 + j0.3 - 1}{j0.3}$

$$= \frac{0.058 + j0.3}{j0.3}$$

$$= \frac{0.3056\angle 79^\circ}{0.3\angle 90^\circ} = 1.0187\angle -11^\circ \text{ pu}$$

Using KVL we can write,

$$E' = j0.2I + V_t$$

$$= j0.2 \times 1.0187\angle -11 + 1.1\angle 15.8^\circ$$

$$= 0.2\angle 90^\circ \times 1.0187\angle -11 + 1.058 + j0.3$$

$$= 0.2037\angle 79^\circ + 1.058 + j0.3$$

$$= 0.039 + j0.2 + 1.058 + j0.3$$

$$= 1.097 + j0.5$$

$$\boxed{E' = 1.2056\angle 24.5^\circ \text{ pu}}$$

$\therefore |E'| = 1.2056$ pu.

$\delta = 24.5^\circ$

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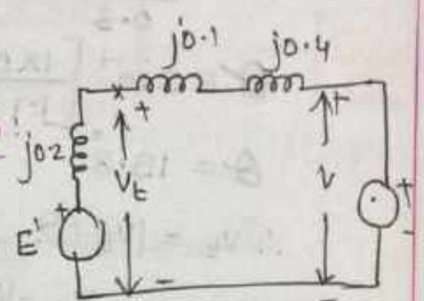
Case(i) Healthy system:

$$X_{12} = 0.5 \text{ pu}.$$

$$\therefore P_{max} = \frac{|E'||V|}{X_{12}} = \frac{1.2056 \times 1}{0.5} = 2.4112 \text{ pu}.$$

$$P_e = P_{max} \sin\delta$$

$$\boxed{P_e = 2.4112 \sin\delta}$$



Case(ii) when one line is open:

$$X_{12} = j0.2 + j0.1 + j0.4$$

$$X_{12} = j0.7 \text{ pu}$$

$$\therefore P_{max} = \frac{|E'||V|}{X_{12}} = \frac{1.2056 \times 1}{0.7} = 1.722 \text{ pu}.$$

$$P_e = P_{max} \sin\delta$$

$$\boxed{P_e = 1.722 \sin\delta}$$

S.92 J.A

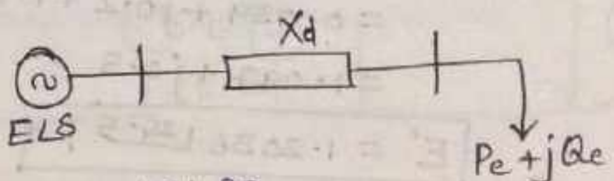
5) A generator having $X_d = 0.7$ pu, delivers rated load @ a p.f. of 0.8 lagging. Find P_e, Q_e, E & δ.

Given: $X_d = 0.7$ pu.

$pf = \cos\phi = 0.8$ lagging

$V = 1\angle 0^\circ$

To find: $P_e, Q_e, E, \delta = ?$



$$S = \frac{1\angle 36.87}{0.8} = 1.25\angle 36.87$$

$$I_a = \frac{S^*}{V^*} = \frac{1.25\angle -36.87}{1\angle 0^\circ} = 1.25\angle -36.87^\circ$$

$E' = V + jX_d' \times I_a$

$= 1.0 + j0.7 \times 1.25 \angle -36.87^\circ$

$= 0.35 + j0.59 = 0.69 \angle 1.03$ pu.

$\boxed{|E'| = 0.69}$

$\boxed{\delta = 1.03}$

$$P_e = \frac{|E'||V|\sin\delta}{X_d'}$$

$$= \frac{0.69 \times 1 \times \sin 1.03}{0.7} = 0.845$$

$$Q_e = \frac{|E'||V|\cos\delta}{X_d'} - \frac{|V|^2}{X_d'}$$

$$= \frac{0.69 \times 1 \times \cos 1.03}{0.7} - \frac{1}{0.7} = -0.92 \text{ pu.}$$

Another Method

6) A generator having $X_d = 0.7$ pu, delivers rated load @ a pf of 0.8 lagging. Find P_e, Q_e, E & δ

Solution:

Let $V = 1\angle 0^\circ$, $I = 1.0(0.8 - j0.6)$

$E = V + jX_d I$

$= 1 + j0.7 \times 1.0(0.8 - j0.6)$

$E = 1.53 \angle 21.5$

$\boxed{|E| = 1.53 \text{ pu}}$, $\boxed{\delta = 21.5^\circ}$

$$\therefore P_e = \frac{|E||V|}{X_d}\sin\delta = \frac{1.53 \times 1}{0.7}\sin 21.5^\circ = 0.8 \text{ pu}$$

$$Q_e = \frac{|E||V|}{X_d}\cos\delta - \frac{|V|^2}{X_d} = \frac{1.53 \times 1}{0.7}\cos 21.5 - \frac{1^2}{0.7}$$

$\boxed{Q_e = 0.6 \text{ pu}}$

30.08.2015

Equal Area Criterion

The stability of single machine connected to infinite bus can be studied by the use of equal area criterion (EAC)

This method is used for the quick prediction of stability. This method is based on the graphical interpretation of the energy stored in the rotating mass as an aid to determine if the machine maintains its stability after a disturbance.

This method is only applicable to a one-machine system connected to an infinite bus or a two-machine system. This method provides a physical insight to the dynamic behaviour of the machine.

Proof: consider a synchronous machine connected to an infinite bus. The swing equation without damping is given by

$$\frac{H}{\pi f_0}\frac{d^2\delta}{dt^2} = P_m - P_e = P_a \quad \text{—— (1)}$$

where P_a is the accelerating power. From the above equation

$$\frac{d^2\delta}{dt^2} = \frac{\pi f_0}{H}(P_m - P_e) \quad \text{—— (2)}$$

Multiplying both sides of eqn (2) by $2\frac{d\delta}{dt}$

$$2\frac{d\delta}{dt}\frac{d^2\delta}{dt^2} = \frac{\pi f_0}{H}(P_m - P_e)\, 2\frac{d\delta}{dt}$$

$$2\frac{d\delta}{dt}\frac{d^2\delta}{dt^2} = \frac{2\pi f_0}{H}(P_m - P_e)\frac{d\delta}{dt} \quad \text{—— (3)}$$

$$\frac{d}{dt}\left[\left(\frac{d\delta}{dt}\right)^2\right] = \frac{2\pi f_0}{H}(P_m - P_e)\frac{d\delta}{dt} \quad \text{—— (4)}$$

Integrating on both sides

$$\int \frac{d}{dt}\left[\left(\frac{d\delta}{dt}\right)^2\right] = \int \frac{2\pi f_0}{H}(P_m - P_e)\frac{d\delta}{dt}$$

$$\left(\frac{d\delta}{dt}\right)^2 = \frac{2\pi f_0}{H}\int_{\delta_0}^{\delta}(P_m - P_e)\,d\delta \quad \text{—— (5)}$$

(or)

$$\boxed{\frac{d\delta}{dt} = \sqrt{\frac{2\pi f_0}{H}\int_{\delta_0}^{\delta_1}(P_m - P_e)\,d\delta}} \quad \text{—— (6)}$$

Equation (6) gives the relative speed of the machine with respect to the synchronously revolving reference frame.

For stability, this speed must become zero at sometime after the disturbance, therefore from equation (6), we have for the stability criterion

$$\int_{\delta_0}^{\delta}(P_m - P_e) = 0 \quad \text{—— (7)}$$

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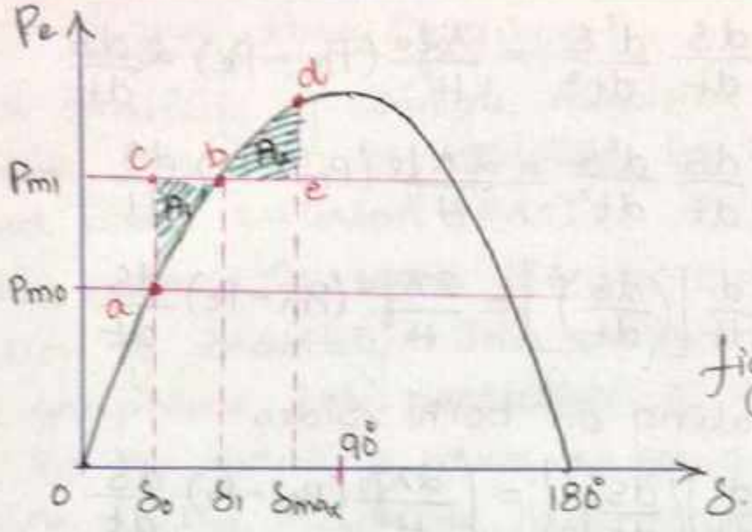


fig(i)

Refering fig(i)

* A machine is operating at δ_0 corresponding to the mechanical power input $P_{m0} = P_{e0}$.

* There is a sudden increase in mech power input which is represented as a horizontal line P_{m1} in the figure.

* Since $\underline{P_{m1} > P_{e0}}$, the accelerating power on rotor is +ve and power angle 'δ' increases from δ_0 to δ_1.

* ∴, The energy stored in the rotor during initial acceleration

$$\int_{\delta_0}^{\delta_1} (P_{m1} - P_e)\, d\delta = \text{Area abc} = \text{Area } A_1$$

* Due to the presence of accelerating power electrical power also increases and matches with P_{m1} when $\delta = \delta_1$ and accelerating power becomes zero at this point.

* Eventhough the accelerating power is zero at this point, the rotor is running above syn speed, hence δ and electrical power P_e will continue to increases.

* When $P_e > P_m$, the rotor decelerate towards syn. speed so the rotor must swing past to point 'b' untill an equal amount of energy is given up by the rotating masses. The energy given up by the rotor as it decelerates back to syn. speed is

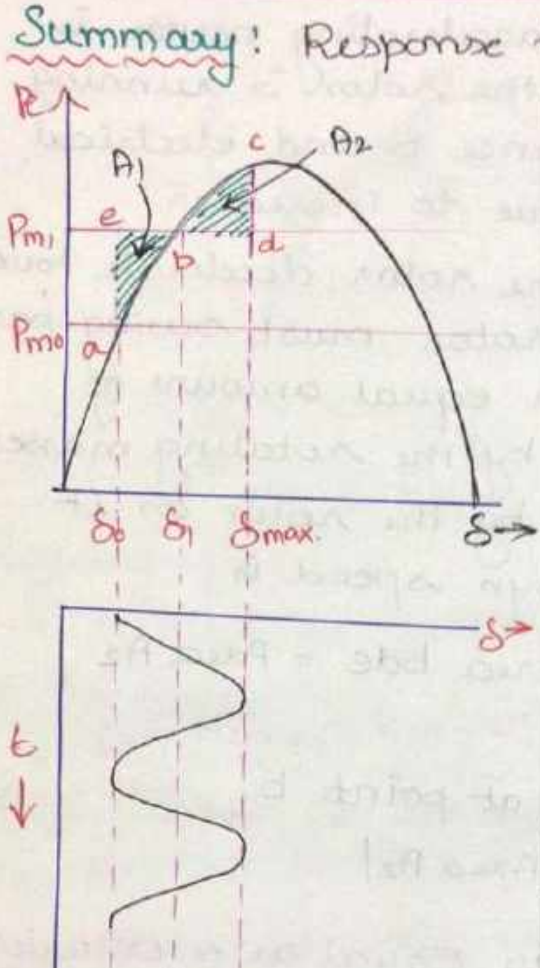
$$\int_{\delta_1}^{\delta_{max}} (P_{m_1} - P_e)\, d\delta = \text{Area bde} = \text{Area } A_2$$

The result is that at point 'b'.

$$|\text{Area } A_1| = |\text{Area } A_2|$$

This is known as Equal area criterion. The rotor angle would then oscillate back and forth between δ_0 and δ_{max} at its natural frequency. The damping present in the machine will cause these oscillations to subside and the new steady state operation would be established at point 'b'.

Summary: Response to step change in P_m.



@ point 'a'

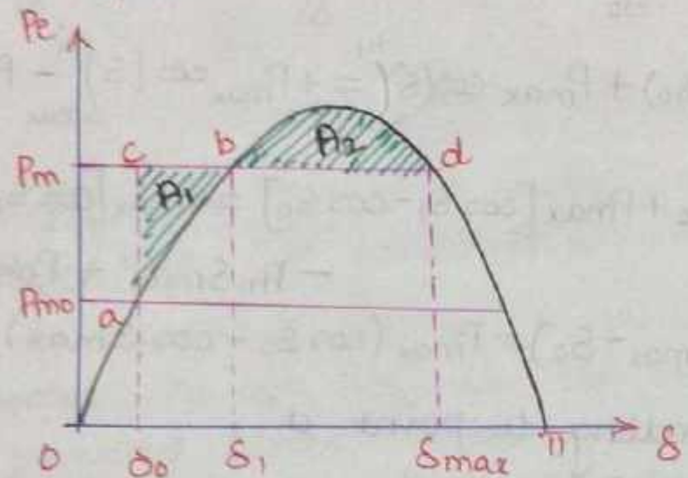
Normal or steady state operating point with $P_{mo} = P_{eo}$ & no accelerating power

@ point 'b'

With ↑se in P_{mo} to P_{m1} the new operating point is 'b' where $P_{m1} > P_{eo}$ & results in accelerating torque and the rotor will not settle in this point 'b' itself because speed of the rotor is greater than syn. speed.

@ point 'c' due to ↑se in speed rotor still change its position from 'b' to 'c' with the ↑se in δ value as δ_{max}. after reaching point 'c', $P_e > P_m \Rightarrow$ decelerating torque is produced and rotor return to point 'b' and oscillates between cab. If sufficient damping force is present rotor will settle to the new equilibrium point 'b' where $|area\ A_1| = |Area\ A_2|$ ie equal area criterion is satisfied

Application of EAC to sudden increase in power input



- EAC is applied to find out how mech. power should be added to maintain stability.
- With sudden ↑se in power input, stability is maintained only if area A_2 at least equal to A_1 can be located above Pm.
- If Area A_2 is less than Area A_1, the accelerating momentum can never be overcome.
- The limit of stability occurs when δ_{max} is at the intersection of line Pm and the power angle curve $90° < \delta < 180°$

Apply EAC to figure,

$$\int_{\delta_0}^{\delta_1} (P_m - P_e)\, d\delta = -\int_{\delta_1}^{\delta_{max}} (P_m - P_e)\, d\delta \quad \text{——} ①$$

$$\int_{\delta_0}^{\delta_1} P_m \, d\delta - \int_{\delta_0}^{\delta_1} P_{max} \sin\delta \, d\delta = \int_{\delta_1}^{\delta_{max}} P_{max} \sin\delta \, d\delta - \int_{\delta_1}^{\delta_{max}} P_m \, d\delta$$

$$P_m(\delta_1 - \delta_0) + P_{max} \cos(\delta)\Big|_{\delta_0}^{\delta_1} = +P_{max} \cos[\delta]_{\delta_{max}}^{\delta_1} - P_m(\delta_{max} - \delta_1)$$

$$\cancel{P_m \delta_1} - P_m \delta_0 + P_{max}[\cos\delta_1 - \cos\delta_0] = P_{max}[\cos\delta_1 - \cos\delta_{max}] - P_m \delta_{max} + \cancel{P_m \delta_1}$$

$$P_m(\delta_{max} - \delta_0) = P_{max}(\cos\delta_0 - \cos\delta_{max}) \quad \text{--- (2)}$$

W.K.T According to point 'd'

$$P_m = P_{max} \sin\delta_{max} \quad \text{— substitute in (2)}$$

$$\cancel{P_{max}} \sin\delta_{max}(\delta_{max} - \delta_0) = \cancel{P_{max}}(\cos\delta_0 - \cos\delta_{max})$$

$$\boxed{(\delta_{max} - \delta_0)\sin\delta_{max} + \cos\delta_{max} = \cos\delta_0} \quad \text{--- (3)}$$

The above non-linear algebric equation can be solved by an iterative technique for δ_{max}. Once δ_{max} is obtained, the max. permissible power on the transient stability is found from

$$\boxed{P_m = P_{max} \sin\delta_1}$$

where $\delta_1 = \pi - \delta_{max}$.

Application of EAC to 3 ϕ fault

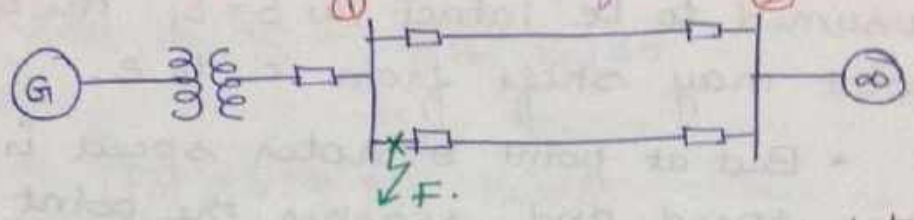
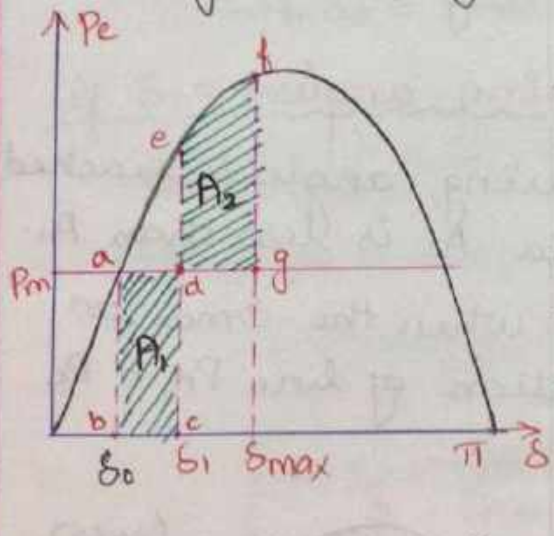


fig - One machine system connected to infinite bus, 3 ϕ fault @ F

In the above figure, a generator is connected to an infinite bus bar through two parallel lines. Assume that the power P_m is constant and the machine is operating steadily, delivering power to the s/m with δ_0



A temporary three ϕ fault occurs @ the sending end of the line @ 'F'

- Since the fault is @ the sending end of the line @ point F, no power is transmitted
- Electrical power o/p is zero. (ie) $P_e = 0$
- Since $P_e = 0$, $P_m > P_e \Rightarrow$ leads to acceleration torque.
- Because of acceleration rotor speed ↑ses. additional kinetic energy is stored and increasing the angle 'δ'.

* When the fault is cleared, both lines are assumed to be intact (ie) $\delta = \delta_1$, the operating point may shift from 'c' to 'e'.

* But at point 'e' rotor speed is above syn. speed and reaches the point 'f'. (the net power is now decelerating and the previously stored KE will be reduced to 0 at point f).

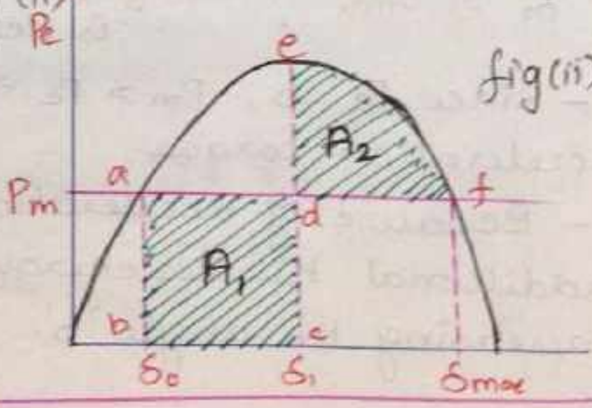
* at point 'f', $P_e > P_m$, the rotor continues to decelerates to 'a' thro 'e' and oscillates between these points. If sufficient damping is provided the operating point 'a' can be maintained.

case(i)

To find critical clearing angle

The critical clearing angle is reached when decelerating area A_2 is less than A_1.

This will happen when the 'δ_{max}' (or) point 'f' is at intersection of line P_m & P_e as shown in fig (ii)



Applying EAC to fig (ii)

$$\int_{\delta_0}^{\delta_1} (P_m - \cancel{P_e}^{\,0})\, d\delta = -\int_{\delta_1}^{\delta_{max}} (P_m - P_e)\, d\delta$$

$$\int_{\delta_0}^{\delta_1} P_m\, d\delta = \int_{\delta_1}^{\delta_{max}} (P_e - P_m)\, d\delta$$

$$P_m(\delta)_{\delta_0}^{\delta_1} = \int_{\delta_1}^{\delta_{max}} P_{max} \sin\delta\, d\delta - P_m(\delta)_{\delta_1}^{\delta_{max}}$$

$$P_m(\delta_1 - \delta_0) = \int_{\delta_1}^{\delta_{max}} P_{max} \underbrace{\sin\delta\, d\delta}_{\substack{-\cos\delta \\ -\cos\delta_{max} + \cos\delta_1}} - P_m(\delta_{max} - \delta_1)$$

$$\cancel{P_m \delta_1} - P_m\delta_0 = P_{max}\left[\cos\delta_1 - \cos\delta_{max}\right] - P_m\delta_{max} + \cancel{P_m\delta_1}$$

$$-P_m\delta_0 = P_{max}\cos\delta_1 - P_{max}\cos\delta_{max} - P_m\delta_{max}$$

if $\delta_1 = \delta_c$

$$P_{max}\cos\delta_c = -P_m\delta_0 + P_{max}\cos\delta_{max} + P_m\delta_{max}$$

$$P_{max}\cos\delta_c = P_m(\delta_{max} - \delta_0) + P_{max}\cos\delta_{max}$$

$$\cos\delta_c = \frac{P_m}{P_{max}}(\delta_{max} - \delta_0) + \frac{\cancel{P_{max}}}{\cancel{P_{max}}}\cos\delta_{max}$$

$$\therefore \boxed{\cos\delta_c = \frac{P_m}{P_{max}}(\delta_{max} - \delta_0) + \cos\delta_{max}}$$

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To find critical clearing time (t_c)

Swing equation must be solved to find the critical clearing time. But for this particular case, due to fault $P_e = 0$. Hence this can be solved analytically.

$$\frac{H}{\pi f_0} \frac{d^2\delta}{dt^2} = P_m - \cancel{P_e}^{\,0} \qquad (1)$$

$$\frac{d^2\delta}{dt^2} = \frac{\pi f_0}{H} P_m \qquad (2)$$

Integrating both the sides

$$\int \frac{d}{dt}\left(\frac{d\delta}{dt}\right) = \int \frac{\pi f_0}{H} P_m$$

$$\frac{d\delta}{dt} = \frac{\pi f_0}{H} P_m \int_0^t dt$$

$$\frac{d\delta}{dt} = \frac{\pi f_0}{H} P_m t \qquad (3)$$

Integrating again, we get

$$\int \frac{d\delta}{dt} = \int \frac{\pi f_0}{H} P_m t$$

$$\delta = \frac{\pi f_0}{H} P_m \int_0^t t$$

$$= \frac{\pi f_0}{H} P_m \frac{t^2}{2} + \delta_0$$

$$\delta = \frac{\pi f_0}{2H} P_m t^2 + \delta_0$$

If $t = t_c \rightarrow$ critical clearing time + $\delta = \delta_c$

$$\delta_c = \frac{\pi f_0}{2H} P_m t_c^2 + \delta_0$$

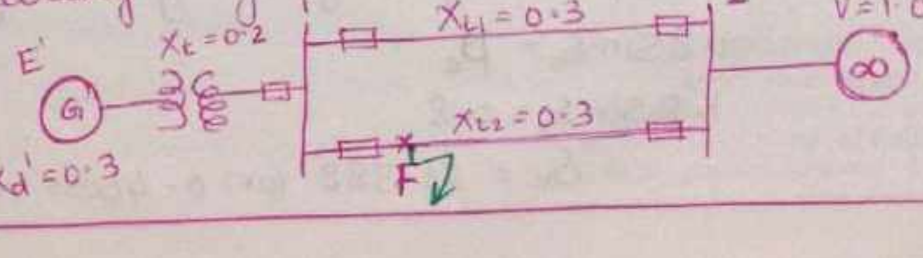
$$\frac{\pi f_0}{2H} P_m t_c^2 = \delta_c - \delta_0$$

$$t_c^2 = \frac{(\delta_c - \delta_0) 2H}{\pi f_0 P_m}$$

$$\therefore \boxed{t_c = \sqrt{\frac{2H(\delta_c - \delta_0)}{\pi f_0 P_m}}}$$

(7) A 60 HZ syn. generator having inertia constant $H = 5$ MJ/MVA and a direct axis transient reactance $x_d' = 0.3$ pu is connected to an infinite bus thro' a purely reactive circuit as shown in figure. Reactances are marked on the diagram on a common s/m base. The generator is delivering real power $P_e = 0.8$ pu and $Q = 0.074$ pu to the infinite bus @ a voltage of $V = 1$ pu.

A temporary 3Ø fault occurs @ the sending end of the line at point F. When the fault is cleared, both lines are intact. Det. the critical clearing angle and the critical fault clearing time.



Solution:

The current flowing into the ∞ bus

$$I = \frac{S^*}{V^*} = \frac{0.8 - j0.074}{1.0\angle 0^\circ} = 0.8 - j0.074 \text{ pu.}$$

The transfer reactance between internal voltage and the infinite bus before fault is

$$X_1 = 0.3 + 0.2 + \frac{0.3}{2} = 0.65$$

The transient internal voltage is

$$E' = V + jX_1 I$$

$$= 1.0 + (j0.65)(0.8 - j0.074)$$

$$\boxed{E' = 1.17\angle 26.387^\circ \text{ pu}}$$

Since both lines are intact when the fault is cleared, power-angle eqn before & after the fault is

$$P_{max} \sin\delta = P_m$$

$$P_{max} \sin\delta = \frac{|E'||V|}{X_1} \sin\delta$$

$$= \frac{(1.17)(1.0)}{0.65} \sin\delta$$

$$= 1.8 \sin\delta$$

The initial operating angle is given by

$$1.8 \sin\delta_0 = P_e$$

$$1.8 \sin\delta_0 = 0.8$$

$$\delta_0 = 26.388^\circ \text{ (or) } 0.46055 \text{ rad.}$$

Referring to fig (ii)

$$\delta_{max} = 180^\circ - \delta_0$$
$$= 180 - 26.388^\circ$$
$$= 153.612^\circ \text{ (or) } 2.681 \text{ rad.}$$

The critical clearing angle is given by

$$\cos\delta_c = \frac{P_m}{P_{max}}(\delta_{max} - \delta_0) + \cos\delta_{max}$$
$$= \frac{0.8}{1.8}(2.681 - 0.46055) + \cos(153.612^\circ)$$
$$\cos\delta_c = 0.09106$$
$$\delta_c = \cos^{-1}(0.09106)$$
$$\boxed{\delta_c = 84.775^\circ}$$
$$\boxed{\delta_c = 1.48 \text{ rad}}$$

The critical fault clearing time is

$$t_c = \sqrt{\frac{2H(\delta_c - \delta_0)}{\pi f_0 P_m}}$$
$$= \sqrt{\frac{2(5)(1.48 - 0.46055)}{\pi(60)(0.8)}}$$
$$\boxed{t_c = 0.26 \text{ seconds}}$$

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Case (ii)

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Application of S.C @ the middle of line 2

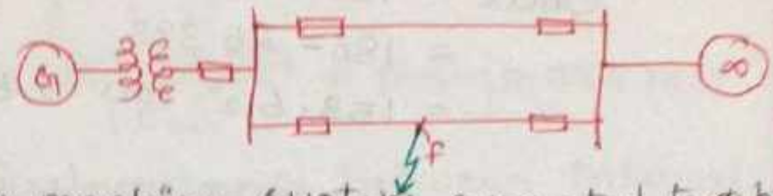
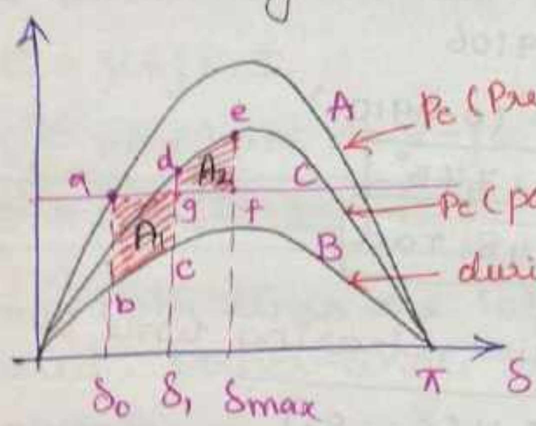


Fig: one machine system connected to ∞ bus three phase fault @ 'F'.

Consider a fault 'F' @ some distance away from the sending end as shown in figure



fig(i): EAC for 3ø fault away from the sending end.

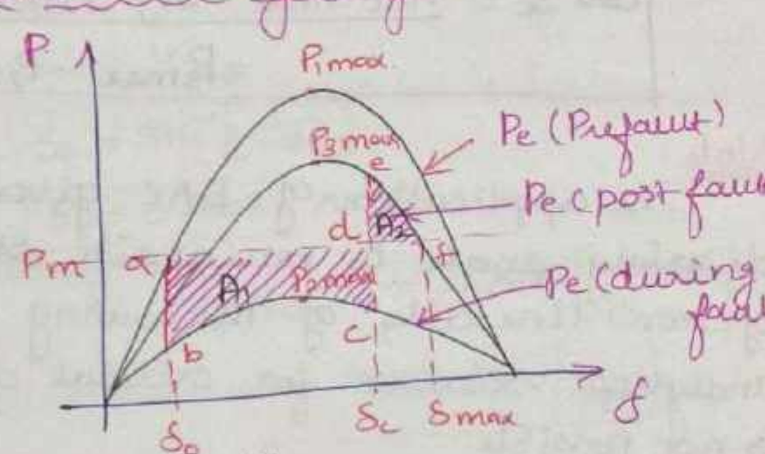
* It is assumed that there is no change in mechanical input power. The power angle curve in prefault condition is shown in fig(i) as curve A, where machine is operating @ point 'a'.
* Due to fault 'F' away from sending end, the equivalent transfer reactance b/w bus bars is increased and lowering the power transfer capability and the power angle curve is represented by curve 'B'.

Finally curve 'c' represents the post fault power angle curve, assuming the faulted line is removed.

Due to acceleration $\delta_0 \rightarrow \delta_1 \rightarrow \delta_{max}$

" " deceleration $\delta_{max} \rightarrow \delta_1 \rightarrow \delta_0$

To find critical clearing angle



Apply EAC. we can write

Area A_1 = Area A_2

$$\int_{\delta_0}^{\delta_c} (P_m - P_e)\, d\delta = -\int_{\delta_c}^{\delta_{max}} (P_m - P_e)\, d\delta$$

$$P_m(\delta_c - \delta_0) - \int_{\delta_0}^{\delta_c} P_{2max} \sin\delta\, d\delta = \int_{\delta_c}^{\delta_{max}} P_{3max} \sin\delta\, d\delta - P_m(\delta_{max} - \delta_c)$$

$$\cancel{P_m \delta_c} - P_m \delta_0 + P_{2max}[\cos\delta_c - \cos\delta_0] = P_{3max}[\cos\delta_c - \cos\delta_{max}] - P_m\delta_{max} + \cancel{P_m\delta_c}$$

$$-P_m\delta_0 + P_{2max}\cos\delta_c - P_{2max}\cos\delta_0 = P_{3max}\cos\delta_c - P_{3max}\cos\delta_{max} - P_m\delta_{max}.$$

$$P_{2max}\cos\delta_c - P_{3max}\cos\delta_c = P_{2max}\cos\delta_0 + P_m\delta_0 - P_{3max}\cos\delta_{max} - P_m\delta_{max}$$

by x^{4} –

$$(P_{3max} - P_{2max})\cos\delta_c = P_m[\delta_{max} - \delta_0] - P_{2max}\cos\delta_0 + P_{3max}\cos\delta_{max}$$

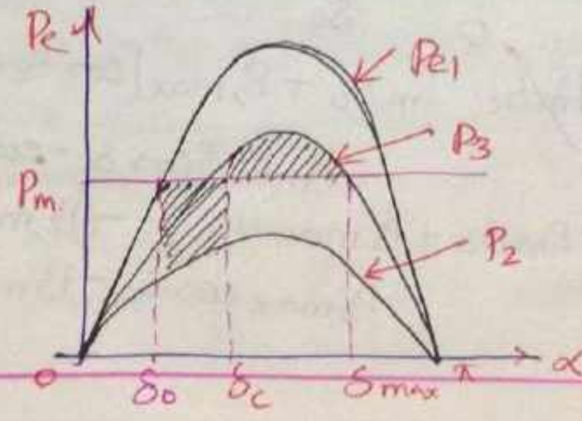
$$\therefore \boxed{\cos\delta_c = \frac{P_m[\delta_{max} - \delta_0] - P_{2max}\cos\delta_0 + P_{3max}\cos\delta_{max}}{P_{3max} - P_{2max}}}$$

Note:

The application of EAC gives the critical clearing angle to maintain stability. B'coz of non-linearity of the swing equation, an analytical solution for critical clearing time is not possible.

(8) A 3 ø generator delivers 1.0 pu, power to an infinite bus thro' a transmission network when a fault occurs. The maximum power which can be transferred during prefault, during fault and post fault conditions is 1.75 pu, 0.4 pu and 1.25 pu. Find the critical clearing angle.

Solution:



$P_1 = 1.75 \sin\delta \quad \Rightarrow P_{1max} = 1.75$

$P_2 = 0.4 \sin\delta \quad \Rightarrow P_{2max} = 0.4$

$P_3 = 1.25 \sin\delta \quad \Rightarrow P_{3max} = 1.25$

Initial loading $P_m = 1.0$ pu

$1.75 \sin\delta_0 = P_m$

$1.75 \sin\delta_0 = 1$

$\sin\delta_0 = 1/1.75$

$\delta_0 = \sin^{-1}\left[\frac{1}{1.75}\right]$

$\boxed{\delta_0 = 0.608}$ rad.

$$\delta_{max} = \pi - \sin^{-1}\left[\frac{P_m}{P_{3,max}}\right]$$

$$= \pi - \sin^{-1}\left[\frac{1}{1.25}\right]$$

$$\boxed{\delta_{max} = 2.214 \text{ rad}}$$

Applying EAC.

Area A_1 = Area A_2.

$$P_m(\delta_c - \delta_0) - \int_{\delta_0}^{\delta_c} P_{e2}\, d\delta = \int_{\delta_c}^{\delta_{max}} P_{e3}\, d\delta - P_m(\delta_{max} - \delta_c)$$

$$\therefore \cos\delta_c = \frac{P_m[\delta_{max} - \delta_0] - P_{2max}\cos\delta_0 + P_{3max}\cos\delta_{max}}{P_{3max} - P_{2max}}$$

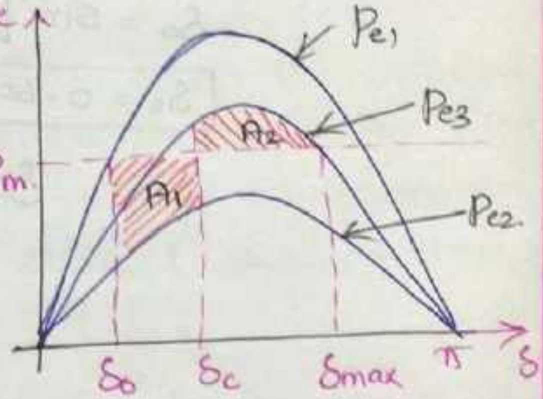
$$= \frac{1.0(2.214 - 0.608) - 0.4\cos 0.608 + 1.25\cos 2.214}{1.25 - 0.4}$$

$\cos\delta_c = 0.6212$ rad

$\delta_c = \cos^{-1}(0.6212) = 0.9$ rad

$\boxed{\delta_c = 51.57^\circ}$

(9) A generator operating @ 50 Hz delivers 1 pu power to an infinite bus through a transmission circuit in which resistance is ignored. A fault takes place reducing the maximum power transferable to 0.4 pu whereas before the fault, this power was 1.6 pu and after the clearance of the fault, it is 1.2 pu. By the use of equal area criterion, determine the critical clearing angle



Solution:

$P_{e1} = 1.6 \sin \delta$

$\therefore \boxed{P_{1,max} = 1.6 \text{ pu}}$

$P_{e2} = 0.4 \sin \delta$

$\boxed{P_{2,max} = 0.4 \text{ pu}}$

$P_{e3} = 1.2 \sin \delta$

$\boxed{P_{3,max} = 1.2 \text{ pu}}$

Initial loading $P_m = 1.0$ pu

$1.6 \sin \delta_0 = P_m$

$1.6 \sin \delta_0 = 1$

$\sin \delta_0 = \frac{1}{1.6}$

$\delta_0 = \sin^{-1}\left[\frac{1}{1.6}\right]$

$\boxed{\delta_0 = 0.675 \text{ rad}}$

$\delta_{max} = \pi - \sin^{-1}\left[\frac{P_m}{P_{3max}}\right]$

$= \pi - \sin^{-1}\left[\frac{1}{1.2}\right]$

$\boxed{\delta_{max} = 2.156 \text{ rad}}$

Applying EAC, we get

$$\text{Area } A_1 = \text{Area } A_2$$

$$\therefore \cos\delta_c = \frac{P_m(\delta_{max}-\delta_0) - P_{2max}\cos\delta_0 + P_{3max}\cos\delta_{max}}{P_{3max} - P_{2max}}$$

$$= \frac{1.0(2.156-0.675) - 0.4\cos 0.675 + 1.2\cos 2.156}{1.2-0.4}$$

$\cos\delta_c = 0.632$ rad

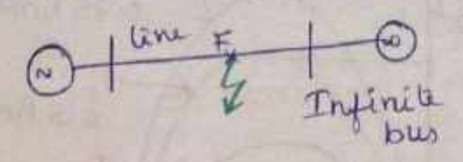
$\delta_c = \cos^{-1}(0.632)$

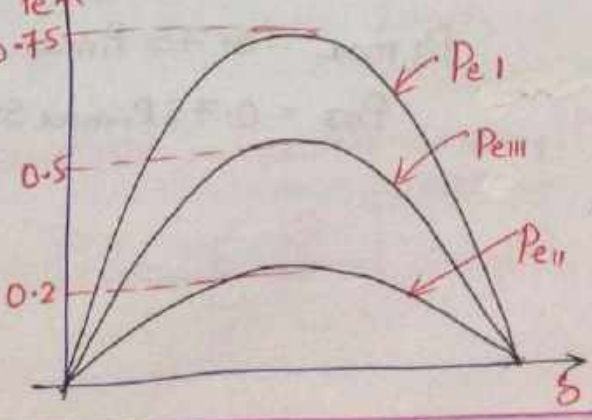
$= 0.887$ radians

$$\boxed{\delta_c = 50.82^\circ}$$

(10) A 50 Hz generator is delivering 50% of the power that it is capable of delivering thro a transmission line to an infinite bus. A fault occurs that ↑ses the reactance between the generator and the infinite bus to 500% of the value. When the fault is isolated, the maximum power that can be delivered is 75% of the original maximum value. Determine the critical clearing angle for the condition described.

Solution:





Prefault condition: Generator is delivering 50% of the power

$$P_{e1} = \frac{|E'||V|}{X_{12}} \sin \delta_0$$

$$P_{e1} = P_{1max} \sin \delta_0$$

$$P_{e1} = 0.5\ P_1 max$$

$$\sin \delta_0 = 0.5$$

$$\delta_0 = \sin^{-1}(0.5)$$

$$\delta_0 = 0.524 \text{ radians}$$

During the fault: Reactance is 500% of the value during the fault

$$\therefore X_{11} = \frac{500}{100} = 5$$

$$P_{e2} = \frac{|E'||V|}{5} \sin \delta$$

$$P_{e2} = \frac{P_{max1}}{5} \sin \delta$$

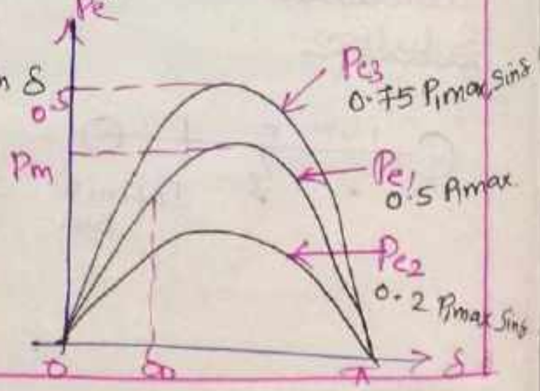
$$P_{e2} = 0.2\ P_1 max \sin \delta$$

Post fault:

P_3 max = 75% of the original max value

$$P_3 max = 0.75\ P_1 max.$$

$$P_{e3} = 0.75 P_1 max \sin \delta$$



$P_m = 0.5$

$$\delta_{max} = \pi - \sin^{-1}\left[\frac{P_m}{P_{3,max}}\right] = \pi - \sin^{-1}\left[\frac{0.5}{0.75}\right]$$

$$\boxed{\delta_{max} = 2.412 \text{ radians}}$$

Critical clearing angle δ_c is given by

$$\cos\delta_c = \frac{P_m(\delta_{max} - \delta_0) + P_{3max}\cos\delta_{max} - P_{2max}\cos\delta_0}{P_{3,max} - P_{2}max}$$

$$= \frac{0.5(2.412 - 0.524) + 0.75\cos 2.412 - 0.2\cos 0.524}{0.75 - 0.2}$$

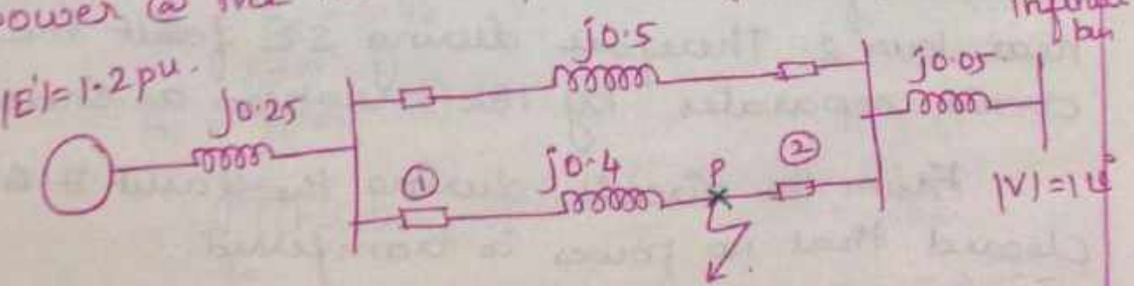
$\cos\delta_c = 0.385$

$\delta_c = \cos^{-1}(0.385)$

$\delta_c = 1.176$

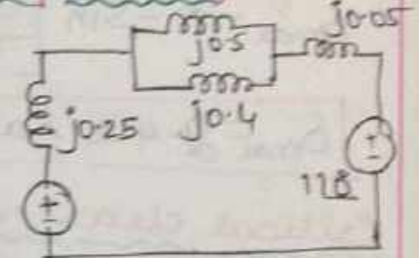
$$\boxed{\delta_c = 67.35^\circ}$$

(11) A three phase fault is applied at the point P as shown in figure. Find the critical clearing angle for clearing the fault with simultaneous opening of the breakers 1 and 2. The reactance values of various components are indicated in the diagram. The generator is delivering 1.0 pu power @ the instant preceding the fault.



Solution :-

I. prefault (under Normal operation)



Reactance

$$X_1 = 0{\cdot}25 + \frac{0{\cdot}4 \times 0{\cdot}5}{0{\cdot}4 + 0{\cdot}5} + 0{\cdot}05$$

$$X_1 = 0{\cdot}522 \text{ pu}$$

$$P_{e1} = \frac{|E'||V|}{X_1} \sin \delta_0$$

Prefault operation $P_{e1} = 1{\cdot}0$ pu

$$1{\cdot}0 = \frac{1{\cdot}2 \times 1}{0{\cdot}522} \sin \delta_0$$

$$1{\cdot}0 = 2{\cdot}3 \sin \delta_0$$

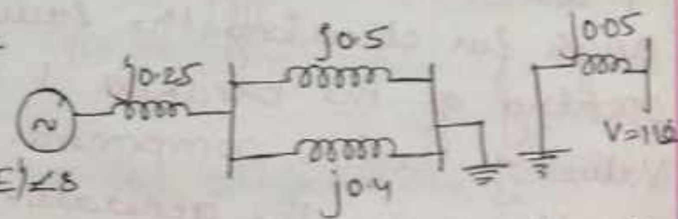
$$\sin \delta_0 = 1/2{\cdot}3$$

$$\delta_0 = \sin^{-1}\left[\frac{1}{2{\cdot}3}\right] = 25{\cdot}8^\circ$$

$$\delta_0 = 25{\cdot}8^\circ$$

$$\delta_0 = 25{\cdot}8 \times \frac{\pi}{180^\circ} = 0{\cdot}45 \text{ radians}$$

II During fault:

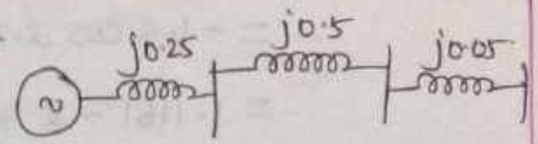


The fault occurs @ the end of the line 2 or near bus 2. Therefore during s.c fault the circuit separates by the breakers as shown.

From the circuit, during the fault it is cleared that no power is transferred.

(ie) $P_{e2} = 0$

III post fault operation:



$$X_{III} = 0.25 + 0.5 + 0.05$$
$$= 0.8$$

$$P_{e3} = \frac{|E'||V|}{X_{III}} \sin\delta$$

$$= \frac{1.2 \times 1.0}{0.8} \sin\delta$$

$$P_{e3} = 1.5 \sin\delta$$

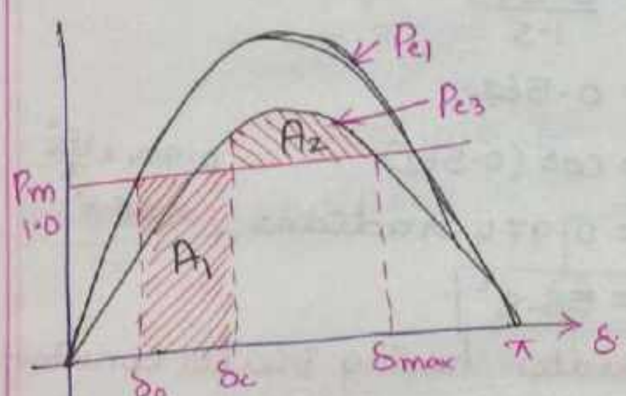
$$P_{3,max} = 1.5$$

The max. possible angle δ_{max} for Area A_1 = Area A_2 is given by

$$\delta_{max} = \pi - \sin^{-1}\left[\frac{P_m}{P_{3\,max}}\right]$$

$$= \pi - \sin^{-1}\left[\frac{1}{1.5}\right]$$

$$\boxed{\delta_{max} = 2.41 \text{ rad.}}$$



Determination of δc

Applying equal area criterion for δ_c

$$A_1 = P_m(\delta_c - \delta_0)$$
$$= 1.0(\delta_c - 0.45)$$
$$A_1 = \delta_c - 0.45$$

$$A_2 = \int_{\delta_c}^{\delta_{max}} (P_{e3} - P_m)\, d\delta$$

$$= \int_{\delta_c}^{2.41} (1.5\sin\delta - 1)\, d\delta = \left[-1.5\cos\delta - \delta\right]_{\delta_c}^{2.41}$$

$$= -1.5\cos 2.41 - 2.41 - [-1.5\cos\delta_c - \delta_c]$$

$= -1.5 \cos 2.41 - 2.41 + 1.5 \cos \delta_c + \delta_c$

$= 1.1161 - 2.41 + 1.5 \cos \delta_c + \delta_c$

$A_2 = -1.293 + 1.5 \cos \delta_c + \delta_c$

Applying EAC, we get

Area A_1 = Area A_2

$\cancel{\delta_c} - 0.45 = -1.293 + 1.5 \cos \delta_c + \cancel{\delta_c}$

$1.5 \cos \delta_c = -0.45 + 1.293$

$\cos \delta_c = \frac{0.843}{1.5}$

$\cos \delta_c = 0.562$

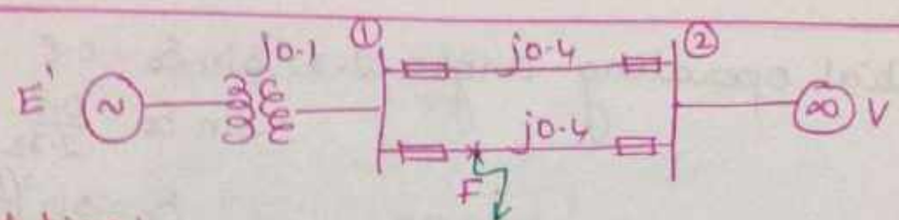
$\delta_c = \cos^{-1}(0.562)$ $\quad 0.974 \times \frac{180^\circ}{\pi} = 55.8^\circ$

$\delta_c = 0.974$ radians

$\boxed{\delta_c = 55.8^\circ}$

12) A 50 Hz syn generator having inertia constant $H = 5$ MJ/MVA and a direct axis tr reactance $X_d' = 0.2$ pu is connected to an infinite bus thro a purely reactive circuit as shown in figure. The generator is delivering real power $P = 0.8$ pu and $Q = 0.6$ pu to the infinite bus @ a voltage of $V = 1$ pu.

A temporary 3Ø fault occurs @ the sending end of the line @ point F. When the fault is cleared, both lines are intact. Determine the critical clearing angle & critical fault clearing time



Solution:

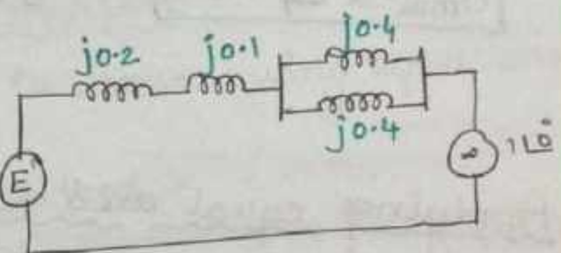
$V = 1.0\angle 0^\circ$

$V^* = 1.0$

Current flowing into the infinite bus $I = \frac{S^*}{V^*}$

$$I = \frac{S^*}{V^*}$$
$$= \frac{P - jQ}{V^*}$$
$$= \frac{0.8 - j0.6}{1.0}$$
$$= 0.8 - j0.6$$

Prefault condition



$$Z_{total} = j0.2 + j0.1 + \frac{j0.4 \times j0.4}{j0.4 + j0.4}$$

ie $X_{total} = j0.5$

Transient internal voltage $E' = V + jX_{total} I$.

$$= 1.0 + j0.5(0.8 - j0.6)$$
$$= 1.3 + j0.4$$
$$\boxed{E' = 1.36\angle 0.299}$$

Prefault + post fault condition

Since both the lines are intact (unimpaired) when the fault is cleared.

$$P_{e1} = P_{e3} = \frac{|E'||V|}{X} \sin\delta$$
$$= \frac{1.36 \times 1.0}{0.5} \sin\delta = 2.72 \sin\delta$$

Initial operating angle $2.72 \sin \delta_0 = 0.8$

$$\sin \delta_0 = \frac{0.8}{2.72}$$

$$\delta_0 = \sin^{-1}\left(\frac{0.8}{2.72}\right)$$

$$\boxed{\delta_0 = 0.299 \text{ rad}}$$

During the fault:

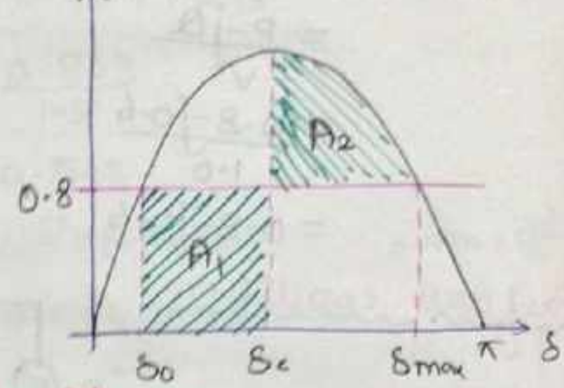
The circuit is separated during the fault @ bus 1, so there is no power transfer takes place

$\therefore P_{e2} = 0$

$\delta_{max} = \pi - \delta_0$

$= 3.14 - 0.299$

$$\boxed{\delta_{max} = 2.84 \text{ rad}}$$



Applying equal area criterion.

Area A_1 = Area A_2

$$\int_{\delta_0}^{\delta_c} P_m \, d\delta = \int_{\delta_c}^{\delta_{max}} (P_{e1} - P_m) \, d\delta$$

$$\int_{\delta_0}^{\delta_c} 0.8 \, d\delta = \int_{\delta_c}^{\delta_{max}} (2.72 \sin \delta - 0.8) d\delta$$

$$[0.8\delta]_{\delta_0}^{\delta_c} = [-2.72 \cos \delta - 0.8\delta]_{\delta_c}^{\delta_{max}}$$

$$\cancel{0.8\delta_c} - 0.8 \times 0.299 = -2.72 \cos 2.84 - 0.8 \times 2.84 + 2.72 \cos \delta_c + \cancel{0.8\delta_c}$$

$$-0.239 = 2.597 - 2.272 + 2.72 \cos \delta_c$$

$$2.72 \cos \delta_c = -0.567$$

$$\cos \delta_c = \frac{-0.567}{2.72}$$

$$\cos \delta_c = -0.207$$

$$\boxed{\delta_c = 1.78 \text{ rad}}$$

Critical clearing time (t_c)

$$t_c = \sqrt{\frac{2H(\delta_c - \delta_0)}{\pi f P_m}}$$

$$= \sqrt{\frac{2 \times 5 \times (1.78 - 0.299)}{\pi \times 50 \times 0.8}}$$

$$\boxed{t_c = 0.343 \text{ seconds}}$$

Solution of Swing Equation:

The methods used to solve swing equations are

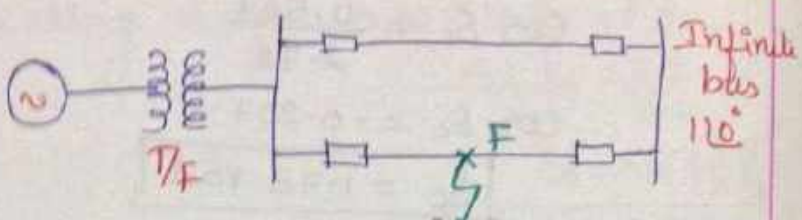
(i) Modified Euler Method.

(ii) Runge-Kutta Method.

(i) Algorithm for numerical solution of swing equation using modified Euler Method.

Numerical integration techniques can be applied to obtain approximate solutions of non-linear differential equations. Euler's method is the simplest method.

Consider a generator connected to an infinite bus through two parallel lines and a 3ϕ fault occurs @ the middle of line 2 as shown in figure.



Let P_m be the I/p power which is a constant.

Prefault condition Under steady state operation power transfer from generator to an infinite bus

$$P_e = P_m$$

$$\frac{E'V}{X_1} \sin \delta_0 = P_{1max} \sin \delta_0 = P_m.$$

$$\sin \delta_0 = \frac{P_m}{P_{1max}}$$

$$\boxed{\delta_0 = \sin^{-1}\left[\frac{P_m}{P_{1max}}\right]}$$

where

$$P_{1max} = \frac{E'V}{X_1}$$

$X_1 \rightarrow$ Transfer reactance for the prefault condition.

The rotor is running @ syn speed $\omega_0 = 2\pi f$ change in angular velocity is Zero (ie) $\Delta\omega_0 = 0$

During the fault: consider a 3Ø fault occurs @ the middle of one line 2 as shown in figure

$$P_{e2} = \frac{|E'||V|}{X_{11}} \sin \delta.$$

$$P_{e2} = P_{2,max} \sin \delta.$$

where $P_{2,max} = \frac{|E'||V|}{X_{11}}$

$X_{11} \rightarrow$ Transfer reactance during fault.

The swing equation is given by

$$\frac{d^2\delta}{dt^2} = \frac{\pi f}{H}\left[P_m - P_{2max}\sin\delta\right]$$

$$= \frac{\pi f}{H} P_a.$$

The above equations are transfered into the state Variable form.

$$\frac{d\delta^{(1)}}{dt} = \Delta\omega$$

$$\frac{d^2\delta}{dt^2} = \frac{d\Delta\omega^{(1)}}{dt} = \frac{\pi f P_a}{H}$$

Compute the first estimate @ $t_1 = t_0 + \Delta t$

$$\delta^P_{i+1} = \delta_i + \left.\frac{d\delta^{(1)}}{dt}\right|_{\Delta\omega_i} \cdot \Delta t$$

$$\Delta\omega^P_{i+1} = \Delta\omega_i + \left.\frac{d\Delta\omega^{(1)}}{dt}\right|_{\delta_i} \Delta t.$$

Compute the derivatives Using the predicted values δ^P_{i+1} and $\Delta\omega^P_{i+1}$, determine the derivaties @ the end of iteration.

$$\left.\frac{d\delta^{(2)}}{dt}\right|_{\Delta\omega^P_{i+1}} = \Delta\omega^P_{i+1}$$

$$\left.\frac{d\Delta\omega^{(2)}}{dt}\right|_{\delta^P_{i+1}} = \left.\frac{\pi f}{H} P_a\right|_{\delta^P_{i+1}}$$

Compute the average derivatives

$$\frac{d\delta}{dt_{ave}} = \frac{\left.\frac{d\delta^{(1)}}{dt}\right|_{\Delta\omega_i} + \left.\frac{d\delta^{(2)}}{dt}\right|_{\Delta\omega^P_{i+1}}}{2}$$

$$\frac{d\Delta\omega}{dt_{ave}} = \frac{\left.\frac{d\Delta\omega^{(1)}}{dt}\right|_{\delta_i} + \left.\frac{d\Delta\omega^{(2)}}{dt}\right|_{\delta^P_{i+1}}}{2}$$

Compute the final estimate (corrected value).

$$\delta^c_{i+1} = \delta_i + \left[\frac{\left.\frac{d\delta}{dt}\right|_{\Delta\omega_i} + \left.\frac{d\delta}{dt}\right|_{\Delta\omega^P_{i+1}}}{2}\right]\Delta t$$

$$\Delta\omega^c_{i+1} = \Delta\omega_i + \left[\frac{\left.\frac{d\Delta\omega}{dt}\right|_{\delta_i} + \left.\frac{d\Delta\omega}{dt}\right|_{\delta^P_{i+1}}}{2}\right]\Delta t$$

(ii) Using Runge-Kutta Method:

The following steps are involved in Runge-Kutta method to determine stability.

I Estimates: $K_1 = \left.\frac{d\delta}{dt}\right|_{\Delta\omega_i} \times \Delta t = \Delta\omega_i \times \Delta t$

$$l_1 = \left.\frac{d\Delta\omega}{dt}\right|_{\delta_i} \times \Delta t = \frac{\pi f}{H}[P_m' - P_e(\delta_i)] \times \Delta t$$

II Estimates: $K_2 = \left[\Delta\omega_i + \frac{l_1}{2}\right]\Delta t$

$$l_2 = \frac{\pi f}{H}\left[P_m' - P_e\left(\delta_i + \frac{K_1}{2}\right)\right] \times \Delta t$$

III Estimates: $K_3 = \left(\Delta\omega_i + \frac{l_2}{2}\right)\Delta t$

$$l_3 = \frac{\pi f}{H}\left[P_m' - P_e\left(\delta_i + \frac{K_2}{2}\right)\right] \times \Delta t$$

IV Estimates: $K_4 = (\Delta\omega_i + l_3)\Delta t$

$$l_4 = \frac{\pi f}{H}\left[P_m' - P_e(\delta_i + K_3)\right] \times \Delta t$$

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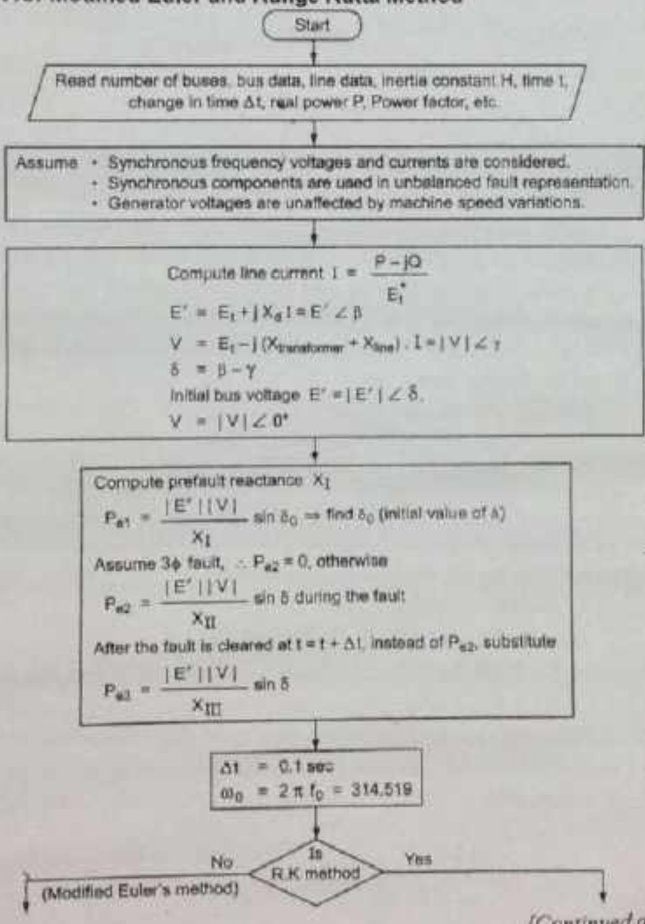
final estimates at $t = t_1$:

$$\delta_{i+1} = \delta_i + \frac{1}{6}\left[k_1 + 2k_2 + 2k_3 + k_4\right]$$

$$\Delta\omega_{i+1} = \Delta\omega_i + \frac{1}{6}\left[l_1 + 2l_2 + 2l_3 + l_4\right]$$

Flow chart for Modified Euler and Runge-Kutta Method.

Flow Chart for Modified Euler and Runge Kutta Method



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State variable form

$$\frac{d\delta^{(1)}}{dt} = \Delta\omega$$

$$\frac{d^2\delta}{dt^2} = \frac{d\Delta\omega^{(1)}}{dt} = \frac{\pi f P_a}{H}$$

Compute the first estimate $t_1 = t_0 + \Delta t$,

$$\delta_{i+1}^{P} = \delta_i + \left.\frac{d\delta^{(1)}}{dt}\right|_{\Delta\omega_i} \cdot \Delta t$$

$$\Delta\omega_{i+1}^{P} = \Delta\omega_i + \left.\frac{d\Delta\omega^{(1)}}{dt}\right|_{\delta_i} \cdot \Delta t$$

Compute the derivatives: Using the predicted values δ_{i+1}^{P} and $\Delta\omega_{i+1}^{P}$, determine the derivatives at the end of iteration

$$\left.\frac{d\delta^{(1)}}{dt}\right|_{\Delta\omega_{i+1}^{P}} = \Delta\omega_{i+1}^{P}$$

$$\left.\frac{d\Delta\omega^{(2)}}{dt}\right|_{\delta_{i+1}^{P}} = \left.\frac{\pi f}{H}\right|_{\delta_{i+1}^{P}}$$

Compute the average derivatives

$$\frac{d\delta}{dt_{ave}} = \frac{\left.\frac{d\delta^{(1)}}{dt}\right|_{\Delta\omega_i} + \left.\frac{d\delta^{(2)}}{dt}\right|_{\Delta\omega_{i+1}^{P}}}{2}$$

$$\frac{d\Delta\omega}{dt_{ave}} = \frac{\left.\frac{d\Delta\omega^{(1)}}{dt}\right|_{\delta_i} + \left.\frac{d\Delta\omega^{(2)}}{dt}\right|_{\delta_{i+1}^{P}}}{2}$$

Compute the final estimate (corrected value),

$$\delta_{i+1}^{C} = \delta_i + \left[\frac{\left.\frac{d\delta}{dt}\right|_{\Delta\omega_i} + \left.\frac{d\delta}{dt}\right|_{\Delta\omega_{i+1}^{P}}}{2}\right]\Delta t$$

$$\Delta\omega_{i+1}^{C} = \Delta\omega_i + \left[\frac{\left.\frac{d\Delta\omega}{dt}\right|_{\delta_i} + \left.\frac{d\Delta\omega}{dt}\right|_{\delta_{i+1}^{P}}}{2}\right]\Delta t$$

I Estimates:

$$K_1 = \left.\frac{d\delta}{dt}\right|_{\Delta\omega_i} \times \Delta t = \Delta\omega_i \times \Delta t$$

$$l_2 = \left.\frac{d\Delta\omega}{dt}\right|_{\delta_i} \times \Delta t$$

$$= \frac{\pi f}{H}[P_m' - P_{e(\delta_i)}] \times \Delta t$$

II Estimates:

$$K_2 = \left[\Delta\omega_i + \frac{l_1}{2}\right]\Delta t$$

$$l_2 = \frac{\pi f}{H}[P_m' - P_{e(\delta_i + (K_1(2))}] \times \Delta t$$

III Estimates:

$$K_3 = \left[\Delta\omega_i + \frac{l_2}{2}\right] \times \Delta t$$

$$l_3 = \frac{\pi f}{H}[P_m' - P_{e(\delta_i + (K_2(2))}] \times \Delta t$$

III Estimates:

$$K_4 = (\Delta\omega_i \times l_3) \times \Delta t$$

$$l_4 = \frac{\pi f}{H}[P_m' - P_{e(\delta_i + (K_3)}] \times \Delta t$$

Find estimates at $t = t_1$:

$$\delta_{i+1} = \delta_i + \frac{1}{6}[K_1 + 2K_2 + 2K_3 + K_4]$$

$$\Delta\omega_{i+1} = \Delta\omega_i + \frac{1}{6}[l_1 + 2l_2 + 2l_3 + l_4]$$

Is $\delta_{i+1} - \delta_i = 0$, $\Delta\omega_{i+1} - \Delta\omega_i = 0$

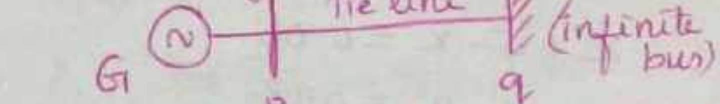
No: iter = 0; iter = iter + 1

Yes

Print critical clearing angle δ and the corresponding critical clearing time t

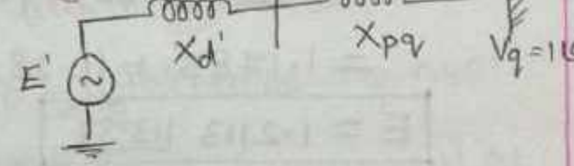
End

(13) Figure shows the generator supplying power to an infinite bus system over a tie line



The generator is supplying 100 MW and 75 MVAR. The tie line reactance is 0.08 pu $X_d' = 0.2$ pu. The inertia constant is 4 pu on 100 MVA base. A 3Ø fault occurs @ bus P for a duration of 0.1 sec. At the end of 0.1 sec, the fault disappears without any circuit breaker opening. Write the swing equation for faulted and post fault cond. Solve these equation by Runge-Kutta Method

Solution:



Real power in pu,

$$P = \frac{\text{Actual Value}}{\text{Base Value}} = \frac{100}{100} = 1 \text{ pu}$$

Reactive power in pu, $Q = \frac{75}{100} = 0.75$ pu.

power from point $p = V_p^* I_{pq}$

$$= V_p^* \left[\frac{V_p - V_q}{X_{pq}}\right]$$

$$= V_p^* \left[\frac{V_p - 1}{0.08}\right]$$

Let $V_p = x + jy$

$$P - jQ = (x+jy)\left[\frac{x+jy-1}{0.08}\right]$$

$$= 1 - j0.75$$

$$1 - j0.75x^2 + jxy - x - jxy + y^2 + jy = j0.08 + 0.06$$

Equating real and imaginary parts.

$$x^2 + y^2 - x = 0.06$$

$$y = 0.08$$

$$x^2 + 0.0064 - x = 0.06$$

$$x^2 - x = 0.0536$$

$$x^2 - x - 0.0536 = 0$$

$$X = \frac{1 \pm \sqrt{1^2 + 4 \times 0.0536}}{2}$$

$$= 1.051 \text{ (or) } -0.051$$

$$\therefore V_p = x + jy$$

$$\boxed{V_p = 1.051 + j0.08}$$

$$E' = V_p + j0.2 \times I_{pq}$$

$$= (1.051 + j0.08) + j0.2 \times \left(\frac{1.051 + j0.08 - 1}{j0.08}\right)$$

$$= 1.1785 + j0.28$$

$$\boxed{E' = 1.2113 \angle 13.37^\circ}$$

$$E' = 1.2113$$

$$\delta_0 = 13.37^\circ = 0.2333 \text{ rad}$$

choose $\Delta t = 0.05$ sec

0   0.05 sec   0.1 sec

$P_e = 0$ for 3ϕ fault.

During the fault:

Step 1: At $t = 0$

$$\delta_0 = 0.2333 \text{ rad.}$$

$$\omega_0 = 2\pi f = 314.159$$

$$\Delta t = 0.05$$

Iteration I

I Estimates: $K_1 = (\omega_{(0)} - \omega_0)\Delta t$.

$= (314.159 - 314.159) \times 0.05 = 0$

$l_1 = \frac{\pi f}{H}[P_m' - P_{e(\delta_0)}]\Delta t$

$= \frac{\pi \times 50}{4}[1-0](0.05) = 1.9635$

II Estimates : $K_2 = \left[\Delta\omega_0 + \frac{l_1}{2}\right]\Delta t$

$= \left[0 + \frac{1.9635}{2}\right] \times 0.05 = 0.0491.$

$l_2 = \left[\frac{\pi f}{H}\left(P_m' - P_{e(\delta_0 + (\frac{K_1}{2}))}\right)\right]\Delta t$

$= \frac{\pi \times 50}{4}[1-0][0.05] = 1.9635$

III Estimates : $K_3 = \left(\Delta\omega_0 + \frac{l_2}{2}\right)\Delta t$

$= \left(0 + \frac{1.963}{2}\right) \times 0.05 = 0.0491.$

$l_3 = \frac{\pi f}{H}\left[P_m' - P_{e(\delta(0) + \frac{K_2}{2})}\right]\Delta t.$

$= \frac{\pi \times 50}{4}[1-0] \times 0.05 = 1.9635$

IV Estimates : $K_4 = (\Delta\omega_0 + l_3)\Delta t$.

$= (0 + 1.963) \times 0.05 = 0.0982.$

$l_4 = \frac{\pi f_0}{H}[P_m' - P_{e(\delta_0 + K_3)}]\Delta t$

$= \frac{\pi \times 50}{4}[1-0]0.05 = 1.9635$

Final estimates:

$\delta_{(0.05)} = \delta_0 + \frac{1}{6}[K_1 + 2K_2 + 2K_3 + K_4]$

$= 0.2333 + \frac{1}{6}[0 + 2(0.0491) + 2(0.0491) + 0.0982]$

$\boxed{\delta_{(0.05)} = 0.2824}$

$$\Delta\omega_{(0.05)} = \Delta\omega_{(0)} + \frac{1}{6}\left[l_1 + 2l_2 + 2l_3 + l_4\right]$$

$$= 0 + \frac{1}{6}\left[1.9635 + 2(1.9635) + 2(1.9635) + 1.9635\right]$$

$$\boxed{\Delta\omega_{(0.05)} = 1.9635}$$

Iteration - 2

I Estimates: $k_1 = \Delta\omega_{0.05} \times \Delta t$

$$= 1.9635 \times 0.05 = 0.098$$

$$l_2 = \frac{\pi f}{H}\left[P_m' - P_{e(\delta_{0.05})}\right]\Delta t$$

$$= \frac{\pi \times 50}{4}\left[1 - 0\right] \times 0.05 = 1.9635$$

II Estimates: $k_2 = \left[\Delta\omega_{0.05} + \frac{l_1}{2}\right]\Delta t$

$$= \left[1.9635 + \frac{1.9635}{2}\right] \times 0.05 = 0.1473$$

$$l_2 = \frac{\pi f}{H}\left[P_m' - P_{e(\delta_{0.05} + \frac{k_1}{2})}\right] \cdot \Delta t$$

$$= \frac{\pi \times 50}{4}\left[1 - 0\right] \times 0.05 = 1.9635$$

III Estimates: $k_3 = \left[\Delta\omega_{0.05} + \frac{l_2}{2}\right]\Delta t$

$$= \left[1.9635 + \frac{1.9635}{2}\right] \times 0.05 = 0.1473$$

$$l_3 = \frac{\pi f}{H}\left[P_m' - P_{e(\delta_{0.05} + \frac{k_2}{2})}\right]\Delta t$$

$$= \frac{\pi \times 50}{4}\left[1 - 0\right] \times 0.05 = 1.9635$$

IV Estimates: $k_4 = (\Delta\omega_{0.05} + l_3) \times \Delta t$

$$= (1.9635 + 1.9635) \times 0.05 = 0.1964$$

$$l_4 = \frac{\pi f}{H}\left[P_m' - P_{e(\delta_{0.05} + k_3)}\right]\Delta t$$

$$= \frac{\pi \times 50}{4}\left[1 - 0\right] \times 0.05 = 1.9635$$

Final Estimates:

$$\delta_{0.1} = \delta_{0.05} + \frac{1}{6}\left[K_1 + 2K_2 + 2K_3 + K_4\right]$$

$$= 0.2824 + \frac{1}{6}\left[0.0982 + 2(0.1473) + 2(0.1473) + 0.1964\right]$$

$$\boxed{\delta_{0.1} = 0.4297}$$

$$\omega_{0.1} = \omega_{0.05} + \frac{1}{6}\left[l_1 + 2l_2 + 2l_3 + l_4\right]$$

$$= 316.1225 + \frac{1}{6}\left[1.9635 + 2(1.9635) + 2(1.9635) + 1.9635\right]$$

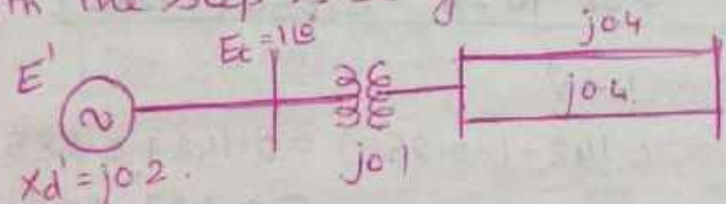
$$\boxed{\omega_{0.1} = 318.086}$$

(14) A 3Ø fault occurs @ the middle of line 2, is cleared by isolating the faulted circuit simultaneously from both ends. Generator is delivering 0.8 pu power @ 0.8 pf lagging. The fault is cleared in 0.1 sec. Obtain the numeric solution of the swing equation upto 0.15 sec using the

(a) Modified Euler Method

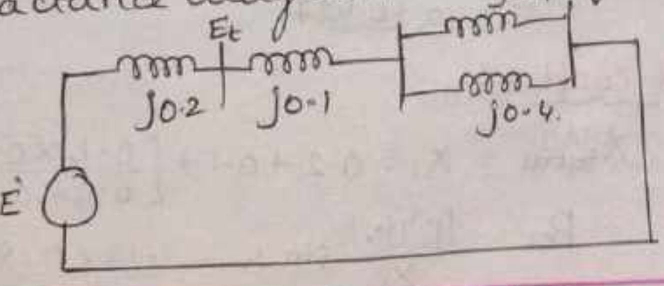
(b) Runge - Kutta Method

with the step size of $\Delta t = 0.05$ sec, Take $H = 5$



Solution:

The reactance diagram is



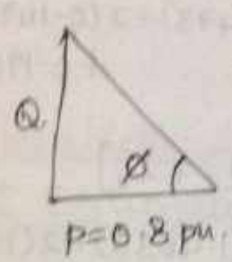
Note: Use calc in radian mode

Take reference as $E_t = 1.0\angle 0^\circ$

$P = 0.8$ pu ; $\cos\phi = 0.8$

$\phi = 0.644$ rad

From figure



$$\tan\phi = \frac{Q}{P}$$

$$Q = p\tan\phi = 0.8\tan(0.644)$$

$$Q = 0.6$$

$$I = \frac{P - jQ}{E_t^*} = \frac{0.8 - j0.6}{1\angle 0} = 0.8 - j0.6$$

$$E' = E_t + jx_d' I$$

$$= 1.0\angle 0 + j0.2(0.8 - j0.6)$$

$$\boxed{E' = 1.12 + j0.16 = 1.13\angle 0.142}$$

$|E'|\angle\beta$

$E = 1.13$

$\beta = 0.142^\circ$

$$V = E_t - j(X_{t/f} + X_{line})I$$

$$= 1.0\angle 0^\circ - j\left[0.1 + \frac{0.4\times 0.4}{0.4+0.4}\right](0.8 - j0.6)$$

$$\boxed{V = 0.82 - j0.24 = 0.854\angle -0.285}$$

$|V|\angle\gamma$

$$\therefore \delta = \beta - \gamma$$

$$= 0.142 - (-0.285) = 0.142 + 0.285$$

$$= 0.427 \text{ radians.}$$

$$\boxed{E' = |E'|\angle\delta}$$

$$= 1.13\angle 0.427$$

Prefault condition:

$$X_{total} = X_1 = 0.2 + 0.1 + \left[\frac{0.4\times 0.4}{0.4+0.4}\right] = 0.5$$

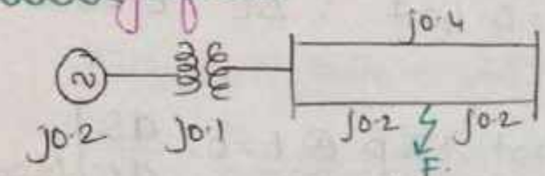
$$P_{e1} = \frac{|E'||V|}{X_1}\sin\delta = \frac{1.13\times 0.854}{0.5}\sin\delta_0 = 0.8$$

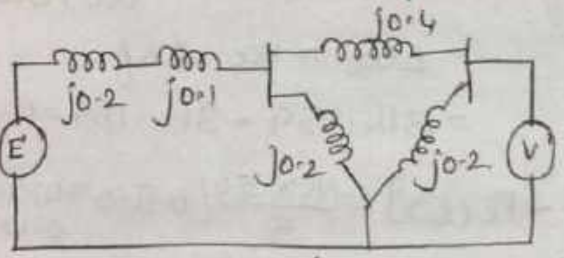
$\sin \delta_0 = 0.414.$

$\delta_0 = \sin^{-1}(0.414)$

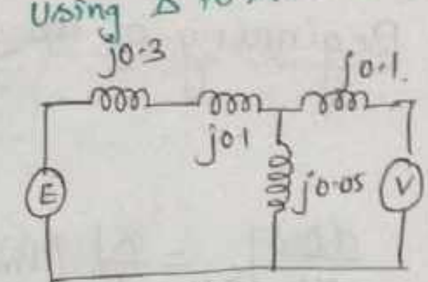
$$\boxed{\delta_0 = 0.427 \text{ radians}}$$

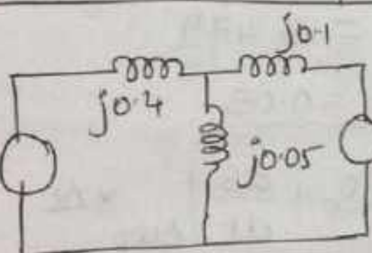
During fault:





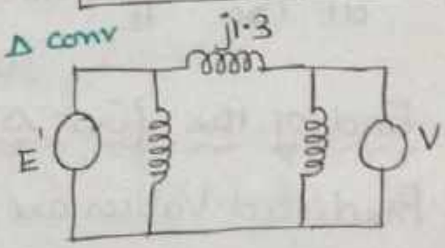
Using Δ to λ conversion.





λ to Δ conv

=>

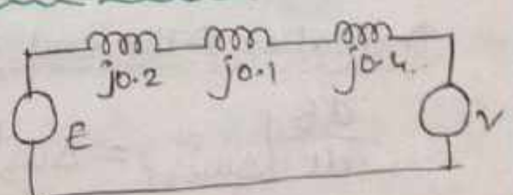


$\therefore X_{11} = 1.3$

$$P_{e2} = P_{2\max} \sin\delta = \frac{|E'||V|}{X_{11}} \sin\delta = \frac{1.13 \times 0.854}{1.3} \sin\delta.$$

$$\boxed{P_{e2} = 0.742 \sin\delta}$$

Post fault condition (line 2 is open)



$X_{111} = j0.2 + j0.1 + j0.4.$

$= j0.7.$

$$P_{e3} = \frac{|E'||V|}{X_{111}} \sin\delta$$

$$= \frac{1.13 \times 0.854}{j0.7} \sin\delta$$

$$\boxed{P_{e3} = 1.378 \sin\delta}$$

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(a) Modified Euler's Method:

During the fault $P_e = 0.742 \sin\delta$

$$\omega_0 = 2\pi f = 2\pi \times 50 = 314.159$$

$$\delta_0 = 0.427 \; ; \; \Delta t = 0.05 \text{ sec}$$

Iteration 1:

Beginning of the first step @ $t=0$. $\left.\frac{d\delta}{dt}\right|_{\Delta\omega_0}$

$$= \Delta\omega_0 = \omega_0 - 2\pi f$$

$$= 314.159 - 314.159 = 0$$

$$\left.\frac{d\Delta\omega}{dt}\right|_{\delta_0} = \frac{\pi f}{H}\left[P_m' - P_{e(\delta_0)}\right] = \frac{\pi \times 50}{5}\left[0.8 - 0.742 \sin 0.427\right]$$

$$= 15\,479$$

End of the first step @ $t = 0.05$.

Predicted values are $\delta_{0.05}^{P} = \delta_0 + \left.\frac{d\delta}{dt}\right|_{\Delta\omega_0} \times \Delta t$

$$= 0.427 + 0 \times 0.05 = 0.427 \text{ rad}$$

$$\Delta\omega_{0.05}^{P} = \Delta\omega_0 + \left.\frac{d\Delta\omega}{dt}\right|_{\delta_0} \times \Delta t$$

$$= 0 + 15.476 \times 0.05 = 0.774 \text{ r/s}$$

Derivaties @ the end of $t = 0.05$

$$\left.\frac{d\delta}{dt}\right|_{\Delta\omega_{0.05}^{P}} = \Delta\omega_{0.05}^{P} = 0.774 \text{ rad/sec}$$

$$\left.\frac{d\Delta\omega}{dt}\right|_{\delta_{0.05}^{P}} = \frac{\pi f}{H}\left[P_m' - P_{e(\delta_{0.05}^{P})}\right]$$

$$= \frac{\pi \times 50}{5}\left[0.8 - 0.742 \sin 0.427\right]$$

$$= 15.479 \text{ r/sec}$$

Corrected Values (Average value of two derivatives)

$$\delta^c_{0.05} = \delta_0 + \frac{\Delta t}{2}\left[\frac{d\delta}{dt}\Big|_{\Delta\omega_0} + \frac{d\delta}{dt}\Big|_{\Delta\omega^p_{0.05}}\right]$$

$$= 0.427 + \frac{0.05}{2}[0 + 0.774] = \underline{0.446 \text{ rad}}$$

$$\Delta\omega^c_{0.05} = \Delta\omega_0 + \frac{\Delta t}{2}\left[\frac{d\Delta\omega}{dt}\Big|_{\delta_0} + \frac{d\Delta\omega}{dt}\Big|_{\delta^p_{0.05}}\right].$$

$$= 0 + \frac{0.05}{2}[15.479 + 15.479]$$

$$\boxed{\Delta\omega^c_{0.05} = 0.774 \text{ rad/sec}}$$

Iteration 2

Beginning of second step at $t = 0.05$.

$$\frac{d\delta}{dt}\Big|_{\Delta\omega^c_{0.05}} = \Delta\omega^c_{0.05} = 0.774.$$

$$\frac{d\Delta\omega}{dt}\Big|_{\delta^c_{0.05}} = \frac{\pi f}{H}[P_m' - P_e(\delta^c_{0.05})]$$

$$= \frac{\pi \times 50}{5}[0.8 - 0.742 \sin(0.446)]$$

$$= 15.077.$$

End of the second step @ $t = 0.1$

Predicted values are $\delta^p_{0.1} = \delta^c_{0.05} + \frac{d\delta}{dt}\Big|_{\Delta\omega^c_{0.05}} \times \Delta t$

$$= 0.446 + 0.774 \times 0.05 = 0.485 \text{ rad}.$$

$$\Delta\omega^p_{0.1} = \Delta\omega^c_{0.05} + \frac{d\Delta\omega}{dt}\Big|_{\delta^c_{0.05}} \times \Delta t.$$

$$= 0.774 + 15.077 \times 0.05 = 1.5278$$

Derivatives @ the end of $t = 0.1$

$$\frac{d\delta}{dt}\Big|_{\Delta\omega^p_{0.1}} = \Delta\omega^p_{0.1} = 1.5278 \text{ r/sec}$$

$$\left.\frac{d\Delta\omega}{dt}\right|_{\delta_{0.1}^{P}} = \frac{\pi f}{H}\left[P_m' - P_e(\delta_{0.1}^{P})\right]$$

$$= \frac{\pi \times 50}{5}\left[0.8 - 0.742\sin 0.485\right] = 14.265 \text{ r/sec}$$

Corrected values:

$$\delta_{0.1}^{c} = \delta_{0.05}^{c} + \frac{\Delta t}{2}\left[\left.\frac{d\delta}{dt}\right|_{\Delta\omega_{0.05}^{c}} + \left.\frac{d\delta}{dt}\right|_{\Delta\omega_{0.1}^{P}}\right]$$

$$= 0.446 + \frac{0.05}{2}\left[0.774 + 1.5278\right]$$

$$\boxed{\delta_{0.1}^{c} = 0.5035 \text{ rad}}$$

$$\Delta\omega_{0.1}^{c} = \Delta\omega_{0.05}^{c} + \frac{\Delta t}{2}\left[\left.\frac{d\Delta\omega}{dt}\right|_{\delta_{0.05}^{c}} + \left.\frac{d\Delta\omega}{dt}\right|_{\delta_{0.1}^{P}}\right]$$

$$= 0.774 + \frac{0.05}{2}\left[15.077 + 14.265\right]$$

$$\boxed{\Delta\omega_{0.1}^{c} = 1.5076 \text{ r/sec}}$$

Fault is cleared @ t = 0.1 sec and the post fault cond $P_e = 1.378 \sin\delta$.

Iteration 3

Beginning of third step @ t = 0.1 sec.

$$\left.\frac{d\delta}{dt}\right|_{\Delta\omega_{0.1}^{c}} = \Delta\omega_{0.1}^{c} = 1.5076 \text{ rad/sec}$$

$$\left.\frac{d\Delta\omega}{dt}\right|_{\delta_{0.1}} = \frac{\pi f}{H}\left[P_m' - P_e(\delta_{0.1}^{c})\right]$$

$$= \frac{\pi \times 50}{5}\left[0.8 - 1.378\sin 0.5035\right]$$

$$= 4.245$$

End of the third step @ t = 0.15, predicted values are

$$\delta_{0.15}^{P} = \delta_{0.1}^{c} + \left.\frac{d\delta}{dt}\right|_{\Delta\omega_{0.1}^{c}} \times \Delta t$$

$$= 0.5035 + 1.5076 \times 0.05$$

$$= 0.579 \text{ rad}$$

$$\Delta\omega^{P}_{0.15} = \Delta\omega^{c}_{0.1} + \frac{d\Delta\omega}{dt}\Big|_{\delta^{c}_{0.1}} \cdot \Delta t$$

$$= 1.5076 + 4.245 \times 0.05 = 1.7197 \text{ r/sec}$$

Derivatives @ the end of $t = 0.15$ sec.

$$\frac{d\delta}{dt}\Big|_{\Delta\omega^{P}_{0.15}} = \Delta\omega^{P}_{0.15} = 1.7197$$

$$\frac{d\Delta\omega}{dt}\Big|_{\delta^{P}_{0.15}} = \frac{\pi f}{H}\left[P_m' - P_e(\delta^{P}_{0.15})\right]$$

$$= \frac{\pi \times 50}{5}\left[0.8 - 1.378 \sin 0.579\right]$$

$$= 1.444 \text{ rad/sec}$$

Corrected Values:

$$\delta^{c}_{0.15} = \delta^{c}_{0.1} + \frac{\Delta t}{2}\left[\frac{d\delta}{dt}\Big|_{\Delta\omega^{c}_{0.1}} + \frac{d\delta}{dt}\Big|_{\Delta\omega^{P}_{0.15}}\right]$$

$$= 0.5035 + \frac{0.05}{2}\left[1.5076 + 1.7197\right]$$

$$\boxed{\delta^{c}_{0.15} = 0.584 \text{ rad.}}$$

$$\Delta\omega^{c}_{0.15} = \Delta\omega^{c}_{0.1} + \frac{\Delta t}{2}\left[\frac{d\Delta\omega}{dt}\Big|_{\delta^{c}_{0.1}} + \frac{d\Delta\omega}{dt}\Big|_{\delta^{P}_{0.15}}\right]$$

$$= 1.5076 + \frac{0.05}{2}\left[4.245 + 1.444\right]$$

$$\boxed{\Delta\omega^{c}_{0.15} = 1.6498 \text{ r/sec}}$$

(b) Runge Kutta Method:

$$\delta_0 = 0.427 \text{ rad} \qquad \Delta t = 0.05.$$

$$\Delta\omega_0 = 314.159 - 314.159 = 0$$

Iteration 1: @ $t = 0$

I Estimates: $k_1 = \Delta\omega_0 \times \Delta t = 0 \times 0.05 = 0$

$$l_1 = \frac{\pi f}{H}\left[P_m' - P_e(\delta_0)\right]\Delta t$$

$$= \frac{\pi \times 50}{5}\left[0.8 - 0.742 \sin 0.427\right] \times 0.05$$

$$= 0.774$$

$\underline{\text{II Estimates}}$: $K_2 = \left(\Delta\omega_0 + \frac{l_1}{2}\right)\Delta t$

$$= \left(0 + \frac{0.774}{2}\right) \times 0.05 = 0.0194.$$

$$l_2 = \frac{\pi f}{H}\left[P_m' - P_e(\delta_0 + k_{1/2})\right] \times \Delta t$$

$$= \frac{\pi \times 50}{5}\left[0.8 - 0.742 \sin\left(0.427 + \frac{0}{2}\right)\right] \times 0.05$$

$$= 0.774.$$

$\underline{\text{III Estimates}}$: $K_3 = \left(\Delta\omega_0 + \frac{l_2}{2}\right)\Delta t$

$$= \left(0 + \frac{0.774}{2}\right) \times 0.05 = 0.0194$$

$$l_3 = \frac{\pi f}{H}\left[P_m' - P_e(\delta_0 + \tfrac{k_2}{2})\right] \times \Delta t$$

$$= \frac{\pi \times 50}{5}\left[0.8 - 0.742 \sin\left(0.427 + \frac{0.0194}{2}\right)\right] \times 0.05$$

$$= 0.764$$

$\underline{\text{IV Estimates}}$: $K_4 = (\Delta\omega_0 + l_3) \times \Delta t$

$$= (0 + 0.764) \times 0.05 = 0.038$$

$$l_4 = \frac{\pi f}{H}\left[P_m' - P_e(\delta_0 + k_3)\right] \times \Delta t$$

$$= \frac{\pi \times 50}{5}\left[0.8 - 0.742 \sin(0.427 + 0.0194)\right] \times 0.05$$

$$= 0.7535$$

$\underline{\text{Final Estimates}}$

$$\delta_{0.05} = \delta_0 + \frac{1}{6}\left[K_1 + 2K_2 + 2K_3 + K_4\right]$$

$$= 0.427 + \frac{1}{6}\left[0 + 2(0.0194) + 2(0.0194) + 0.038\right]$$

$$\boxed{\delta_{0.05} = 0.46 \text{ rad}}$$

$$\Delta\omega_{0.05} = \Delta\omega_0 + \frac{1}{6}\left[l_1 + 2l_2 + 2l_3 + l_4\right]$$

$$= 0 + \frac{1}{6}\left[0.774 + 2(0.774) + 2(0.764) + 0.7535\right]$$

$$\boxed{\Delta\omega_{0.05} = 0.767 \text{ rad/sec}}$$

Iteration - 2

I Estimates: $K_1 = 0.0384$ & $l_1 = 0.754$

II Estimates: $K_2 = 0.0572$ & $l_2 = 0.734$

III Estimates: $K_3 = 0.0567$ & $l_3 = 0.724$

IV Estimates: $K_4 = 0.0746$ & $l_4 = 0.695$

Final Estimates

$$\delta_{0.1} = \delta_{0.05} + \frac{1}{6}\left[K_1 + 2K_2 + 2K_3 + K_4\right]$$

$$= 0.446 + \frac{1}{6}\left[0.0384 + 2(0.0572) + 2(0.0567) + 0.0746\right]$$

$$\boxed{\delta_{0.1} = 0.503 \text{ rad.}}$$

$$\Delta\omega_{0.1} = \Delta\omega_{0.05} + \frac{1}{6}\left[l_1 + 2l_2 + 2l_3 + l_4\right]$$

$$= 0.76 + \frac{1}{6}\left[0.754 + 2(0.734) + 2(0.724) + 0.695\right]$$

$$\boxed{\Delta\omega_{0.1} = 1.495 \text{ r/sec}}$$

Iteration - 3

I Estimates: $K_1 = 0.0748$ & $l_1 = 0.213$

II Estimates: $K_2 = 0.08$ & $l_2 = 0.413$

III Estimates: $K_3 = 0.0783$ & $l_3 = 0.1382$

IV Estimates: $K_4 = 0.0817$ & $l_4 = 0.068$.

Final Estimates

$$\delta_{0.15} = \delta_{0.1} + \frac{1}{6}\left[K_1 + 2K_2 + 2K_3 + K_4\right]$$

$$= 0.503 + \frac{1}{6}\left[0.0748 + 2(0.08) + 2(0.0783) + 0.0817\right]$$

$$\boxed{\delta_{0.15} = 0.582 \text{ rad}}$$

$$\Delta\omega_{0.15} = \Delta\omega_{0.1} + \frac{1}{6}\left[l_1 + 2l_2 + 2l_3 + l_4\right]$$

$$= 1.495 + \frac{1}{6}\left[0.213 + 2(0.413) + 2(0.1382) + 0.068\right]$$

$$\boxed{\Delta\omega_{0.15} = 1.636 \text{ rad/sec}}$$